Quantifying the limitations of far-field imaging by a left-handed planar lens

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Abstract

We demonstrate that the transition between the diffraction limited and subwavelength imaging regimes in a left-handed planar lens is governed by a universal parameter combining absorption, light wavelength, and lens thickness. We show that the limits of lossless media and zero wavelength do not commute; the former limit yields sub-diffraction imaging, while the latter leads to diffraction limited behavior, which typically dominates the far-field image of realistic planar lenses. Finally we discuss the relationship between the universal lens parameter and the resonant excitation of surface plasmonic waves.

Materials in which both the real parts of ϵ and μ are simultaneously negative enabled the first model of the left-handed medium (LHM) lens, which utilizes negative refraction at an interface between LHM and regular media.[1] Considerable interest in the planar (LHM) lens arose after it was suggested that this imaging system could in principle focus all fourier components of an object.[2] Physical motivation for the use of LHM systems includes the imaging of protein molecules using visible light, integrated photonic devices, and interesting cloaking applications.[3, 4, 5, 8] Many authors have rigorously shown the subwavelength imaging capabilities of the left-handed planar lens system in the near-field regime in the limit of small absorption[6], however recent work demonstrates that the planar left-handed lens does not exhibit superlensing in the far-field regime.[7] In this work we analyze the resolution capabilities of the left-handed planar lens – sometimes called the “superlens”– in the far-field regime, and resolve the controversy regarding it’s resolution by introducing a fundamental parameter that combines wavelength, lens thickness, and absorption.

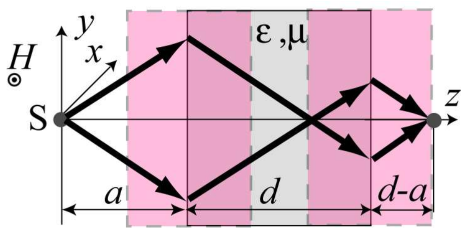

Figure 1: Planar LHM-based lens with two real foci outside and inside the slab. S is the source, a is the distance from the source to the slab, d is the width of the slab. Dark (Red) regions with dashed boundaries are the resonant regions. The fields diverge in these regions when the absorption tends to zero.

The elusiveness of LHM is due to no known naturally occurring isotropic materials having simultaneously negative ϵ and μ at the same frequency. However, negative permittivity and induced magnetic resonance have been used in arrays of metallic split rings to achieve negative refraction at 100THz.[8] There have been experimental observations of negative refraction at 10.5 GHz in metamaterials.[9] Structures containing metal-dielectric layers and split ring resonators have been engineered to achieve negative ϵ and μ at microwave, near-infrared, and visible frequencies.[10, 11] Since actual LHMs are typically metamaterials with periodic or quasi-periodic structure[12], their effective parameters ϵ and μ exhibit strong spatial dispersion $[\epsilon, \mu = \epsilon, \mu(\omega, \vec{k})]$ [7, 13, 14], which potentially suppresses the imaging performance of the planar lens.[7, 14] In the present work we neglect the nonlocalities in both dielectric permittivity ϵ and magnetic permeability μ and consider the basic model – a homogeneous and isotropic "hypothetical" slab with

$$\epsilon = \mu = -1 + i\delta, \tag{1}$$

where $\delta \ll 1$. Thus, our model represents the best case scenario for the superresolution.

To provide a general solution to the planar lens imaging system we use a Green's function method to describe the system response to a delta source with fixed point like shape. Instead of performing conventional resolution calculations using the uncertainty principle, we define a more quantitative parameter describing subwavelength imaging,

$$P = H^f_{ev}/H^f_p, \tag{2}$$

with H^f_{ev} and H^f_p being contributions of evanescent and propagating fields at the focal point of the system respectively (excitation with TM-polarized light is assumed). When $|P| \geq 1$ superresolution is possible while $|P| \ll 1$ corresponds to the diffraction regime. In the limit of small system loss the propagating spectrum has been shown to be completely restored in the vicinity of the focal point. [6, 15]

The magnitude of $k_0 d$ does not provide sufficient information for the applicability of diffraction theory in the far-field regime for the planar lens due to resonant interaction of surface plasmon waves.[16] As described in details in [17], the fundamental parameter S which is responsible for the applicability of diffraction theory can be defined as a function of the free space wavevector k_0, lens thickness d, and system absorption δ.

$$S = k_0 d\sqrt{\mathrm{Im}(\epsilon + \mu)/2} = k_0 d\sqrt{\delta} \tag{3}$$

At $S \gg 1$ the system is always described by diffraction theory,

$$P(S) = \frac{8}{i\pi k_0 d}\frac{(i\sin S - \cos S)\exp(-S)}{S(i-1)}, \tag{4}$$

and $|P(S)| \ll 1$.

When $S \ll 1$ the system exhibits superresolution with quasistatic-like behavior

$$P(S) \simeq \frac{2}{i\pi}\ln\left[-\frac{2\ln(\delta/2)}{k_0 d}\right] \propto \ln\ln(1/S), \tag{5}$$

$|P(S)| \to \infty$ as $S \to 0$. However, divergence of $|P(S)|$ is extremely slow, and in practice $|P(S)| \geq 1$ is unachievable in realistic far-field structures.

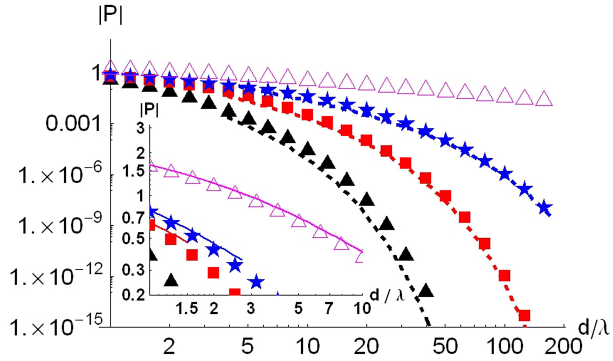

Figure 2: Dependence of the parameter $|P|$ on normalized lens thickness for $\delta = 10^{-2}$ (solid triangles), $\delta = 10^{-3}$ (boxes), $\delta = 10^{-4}$ (stars), and $\delta = 10^{-18}$ (empty triangles); symbols correspond to results of numerical integrations; dashed and solid lines correspond to approximate Eq.(4) and Eq.(5) respectively; note that quasistatic Eq.(5) adequately describes the far-field behavior of the system in the limit of vanishingly small absorption

The resonant interaction of surface plasmon waves is why the condition $k_0 d \gg 1$ is not sufficient for describing the diffraction limited regime. These surface plasmon waves exist when the sign of k_z/ϵ changes at the interface between vacuum and LHM.[18, 19] At $\delta \ll 1$ it reads

$$\begin{aligned} k_1 &= -\frac{k_2}{-1+i\delta} \\ \sqrt{k^2-k_0^2} &= \sqrt{k^2-k_0^2+k_0^2 2i\delta}. \end{aligned} \tag{6}$$

The mismatch responsible for the breakdown of resonant excitation of surface modes can be estimated as

$$D = \frac{\sqrt{k^2-k_0^2+k_0^2 2i\delta}-\sqrt{k^2-k_0^2}}{\sqrt{k^2-k_0^2}} \simeq \frac{i\delta k_0^2}{k^2-k_0^2}. \tag{7}$$

The condition $D \ll 1$ must hold for the resonance to exist. When $S \gg 1$ there are no traces of the resonance and regular diffraction theory is applicable. The violation due to the absorption is small at $S \ll 1$, and the main features of the resonance should be preserved given these conditions.

In conclusion our resolution performance analysis of the planar LHM lens in the far-field regime provides the physical criterion responsible for the imaging capabilities of this ideal system. Wavelength, lens thickness, and absorption are the parameters which define the imaging performance of the planar LHM lens, and must all be considered in future analysis and design of these photonic devices.

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