

# Broadband Negative-Refractive-Index Media: Analytical Modeling and Free Space Measurements

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Since the realization of negative-refractive-index (NRI) media, the narrow bandwidth of these materials has continually hindered their practical use. The bandwidth of traditional NRI media is limited by the narrowband response of the split-ring resonator (SRR) that gives rise to negative effective permeability. Typical SRRs exhibit negative permeability over a fractional bandwidth of approximately 10%. Increasing the bandwidth over which negative permeability can be achieved greatly enhances the utility of NRI media in real-world applications.

A volumetric NRI medium exhibiting a large backward-wave bandwidth was proposed in [1] and analyzed in [2]. The wide bandwidth of operation of the proposed structure can be attributed to its broadband negative permeability response, which results from contradirectional coupling between a forward free-space wave and a backward wave guided by a transmission line. The NRI structure in [2] exhibited a backward-wave bandwidth of 60%, and subsequent parametric analysis of a similar structure showed that even larger bandwidths of operation were possible [3]. The proposed broadband NRI medium was experimentally verified in [4], for frequencies between 1.73GHz and 2.85GHz. A four-cell slab of the NRI medium was tested in a parallel plate waveguide and used as a flat lens to image an electric line current. A super-resolved image of the source with resolution enhancement  $R_e=2$  was observed at 2.45 GHz in these experiments.

In this paper, we show how the dispersion characteristics of an infinite, broadband NRI medium can be derived analytically using multiconductor transmission line (MTL) theory. In addition, the scattering parameters of a finite-thickness slab of the medium are computed using the same technique. Two-port scattering parameters of the slab are generated from a four-port MTL model of the NRI medium by appropriately terminating two of its ports. From the two-port scattering parameters, the dispersion characteristics of the medium are extracted and compared with those of the infinite structure.

An experimental realization of a broadband NRI medium is also presented. A slab of a NRI medium is designed to be well-matched to free-space. The slab is constructed by stacking multiple layers of microwave substrates to form a lens that can focus microwaves in free space. Capacitively-loaded transmission-line grids are printed on either side of each microwave substrate and connected by inductive vias in order to form a cage-like structure that establishes broadband negative permeability. To achieve negative permittivity, wires are threaded through the center of each unit cell. Free-space measurements of the scattering parameters at normal incidence to the slab are performed using a four-lens system referred to as a Gaussian beam telescope. The measured data are then compared to that computed using full-wave and MTL analysis.

## References

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