

# ASSESSING THE PHYSICAL LIMITATIONS OF SYNTHESIZED METAMATERIALS THROUGH RIGOROUS NUMERICAL MODELING

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Metamaterials have attracted considerable recent attention because they promise to open up new vistas and realize material characteristics never envisioned only a few years ago. In this paper we begin by reviewing some of the interesting and novel applications of metamaterials that have been conjectured on the basis of their effective medium characteristics. Three of these, which have featured prominently in recent publications [1], are:

- (a) Lens effect produced by DNG slabs that are useful for enhancing the directivities of small antennas, e.g., dipoles and microstrip patches, by collimating the cylindrical waves emanating from these antennas and focusing them at infinity.
- (b) Improvement in the performance of a small monopole antenna, realized via the use of an ENG envelope that compensates for its high capacitive reactance.
- (c) Plasmonic effects that manifest into unusual transmission characteristics.

The use of effective medium type of description of artificially synthesized media, which include metamaterials, is widely used to research into the characteristics of such media. This is certainly understandable, since the effective medium description is very appealing from the point of view of interpreting the physical characteristics of such materials, by relating the  $\epsilon$  and  $\mu$  of these artificial materials to equivalent homogeneous media with the same permittivities and permeabilities. Once this has been done, it becomes relatively straightforward to analyze them by using conventional techniques and simulation tools. It is very common to use periodic inclusions, embedded in a background medium, to realize metamaterial characteristics, e.g., either negative  $\epsilon$ , negative  $\mu$ , or both. Also, it is not unusual to choose elements (inclusions) that have fine features to realize the desired metamaterial characteristics, and such a choice places a heavy burden on the CPU time and memory when one attempts to simulate an antenna/metamaterial composite with a view to predicting its performance. The same exercise is greatly simplified, however, and a physical interpretation of these characteristics becomes feasible, once an equivalent effective medium is found to replace the original configuration, essentially via the artifice of homogenization.

Let us now assume that we have carried out this important step of replacing the original metamaterial medium with its equivalent medium, and have gone on to the next step of predicting the performance of devices that integrate this medium to achieve certain performance characteristics. We then pose an all-important question, which may go like this: "Are the predictions that we have made realistic and believable, or do we need to critically examine the conclusions we have arrived at by revisiting the original physical structure instead, and simulating it directly?"

As an example, let us suppose, for instance, we have concluded that a small antenna whose dimensions are small fractions of a wavelength can provide us a very high directivity, once we have integrated it with a

metamaterial medium with the “right” type of effective parameters. A natural question to ask, then, is: “Are there any Physical Limitations that would inhibit us from realizing the performance as predicted via the use of effective medium approach?” Perhaps the next logical question would be: “How should we proceed to answer this question, in a manner such that we can be confident in the response we offer?” The principal objective of this paper is to provide a systematic, reliable, and rigorous approach to the latter question. Once we have established the roadmap we should follow to accomplish the above goal, the answer to the first question—which is the central theme of this presentation—will naturally follow, as a next step.

Let us now turn to the problem of extracting the effective medium parameters of a metamaterial slab. The usual approach to modeling metamaterials is to begin with a thin slab of the artificial dielectric, which typically has metallic or dielectric inclusions, embedded in a background dielectric medium whose material parameters are different from those of the inclusions. The characteristics of metamaterials differ significantly from the conventional media found in nature—hence its name. To characterize them, we first calculate (or measure) the scattering properties ( $S_{11}$ ,  $S_{21}$ ), of a slab of metamaterial, and then use the formulas available in the literature [2,3] to back out the effective parameters  $\epsilon$  and  $\mu$  of the metamaterials. As mentioned earlier, one of the primary motivations for going through this process is that replacing the metamaterials with their effective parameters enables us to simplify the analysis of the composite structures, comprising of antennas and metamaterials. Some examples of such structures—also alluded to above—include small antennas encased within a metamaterial shell; antennas placed above high impedance substrates, which behave like artificial magnetic ground plane (AMGs); and, small antennas covered by superstrates that enhance their directivities.

This paper begins by examining the techniques available in the literature for deriving the effective parameters of metamaterials. All simulations in this paper have been carried out by using GEMS [4], which is a 3-D parallel FDTD solver capable of handling a large number of DoFs (upward of  $10E+9$ ). We investigate single and multilayer slabs (see Figs. 2 (a)-(b)) and examine the consistency of the effective parameters we extract for them (see Fig. 3), as we vary the thickness of the slab and the angle of incidence of the interrogating plane wave. We also study the field behaviors inside the slab, both for the single and multilayer cases, to develop an understanding of the characteristics of these materials, and to see how these characteristics relate to those predicted on the basis of the effective  $\epsilon$  and  $\mu$  parameters.

Next, we investigate a number of physical metamaterial configurations (see Fig. 2(c)) with the objective of comparing the performances predicted by the effective medium approach with that actually realized on the basis of rigorous numerical simulations. The first of these is the case of a Gaussian Beam impinging upon a multilayer metamaterial slab, containing split-ring resonators and dipoles, over a frequency range within which the slab has DPS, SNG, DNG properties in different frequency bands. We track the propagation of the beam within the metamaterial slab as well as in the air region outside (see Fig. 4). We also compare the behavior of the beam propagation in a conventional dielectric slab, which serves as a reference structure (see Fig. 5).

Next, we turn to antennas covered by metamaterial substrates and superstrates [5,6], again from the point of view of examining the feasibility of employing the effective parameters to model the antenna/metamaterial composites (see Fig. 1).

Finally, we consider the behavior of metamaterial slabs when used as lenses, as proposed by Veselago. Our objective is to investigate the field behaviors in the slabs of this type as predicted by using rigorous simulations (see Fig. 6). All of these three studies compare the rigorous simulation results with the corresponding behaviors as predicted by using their effective media representations.

What we have learnt from the above comparisons has led us to conclude that the effective medium approach may not be reliable for assessing the physical limitations of a proposed device containing metamaterials, and only rigorous simulations can provide us the correct answers.

**References:**

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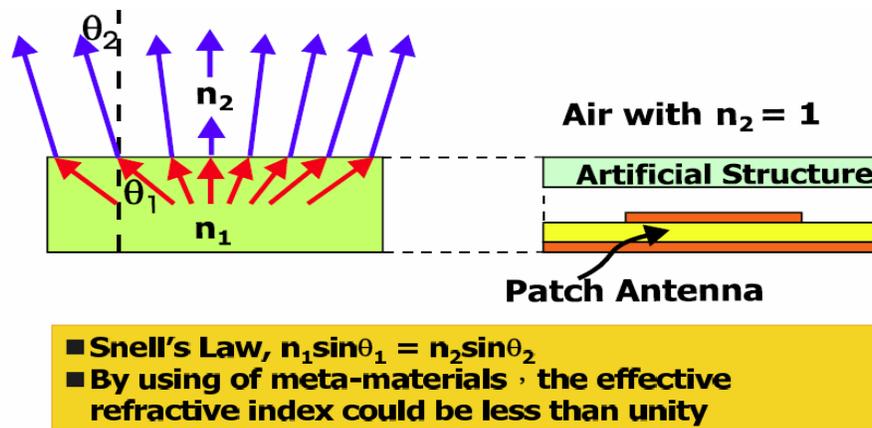


Figure 1. Concept of the high gain patch antenna achieved by the use of metamaterial superstrate.

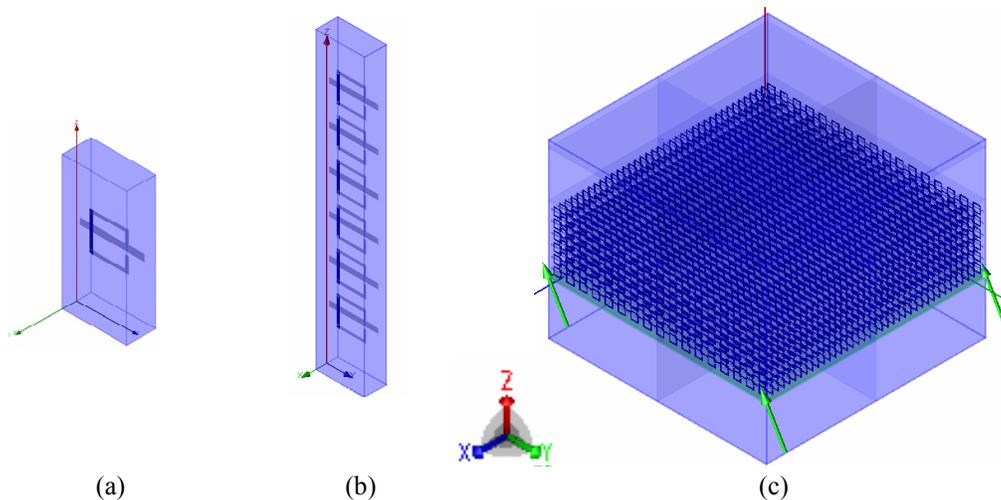


Figure 2. Geometries of unit cells for (a) single-layer and (b) six-layer slab, and a finite six-layer metamaterial slab.

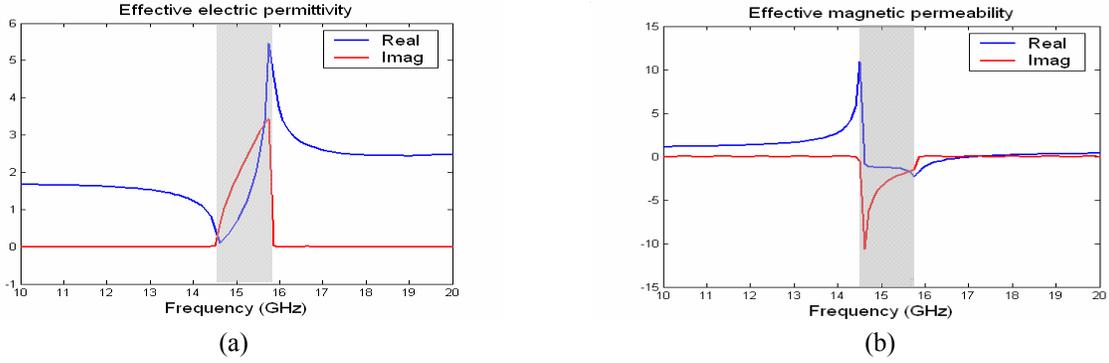


Figure 3. Effective material parameters of the 1-layer slab extracted by using the inversion approach: (a)  $\epsilon$ ; and (b)  $\mu$ .

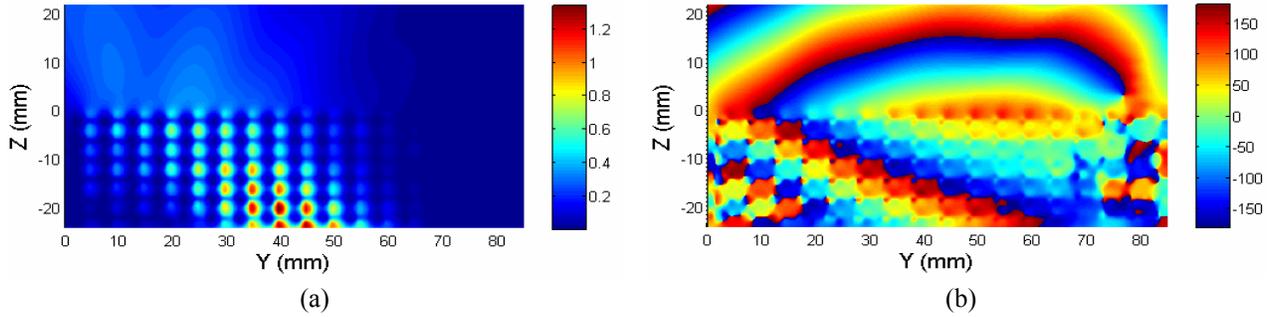


Figure 4. (a) Magnitude and (b) phase at 15.3 GHz on the YZ plane (plane of incidence) at oblique  $TM_z$  incidence ( $30^\circ$  off-normal). The metamaterial slab occupies the region between  $z = -23.75$  and  $0$  mm.

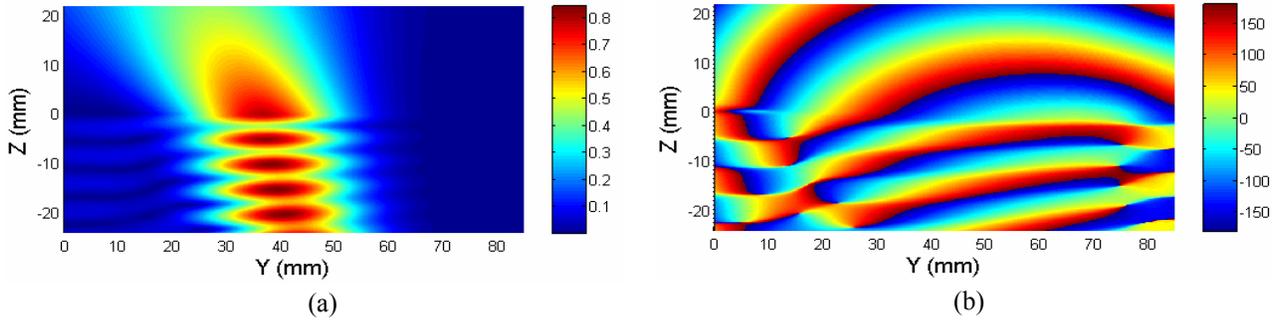


Figure 5. (a) Magnitude and (b) phase at 15.3 GHz on the YZ plane (plane of incidence) at oblique  $TM_z$  incidence ( $30^\circ$  off-normal). The metamaterial slab is replaced with a dielectric slab with  $\epsilon_r = 4$ .

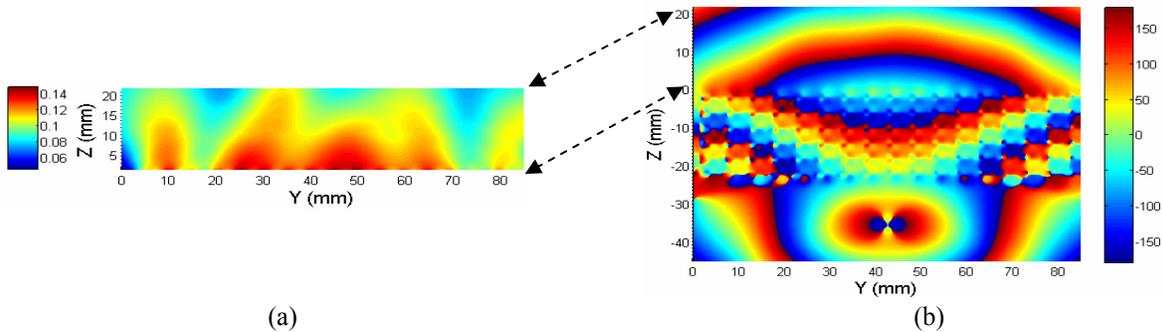


Figure 6. (a) Magnitude and (b) phase at 15.3 GHz on the YZ plane (E-plane) when a y-aligned dipole is placed at 8.5 mm away from the metamaterial slab.