Method of Moments simulation of infinitely periodic structures involving connected dielectric objects

X. Dardenne and C. Craeye
Université catholique de Louvain, Laboratoire TELE,
Place du Levant, 2, 1348, Louvain-la-Neuve, Belgium

Abstract

This paper describes a Method of Moments technique for the simulation of infinitely periodic structures involving dielectric objects whose unit cell can be connected or not. A numerical approach, based on the Method of Moments was developped, using the surface equivalence principle and the PMCHWT formulation of the integral equations. The use of periodic Green's functions for both inside and outside problems enables the analysis of structures where the dielectric volumes of successive cells are connected. A fast algorithm for the computation of the doubly periodic Green's function and its gradient is also implemented. The accuracy of the MoM approach is assessed in several cases.

1 Introduction

Periodic dielectric structures are now intensively considered for traditional applications, like phased arrays, as well as for novel structures, like metamaterials. For instance, a dielectric EBG superstrate made of stacked arrays of cylindrical rods was proposed in [1] to strongly increase the directivity of patch antennas. In a first instance, the design process can be dramatically accelerated by resorting to infinite-array simulations [2]. In order to simulate infinitely periodic dielectric structures, we developped an efficient approach based on the Method of Moments and the PMCHWT [3]-[4] formulation of the surface integral equations. The orginality of this approach mainly relies on the use of doubly periodic Green's function outside, as well as singly or doubly periodic Green's function inside the dielectric medium. Overlapping basis functions allow equivalent currents to flow between successive unit cells. For reasons of computational efficiency, the Green's functions as well as their gradients are tabulated before filling the MoM impedance matrix and this tabulation relies on an efficient algorithm [5], based on series accelerators to further reduce the computation time. This paper is organized as follows: Section 2 recalls the basics of the PMCHWT formulation and describes how it can be used in conjunction with periodic Green's functions to analyze periodic dielectric objects. In Section 3, this method is applied to compute the transmission properties of an EBG structure made of infinite dielectric rods (i.e. the unit cell is connected along only one direction). Section 4 demonstrates the accuracy of this method in the case where the unit cell of the structure is connected along two directions. Finally, conclusions are drawn in Section 5.

2 Integral equation formulation

When analyzing the scattering by dielectric bodies, these bodies are removed and replaced by fictitious equivalent currents on the interface [6]. These equivalent currents are expressed respectively as $\vec{J} = \vec{n} \times \vec{H}$ and $\vec{M} = -\vec{n} \times \vec{E}$, where $\vec{n}$ is the unit normal pointing outward the object and $\vec{E}$ and $\vec{H}$ are the electric and magnetic fields. The knowledge of

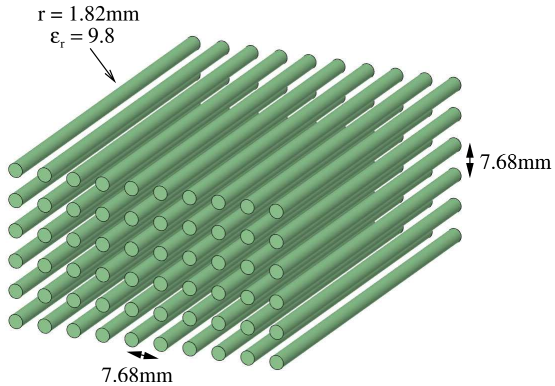

Figure 1: Dielectric EBG structure

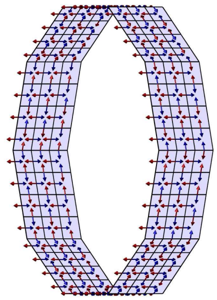

Figure 2: Unit cell for the single layer EBG. Small arrows represent the basis functions.

these currents is sufficient to compute the fields scattered by the object. They can be determined by ensuring the continuity of tangential electromagnetic fields along the surface of the dielectric object (PMCHWT approach).
Assuming incident fields $\vec{E}_{inc}$ and $\vec{H}_{inc}$, the integral equations for the electric and magnetic fields read:

$$\vec{n} \times \left(\vec{E}_{inc} + \frac{k_{in}^2 + \nabla\nabla\cdot}{j\omega\epsilon_{in}} \int_S G_{in}\vec{J}dS + \frac{k_{out}^2 + \nabla\nabla\cdot}{j\omega\epsilon_{out}} \int_S G_{out}\vec{J}dS - \fint_S \vec{M} \times \nabla'(G_{in} + G_{out})dS \right) = 0 \tag{1}$$

$$\vec{n} \times \left(\vec{H}_{inc} + \frac{k_{in}^2 + \nabla\nabla\cdot}{j\omega\mu_{in}} \int_S G_{in}\vec{M}dS + \frac{k_{out}^2 + \nabla\nabla\cdot}{j\omega\mu_{out}} \int_S G_{out}\vec{M}dS + \fint_S \vec{J} \times \nabla'(G_{in} + G_{out})dS \right) = 0 \tag{2}$$

Where $\fint$ stands for principal value integration, $\vec{J}$ and $\vec{M}$ are the unknown equivalent electric and magnetic currents, S is the interface between the dielectric body and the outside medium, G_{out} is the free-space Green's function for the outside medium, and G_{in} for the inside medium.
These equations are then solved by the Method of Moments, with RWG [7] and Roof-Top basis functions and Galerkin testing. More details about how these equations are obtained can be found in [5] and [8].
In order to reduce the computational cost, the Green's functions and their gradients are tabulated before filling the impedance matrix, and this tabulation relies on efficient algorithms [5] and on the use of series convergence accelerators.

3 Periodic dielectric structures connected in one direction

In order to simulate an infinitely periodic dielectric structure whose unit cell is connected in only one direction, such as the EBG structure presented in Fig. 1, the unit cell is made of thin slices of dielectric rods (Fig. 2). As seen from the exterior medium, these slices are periodized in both directions and the doubly periodic Green's function is used. However, as seen from inside the rods, the unit cell must be periodized in only one direction, along the rods. Hence, for interior integrals, the singly periodic Green's function is used. The rods are discretized using roof-top basis functions defined on rectangular domains, as shown in Fig. 2 and overlapping basis functions are added along one edge of the unit cell to allow the current to flow from one cell to the next. The transmission and reflection coefficients of a plane wave through 6 layers of rods are presented in Fig. 3 where the

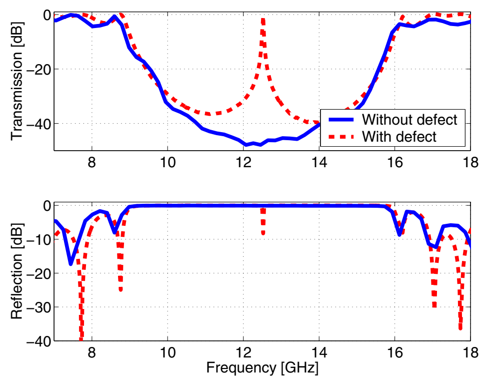

Figure 3: Transmission and reflection coefficients of a plane wave through the 6-layers EBG with and without a defect.

typical band-gap exhibited by these structures is shown. This figure also compares those coefficients for a 6 layers EBG with and without a defect. The defect introduced consists of an increased distance between the 3rd and 4th layers of rods. When this distance is increased to 26.1mm, a transmission peak is observed in the middle of the band-gap. These results are in excellent agreement with those presented in [1], which were obtained with the help of the FDTD method.

4 Periodic dielectric structures connected in two directions

In order to simulate an infinitely periodic dielectric structure whose unit cell is connected in two directions, the doubly periodic Green's function is used for both inside and outside problems. This approach is validated in the trivial case of an infinite dielectric slab. The simulated transmission and reflection coefficients at normal incidence of a plane wave through a slab of thickness 0.33 cm and $\epsilon_r = 8$ is compared in Fig. 5 with analytical results. An excellent agreement is obtained. Fig. 5 also shows the simulated coefficients for a periodicaly perforated slab, whose unit cell is depicted in Fig. 4, for $\epsilon_r = 8$ and $\epsilon_r = 1$. It is worth noting that, as expected, the magnitude of the reflection coefficient obtained for $\epsilon_r = 1$ is constant at 0 dB over the whole frequency range. In both cases, energy conservation, although not explicitely imposed by the Method of Moments, is fufilled with an excellent accuracy: the error does not exceed 0.05 dB and is even smaller than 0.01 dB most of the time.

5 Conclusion

A Method of Moments approach has been proposed for the simulation of infinitely periodic structures made of dielectric objects whose unit cells can be connected. This approach is based on the surface equivalence principle and the PMCHWT formulation of the integral equations. These equations are discretized and solved with the Method of Moments, using the Galerkin testing procedure. To this end, a fast computation method for the singly and doubly periodic Green's functions and for their gradient is used. Unit cells connected with each other through the dielectric material could be treated through the use of periodic Green's functions for both regions inside and outside the dielectric material;

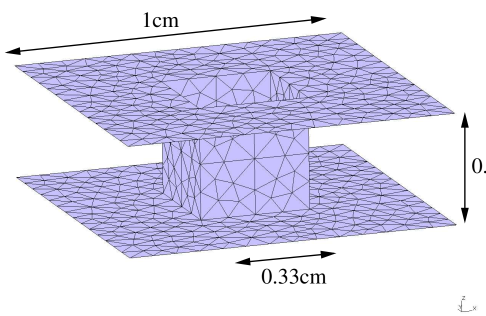

Figure 4: Unit cell of a perforated dielectric slab.

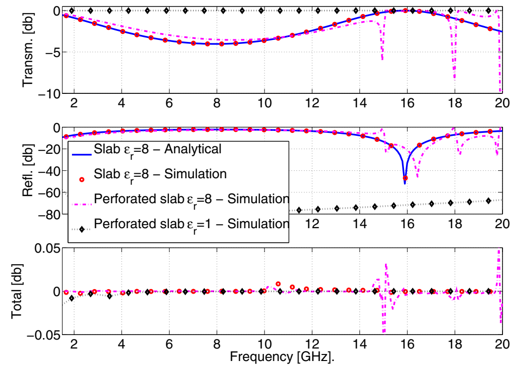

Figure 5: Transmission coefficient through dielectric slabs and perforated slabs (unit cell in Fig. 4) for normal plane-wave incidence. Slab height : d=0.33cm.

singly periodic functions for connection in one direction, doubly periodic functions for connection in two directions. Basis functions overlapping consecutive cells, were used to properly implement the method for connected structures. This connection method was demonstrated on several examples and the accuracy of this approach was assessed through energy conservation checks.

References

[1] Y. Lee, J. Yeo, R. Mittra, and W. Park, "Application of Electromagnetic Bandgap (EBG) Superstrates With Controllable Defects for a Class of Patch Antennas as Spatial Angular Filters," *IEEE Trans. Antennas Propagation*, vol.53, no.1, pp.224–235, 2005.

[2] C. Craeye and X. Dardenne, "Element pattern analysis of wide-band arrays with the help of a finite-by-infinite array approach,"*IEEE Trans. Antennas Propagation*, vol. 54, no. 2, pp. 519-526, Feb. 2006.

[3] A.J. Poggio and E.K. Miller, *Integral equation solutions of three-dimensional scattering problems*, Chap. 4, U. K. permagon, Oxford, 1973.

[4] T.K. Wu and L. L. Tsai, "Scattering from arbitrarily-shaped lossy dielectric bodies of revolution," *Radio Sci.*, vol. 12, no. 5, pp.709-718, 1977.

[5] X. Dardenne, N. Guérin, and C. Craeye, "Efficient MoM analysis of metamaterials involving dielectric structures," *Proc. of the ACES 2007 Conference,* Verona, Italy, 2007.

[6] R. F. Harrington, *Time-Harmonic Electromagnetic Fields,* McGraw-Hill Book Company, 1961.

[7] S.M.Rao, D.R.Wilton and A.W.Glisson, "Electromagnetic scattering by surfaces of arbitrary shape", *IEEE Trans. Antennas Propagation*, Vol.AP-30, pp. 409–418, May 1982.

[8] P. Yla-Oijala, M. Taskinen, and J. Sarvas, "Surface integral equation method for general composite metallic and dielectric structures with junctions," *Progress Electromagn. Research*, vol. 52, pp. 81-108, 2005.