Spatial Resolution in Tomographic Imaging with Diffracted Fields

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Abstract

The spatial resolution that is achievable using non-uniformly sampled data in the Fourier domain is examined. In particular, we focus on the typical non-uniform sampling that arises when diffraction tomography is used. We firstly introduce an effective conversion from non-uniform to uniform sampling grids using a transformation that minimizes an error measure in the frequency domain and that is particularly suited for the sampling location patterns encountered in diffraction tomography. Furthermore, for this scenario, we investigate the impact of sampling patterns on spatial resolution changes and how the sampling pattern can be varied to improve the spatial resolution. Finally, simulations are performed to show how spatial resolution varies when the number of transmitters or receivers changes.

1. Diffraction Tomography

Diffraction Tomography is an extension of the classical tomography technique that emerged over the past twenty-five years as a linear approach to the problem of quantitatively determining the structure of an unknown object from measurements of the waves diffracted by the object. It differs from classical tomography because a full wave propagation (albeit scalar) analysis is considered, instead of attenuation coefficients or time delays [1-3]. The structure to be reconstructed by diffraction tomography is usually the spatial distribution of the complex-valued refraction index inside the object. Therefore, diffraction tomography can be considered a problem of inverse scattering and it is applicable to many different scientific disciplines, such as crystal structure determination [4], medical ultrasound tomography [5], optical and coherent X-Ray microscopy [6], elastic wave inverse scattering [7], acoustic [12,13] and electromagnetic underground surveying [8-11,14-18].

2. Limits for Diffraction Tomography

The success of linearized diffraction tomography relies on three assumptions:

- Linearity: linearized diffraction tomography is based on linear approximation of the scattered field (such as Born or Rytov). In many cases, the linearity assumption fails; however, for geophysical applications and other cases, the target can be considered a weak scatterer and therefore the linear assumption holds.
- Available samples: in general, different constraints (economic, safety, operating, geometric or physical) limit the number of receiving and/or transmitting units, leading to insufficient or incomplete data sampling which reduces the resolution and/or fidelity of the reconstructed image.
- Frequency: although frequency is not an intrinsic problem of diffraction tomography, it drastically limits the resolution attainable in the reconstruction process. Two main reasons are discussed as follows. On one hand, Born and Rytov approximations are valid only if the inhomogeneities in the medium are relatively small compared with the wavelength of the incident field. On the other hand, attenuation in a lossy medium is heavily dependent on frequency: higher frequencies yield higher losses. In some applications, such as geophysical and biomedical imaging, the choice of the frequency is essentially based on the highest possible value such that scattered fields can still be detected above a certain SNR.

From a resolution point of view, the best way to implement diffraction tomography is by using the highest possible number of receiver/transmitter units, together with the highest number of frequencies available. Practical engineering design, however, imposes limitations, and designers need to perform a trade-off between benefits acquired by using more transceivers or frequencies and how it relates to the resolution of the reconstrued images. However, to the best knowledge of the authors, no explicit work aimed at quantifying the expected resolution of a diffraction tomography system has yet been published. Our work is essentially aimed to provide the designer with a quantitative analysis of the expected resolution on the reconstructed image when frequencies and number of transmiter/receiver are known a priori. This relation can also be inverted: if the designer requires a certain resolution, our analysis provides a way to determine the number of receivers or transmitters needed and frequencies to be used.

3. Image Reconstruction Approach

Our approach for the image reconstruction is essentially based on the Fourier generalized diffraction theorem: the object function (i.e. the variation of the permittivity) is non-uniformly sampled in a 2D Fourier space. A rectangular lattice is henceforth synthesized by appropriately weighting the available samples. After performing an inverse Fourier transformation we derive a simpler expression in the spatial domain to be minimized. Least Mean Squares (LMS) or Projection on Convex Set (POCS) algorithms can be effectively used to solve this minimization problem and hence obtain a reconstruced image. We perform several simulations aimed at showing how spatial resolution changes when the number of Tx/Rx/Tnes changes.

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5. References

1. E.Wolf , "Three Dimensional Structure Determination of Semitransparent Objects from Holographic Data,", *Optical Communications.* 1969, Vol. 1, pp. 135-156.
2. A.J. Devaney, "A Filtered Backpropagation Algorithm for Diffraction Tomography," *Ultrasonic Imaging,* 1982, Vol. 4, pp. 336-350.
3. G.A. Tsihrintzis and A.J. Devaney, "High Order (Nonlinear) Diffraction Tomography : Inversion of the Rytov Series," *IEEE Transactions on Information Theory.* Aug. 2000, Vol. 46, 5, pp. 1748-1761.
4. H. Lipson and W. Cochran, *The Determination of Crystal Structures.* Ithaca : Cornell University Press, 1966.
5. J.F. Greenleaf, "Computerized Tomography with Ultrasound," *Proceedings IEEE.* 1983, Vol. 71, p. 330.
6. M.H. Maleki, A.J. Devaney and A. Schatzberg, "Tomographic Reconstruction from Optical Scattered Intensities," *Journal of Optical Society of America.* 1992, Vol. 9, pp. 1356-1363.
7. S.K. Datta, J.D. Achenbach and Y.S. Rajapakse , *Elastic Wave Inverse Scattering.* New York : Elsevier, 1990.
8. A.J. Devaney, "Geophysical Diffraction Tomography," *IEEE Transactions on Geological Science, Special Issue on Remote Sensing.* 1984, Vol. 22, pp. 3-13.
9. J.E. Molyneux and A.J. Witten, "Diffraction Tomographic Imaging in a Monostatic Measurement Geometry," *IEEE Transactions on Geoscience and Remote Sensing.* 1993, Vol. 31, pp. 507-511.
10. A. Witten and E. Long, "Shallow Applications of Geophysical Diffraction Tomography," *IEEE Transactions on Geoscience and Remote Sensing.* 1986, Vol. 24, p. 654.
11. A. Witten, J. Tuggle and R.C. Waag, "A Practical Approach to Ultrasonic Imaging Using Diffraction Tomography," *Journal of Acoustic Society of America.* 1988, Vol. 83, p. 1645.
12. N. Sponheim et al. , "Quantitative Results in Ultrasonic Tomography of Large Objects Using Line Sources and Curved Detector Arrays," *IEEE Transactions on Ultrasonics, Ferroelectrics, Frequency Control.* 1991, Vol. 38, p. 370.

13. G.A. Tsihrintzis and A.J. Devaney, "Application of a Maximum Likelihood Estimator in an Experimental Study of Ultrasonic Diffraction Tomography," *IEEE Transactions on Medical Imaging.* 1993, Vol. 12, pp. 545-554.
14. R.B. Pratt and M.H. Worthington, "The Application of Diffraction Tomography to Cross-Hole Seismic Data," *Geophysics.* 1988, Vol. 53, p. 1284.
15. A. Witten and W.C. King, "Acoustical Imaging of Subsurface Features," *Journal of Environmental Engineering.* 1990, Vol. 116, p. 166.
16. A.J. Witten, J.E. Molyneux and J.E. Nyquist, "Ground Penetrating Radar Tomography: Algorithms and Case Studies," *IEEE Transactions on Geoscience and Remote Sensing.* 1994, Vol. 32, pp. 461-467.
17. E.J. Witterholt, J.L. Kretzeshmar and K.L. Jay, "The Application of Crosshole Electromagnetic Wave Measurements to Mapping of a Steam Flood," *Proceedings Petroleum Society CIM.* 1982.
18. A. Witten et al., "Gephysical Diffraction Tomography at a Dinosaur Site," *Geophysics.* 1992, Vol. 57, pp. 187-195.