Time-domain Beam Shaping of Pulsed Arrays

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Abstract

Pulsed arrays are becoming popular in new Ultra-wideband applications to enhance the robustness of transmitted and received signals in complex environments and to identify the angle of arrival of multiple echoes. A global synthesis technique is here proposed to shape the array field in accordance to a given angle-time mask. The synthesis problem is cast as the inverse Radon transform of a desired array mask, applying the Alternate Projections method to include constraints on the input signal's waveforms and to improve the synthesis resolution.

1. Introduction

Ultra-Wideband (UWB) pulsed arrays [1] consist of an arrangement of antennas having more than 25% bandwidth, which are sourced by base-band carrier-free input signals. Several applications of UWB arrays have been recently proposed for communications, radar, precise indoor positioning and tracking [2].

As discussed in recent theoretical investigations, the interest in the UWB arrays relies on their unique features of grating lobe cancellation [3], sparse array design [4] and mutual coupling reduction [5].

Actual applications of pulsed arrays mainly adopt excitation scheme borrowed from narrow-band arrays: Gaussian, modulated Gaussian or Hermite-Rodriguez pulses are weighted and delayed [6] with the main objective to produce a frequency-independent beam steering. Nevertheless unconventional performance could be pursued by actual and future UWB arrays whether both the angular and the temporal dimensions were fully exploited to shape, discriminate or multiply the transmitted or radiated pulses. Pulsed radiation phenomenology differs from the conventional narrow-band one since the temporal dimension, replacing the frequency domain, adds a new degree of freedom to array excitation design: the input signals' waveform.

In this paper, a more general time-angle synthesis technique is introduced for a full shaping of the radiated field which is able to generate the array input signal waveforms together with the beamforming network's topology. The synthesis is formulated as a time-angle constrained inversion problem based on the fact that the time-domain antenna far field can be related to the Radon transform of the space-time-varying currents [7]. Thus, given a desired far field, for instance specified through a mask, a discrete Radon transform [8], relating the array port currents to far field pattern, needs to be formally inverted to find the optimum time-domain currents waveforms. The beamforming network constraints are included by mapping the unknown currents onto a proper set of time-dependent basis functions.

Finally, to improve the robustness and the efficiency of this idea, the constrained Radon inversion is hence performed by successive iterations within the framework of the Alternating Projections method, proposed in [9] for monochromatic arrays, and here extended to the time domain.

2. The time-domain synthesis problem

Given a linear array of N elements at positions $x_n = nd$, where d is the inter-element distance, and sourced by a set of currents, hereafter indicated in a compact notation $i^+(x_n, \tau)$, $n = 1..N$, the time-domain array factor is expressed as [3]

$$AF(\theta,\tau) = \sum_{n=1}^{N} i^+(x_n, \tau + t_n(\theta)) \tag{1}$$

where $t_n(\theta) = nd/c\cos\theta$ and c is the speed of light. This expression can be interpreted as the Discrete Radon Transform of the current distribution [8], [10] The mutual coupling among radiating elements in the array is here neglected since it plays a secondary role in the case of pulsed arrays as demonstrated in [5]. It is also supposed that all the radiating elements have the same radiation pattern and the total radiated field of the array is therefore the convolution between the array factor and the element effective height $\underline{h}^t$:

$$\underline{E}(r,\theta,\phi,t) = -\tfrac{\eta_0}{4\pi rc}[\underline{h}^t(\theta,\phi,\bullet) * AF(\theta,\bullet)](t-\tfrac{r}{c}) \qquad (2)$$

The synthesis problem is defined as the retrieval of the excitation currents $i^+(x_n,\tau)$ once the array elements, the array geometry, the desired angle-time radiation pattern, and the beamforming network topology have been specified. The array factor and the radiated pattern corresponding to the retrieved currents are hereafter denoted to as *realized array factor* and *realized pattern*, respectively.

3. Constraints over field pattern

In the following, it is assumed that the array radiation requirements are specified over the only co-polar component of the total radiated field, (say E_θ), which has to be bounded between upper and lower angle-time masks $M_U^E(\theta,\tau)$ and $M_L^E(\theta,\tau)$ so that

$$M_L^E(\theta,\tau) \le E_\theta(\theta,\phi,\tau) \le M_U^E(\theta,\tau) \qquad (3)$$

The synthesis method will be instead referred to the array patterns masks $M_U^{AF}(\theta,\tau)$ and $M_L^{AF}(\theta,\tau)$ which, according to (2), are obtained from $M_{U,L}^E$ by means of deconvolution of the element effective height:

$$M_{U,L}^{AF}(\theta,\tau) = h_\theta^T(\theta,\cdot) \otimes M_{U,L}^E(\theta,\cdot)(\tau) \qquad (4)$$

where $a = b \otimes c$ means the deconvolution of the function b out of the function c, e.g. such that $a * c = b$.
Since the cross-polar component is neglected in the synthesis process (as in the monochromatic case), the term radiated field will refer, in the rest of the paper, to the only co-polar component.
We introduce the set, $\mathcal{M}$, of all the time-angular functions $g(\theta,\tau)$ which fulfill the mask constraints, e.g. such that:

$$\mathcal{M} = \{g(\theta,\tau): \ M_L^{AF}(\theta,\tau) \le g(\theta,\tau) \le M_U^{AF}(\theta,\tau)\} \qquad (5)$$

4. Constraints over the beamforming

Let $\mathcal{B}$ denote the set of all the array factors which can be radiated by the considered array arrangement for any choice of the input current set. Constraints over the beamforming network are introduced by enforcing the array current waveforms to be linear combination of a prescribed set of basis functions which are simply generated. In particular the Hermite-Rodriguez [11] orthogonal basis functions $\{w_{\lambda,k}(t)\}_k$ are considered. These are defined as

$$w_{\lambda,k}(t) = \frac{1}{\sqrt{2^k k!}} H_k(t/\lambda) \frac{1}{\sqrt{\pi}\lambda} e^{-t^2/\lambda^2} \qquad (6)$$

where $\{\alpha_{n,\lambda,k}\}$ is the set of mapping coefficients, λ is a scale parameter and $H_k(\tau)$ is the Hermite polynomial of order k. These functions are typically used in UWB applications since they can be produced by a single gaussian pulse generator plus modules performing derivatives, attenuation and delay. The complexity of the network, e.g. the number of processing modules, can be a-priori defined by choosing a subset of K Hermite-Rodriguez functions, which corresponds to enforce a topology constraint onto the beamforming scheme. Such a constraint will produce a subset of array factors $\mathcal{W} \subset \mathcal{B}$ generated by those antenna currents which may be expressed onto the $\{w_{\lambda,k}(t)\}_k$ base.

5. Intersection-finding problem

The solution of the synthesis problem is an array factor which belongs to the intersection $\mathcal{W} \bigcap \mathcal{M}$. One of the most effective methods for the solution of the intersection problem is based on the Alternate Projection, as in [9], which is here properly extended to time-domain arrays. A projector P_A over the closed subset $\mathcal{A}$ of a normed space $\mathcal{H}$ is the operator defined by

$$P_A : x \in \mathcal{H} \to y_0 \in \mathcal{A} : \|x - y_0\| \leq \|x - y\|, \forall y \in \mathcal{A} \tag{7}$$

The point y_0, which is the point of $\mathcal{A}$ nearest to x, is called the projection of x over $\mathcal{A}$. It is hence required to define two projection operators. The first one, denoted as $P_{\mathcal{M}}$, projects waveform-constrained array factors to the set $\mathcal{M}$ of the pattern-mask constrained functions. As in the frequency domain formulation [9], $P_{\mathcal{M}}$ crops the realized array factor outside the mask. The second projector, $P_{\mathcal{W}}$, projects mask-constrained functions to the set $\mathcal{M}$ of the waveform-constrained array factors and is specifically introduced for the time-domain synthesis as the cascade of three operators

$$P_{\mathcal{W}} = \mathcal{R}_n \circ C_w \circ \mathcal{R}^{-1} \tag{8}$$

$\mathcal{R}^{-1}$ is the inverse Radon transform which can be computed by means of a sequence of direct and inverse Fourier transforms via the Projection Slice Theorem [12] and produces a continuous current-line source $i^+(x,t)$. $C_{\mathcal{W}}$ applies to $i^+(x,t)$, samples such a current at the array ports ($x = nd$) and produces the approximation (projection) of so obtained input currents onto the Hermite Rodriguez base:

$$C_w \circ i^+(x,t) = \{i_n^+(t)\}_n = \left\{ \sum_{k=0}^{K} \alpha_{n,k} w_{\lambda,k}(t - \tau_n \right\}_{n=1..N} \tag{9}$$

where τ_n is a time shift of the n th current and the $\alpha_{n,k}$ coefficients are calculated as:

$$\alpha_{n,k} =< i^+(x_n,t), w_{\lambda,k}(t) >= \frac{1}{\sqrt{2^k k!}} \int_{-\infty}^{+\infty} i^+(x_n,t) H_k(t/\lambda) dt \tag{10}$$

Finally, $\mathcal{R}_n$ is the discretized Radon transform which applies to a set of current waveforms and produces an array-factor like function.

Starting from an initial guess for the array factor, the iterations of the two projection operations permits to iteratively approach the optimum excitation solution. The mth estimation of the source is

$$\begin{gathered} \{i^{+,(m)}(x_n,t)\}_{n=1..N} = C_w \circ \mathcal{R}^{-1} \circ [P_{\mathcal{M}} \circ AF^{(m)}(\theta,t)], \\ AF^{(m)}(\theta,t) = \mathcal{R}_n \circ \{i^{+,(m-1)}(x_n,t)\}_{n=1..N} \end{gathered} \tag{11}$$

and the iterations stop after a prescribed number of runs or when the convergence has been achieved ..The first iteration involves the user-defined upper mask $M_U^{AF}(\theta,\tau)$ as initial guess for $AF(\theta,\tau)$ and therefore the first step of the synthesis algorithm performs a simple Radon inversion of the desired mask with a proper mapping of the excitations.

6. A numerical Example

The performances of the proposed time-domain synthesis are here discussed by means of an example involving multiple-monocycle pulses. The synthesis is applied to a linear array of $N = 20$ equi-spaced radiating elements having inter-element distance $d = 5cm$. The co-polar component of the element's effective height was supposed to be an isotropic differentiator $h_\theta^t = \delta'(\tau)$.

The radiated field to achieve includes two independent beams, and different waveforms are required to be radiated along the two angular directions: three oscillations in the first beam and two oscillations in the other (Fig.1).

After 20 iterations the realized field pattern exhibits a good match to the mask despite of the presence of some extra oscillations.

The optimum Hermite-Rodriguez coefficients and the related array currents are shown in Fig.2. Because of the asymmetry of the desired pattern, the currents have different waveforms and quite different coefficient sets. In particular, the array seems divided into two smaller arrays, with the stronger excitations on the edge elements.

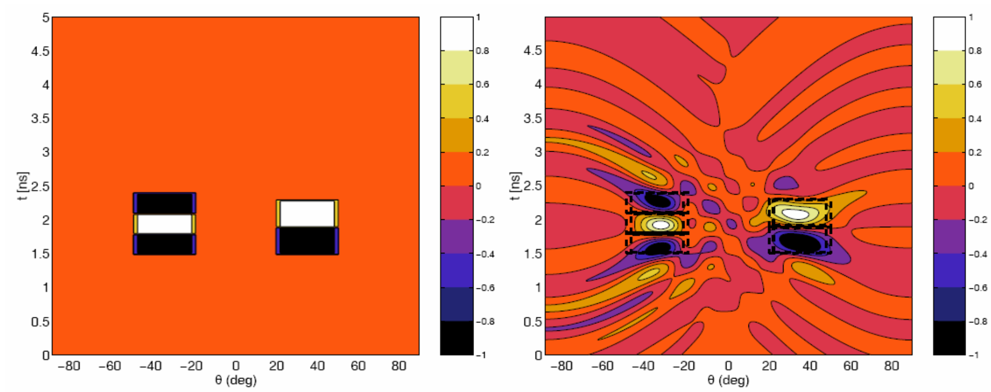

Fig.1: Time-domain desired radiated field mask and realized pattern after 20 iterations.

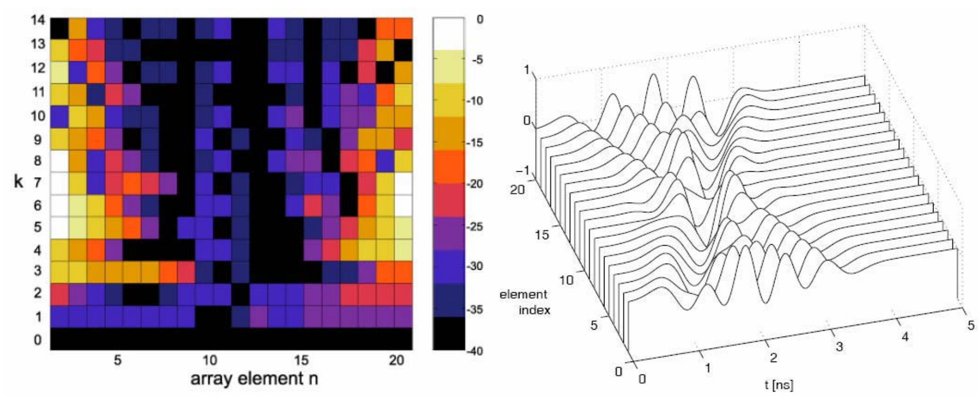

Fig.2: Optimized Hermite-Rodriguez coefficients and realized TD input currents after 20 iterations

7. References

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