Filling Time Reduction in RLSA MoM Analysis by using Asymptotics

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Abstract

The accurate design and further optimization of a high-gain RLSA requires a full wave analysis of the entire structure, that results electrically large. In this paper an approximate asymptotic formula for the slot mutual admittance evaluation is presented and applied to a MoM algorithm. The formula accuracy to the asymptotic order $O(r^{-3/2})$ permits, in a typical RLSA analysis, avoiding the numerical integration in more than 90% of the slot pairs, with a save of CPU time of the same order.

1. Introduction

Radial Line Slot Array (RLSA) antennas are good candidates for planar high gain and low cost solution at millimeter waves [1-5]. Most of the proposed designs are based on a simplified model, however, a full wave analysis is desirable, as proposed in [6]. There, a Method of Moments (MoM) analysis is developed with special care to an efficient evaluation of the Green’s function. High gain RLSAs present many hundreds of slots and on each slot the unknown magnetic current is expanded in terms of MoM basis functions, thus resulting in a medium size MoM matrix dimension (some thousands). The use of entire domain basis functions on the slots (rectangular waveguide modal current distributions) allows an accurate description of the current with a few unknowns per slot, thus minimizing MoM matrix dimension. However, the filling time of such a matrix remains the most time consuming operation in the algorithm. As a matter of facts, the evaluation of the mutual admittance between surface magnetic current unknowns requires the numerical calculation of a four-fold integral extended on the two slots. An asymptotic approximation of the Green’s function has been introduced in [7] and [8] for reducing this four-fold integral to a surface integral and, consequently, the MoM matrix filling time. In this paper, instead, we present an approximate asymptotic expression that allows us to evaluate directly the slot mutual admittance. Since the leading and the first order terms are retained in the asymptotic approximation, the closed form expression can be satisfactorily used in place of numerical integration for slot distance greater than 1.5-2 wavelength and for any reciprocal orientation. This permits, in a typical RLSA analysis, avoiding the numerical integration in more than 90% of the slot pairs, with a save of CPU time of the same order.

2. Formulation

We suppose that the metallic rim that bounds the PPW is not present since we are interested in practical applications where RLSAs are electrically large [1-3]. Indeed, in the design optimization process of this type of antennas the metallic rim is supposed not present and the power flowing in the PPW beyond the slots minimized, thus rendering the rim effect negligible. By applying the equivalence principle, the slots are metalized and covered by an unknown magnetic current distribution; in this way the solution domain is split into an internal and an external region; namely a parallel plate waveguide (PPW) and (under an infinite ground plane approximation) a grounded half-space. By imposing identical amplitude and opposite flowing direction for the magnetic current on the two sides of each slot, we automatically imply the continuity of the tangential electric field through the slots. Analogous continuity condition for the magnetic field is imposed by writing the Magnetic Field Integral Equation (MFIE)

$$\mathbf{n} \times \iint_{\text{Slots}} \left[ Y^{\text{int}}(\mathbf{r}, \mathbf{r}') + Y^{\text{ext}}(\mathbf{r}, \mathbf{r}') \right] \cdot \mathbf{M}(\mathbf{r}') dS' = \mathbf{n} \times \mathbf{H}^{\text{inc}}(\mathbf{r})$$

(1)

with $\mathbf{r} \in \text{Slots}$, in which the magnetic field radiated by the unknown magnetic current distribution is expressed via convolution with the proper ambient (internal/external) Green’s function. In (1) $\mathbf{H}^{\text{inc}}$ is a feeding internal excitation and $\mathbf{n}$ the unit vector normal to slot surface. The MFIE (1) is then solved using a standard MoM approach. The magnetic
current on each slot is expanded by using rectangular waveguide modal \( TE_{mn} \) current basis functions \( b_i(\mathbf{r}) = b_i(u,v)\mathbf{u}, \) where

\[
b_i(u,v) = \begin{cases} 
\sin\left(n\pi\left(\frac{u}{w_i} + \frac{1}{2}\right)\right), & |u| \leq \ell_i \text{ and } |v| \leq \frac{w_i}{2} \\
0, & \text{otherwise}
\end{cases}
\]

and \( u,v \) is a local Cartesian coordinate system (Fig. 1) on the slot plane (i.e., \( \mathbf{r} = r_i + uu + vv \)) with \( r_i \) denoting the center of the \( i \)-th basis function) while \( n \) is the \( TE_{mn} \) mode order.

The same sinusoidal shape is also assumed for weighting functions (Galerkin method), obtaining the following MoM algebraic system of equations

\[
\left[ Y_{ij}\right]_{N \times N} \left[ V_i\right]_N = \left[ I_j\right]_N,
\]

whose solution allows to determine the voltage expansion coefficients for the magnetic current.

Introducing Fourier transform of quantities and convolution theorem, each admittance matrix entry \( Y_{ji} \) can be calculated as the sum of an internal and external term

\[
Y_{ji}^{\text{int/ext}} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} B_j(\mathbf{h} \cdot \mathbf{u}, \mathbf{h} \cdot \mathbf{v}) B_i(\mathbf{h} \cdot \mathbf{u}', \mathbf{h} \cdot \mathbf{v}') \left[ k^2 (\mathbf{u} \cdot \mathbf{u}') - (\mathbf{h} \cdot \mathbf{u}) (\mathbf{h} \cdot \mathbf{u}') \right] G_{ji}^{\text{int/ext}} (|\mathbf{h}|) e^{j|\mathbf{h} \cdot \rho} \frac{d\mathbf{h}}{(2\pi)^2}
\]

in which the Fourier transforms of basis functions are defined as

\[
B_j(\mathbf{h} \cdot \mathbf{u}, \mathbf{h} \cdot \mathbf{v}) = \frac{2j^{j+1} \pi \sin\left(\frac{\mathbf{h} \cdot \mathbf{u}}{2}\right) \sin\left(\frac{\mathbf{h} \cdot \mathbf{v}}{2}\right)}{\left(\frac{\mathbf{h} \cdot \mathbf{v}}{2}\right)^3 - (\mathbf{h} \cdot \mathbf{u})^2}.
\]

In (4) we denote with

\[
G^{\text{ext}} (|\mathbf{h}|) = \frac{-1}{k_0 \sqrt{k_0^2 - h \cdot h}}
\]

the Fourier transform of the of the grounded half-space Green’s function, while with

\[
G^{\text{int}} (|\mathbf{h}|) = \frac{1}{jhk_0} \sum_{m=-\infty}^{\infty} \frac{\mathcal{E}_m}{k_{jm}^2 - h \cdot h}
\]

the Fourier transform of the PPW Green’s function, where \( k_{jm} = \sqrt{k^2 - k_{zm}^2}, \) with \( \Re m\{k_{zm}\} \leq 0, \) and \( k_{zm} = m\pi / h. \)

### 2.1 Asymptotic approximation

Inserting (6) in eq. (4), exchanging the order of summation and integration and choosing two axis parallel and perpendicular to slot center joining vector \( \mathbf{h} = h_1 \hat{\mathbf{r}}_1 + h_2 (\hat{\mathbf{z}} \times \hat{\mathbf{r}}_1) \), for the admittance matrix of the PPW we obtain
\[
Y_{ij}^{\text{int}} = \frac{1}{4\pi^2 jh_x} \sum_{n=0}^\infty e_n \int \int M(h_x, h_y) e^{-j\rho_y} \frac{1}{k^2_{jm} - h_x^2 - h_y^2} dh_x dh_y ,
\]
with
\[
M(h_x, h_y) = B_i(\mathbf{h} \cdot \mathbf{u}, \mathbf{h} \cdot \mathbf{v}) B_j(-(\mathbf{h} \cdot \mathbf{u}^*),(\mathbf{h} \cdot \mathbf{v})) \left[ k^2 (\mathbf{u} \cdot \mathbf{u}^*) - (\mathbf{h} \cdot \mathbf{u})(\mathbf{h} \cdot \mathbf{u}^*) \right].
\]
In the \( h_x \) complex plane the integrand function exhibits pole singularities at \( \pm \sqrt{k^2_{jm} - h_x^2} \) and an exponentially vanishing behaviour as \( \Im\{h_x\} \rightarrow \infty \). By invoking the Jordan’s lemma, the original integration path onto the real axis is deformed and reduced to residue contributions at poles in the lower half-plane, thus
\[
Y_{ij}^{\text{int}} = \frac{1}{4\pi^2 h_x} \sum_{n=0}^\infty e_n \int \frac{M(h_x, h_y)}{\sqrt{k^2_{jm} - h_x^2 - h_y^2}} e^{-j\rho_y} dh_y .
\]
For each \( m \) the original integration path along the real axis is deformed onto the steepest descent path through the saddle point. Thus, the asymptotic expression for the mutual admittance is
\[
Y_{ij} \sim g(\rho_y) \left[ M(k_{jm}, 0) + \frac{1}{2k_{jm}\rho_y} \left( k_{jm} \frac{\partial M(k_{jm}, 0)}{\partial h_x} - k_{jm}^2 \frac{\partial^2 M(k_{jm}, 0)}{2\partial h_y^2} - M(k_{jm}, 0) \right) \right]
\]
where
\[
g(\rho_y) = \frac{1}{4\pi^2 h_x} \sum_{n=0}^\infty e_n H_n^{(1)}(k_{jm}\rho_y)
\]
is the spatial Green’s function of the internal region, and \( e_n = 2 \) or 1 for \( m = 0 \) or \( m > 0 \), respectively.

Acting similarly for the grounded half-space Green’s function, we obtain
\[
Y_{ij}^{\text{int}} \sim g(\rho_y) \left[ M(k_{jm}, 0) + \frac{1}{2k_{jm}\rho_y} \left( 2k_{jm} \frac{\partial M(k_{jm}, 0)}{\partial h_x} - k_{jm}^2 \frac{\partial^2 M(k_{jm}, 0)}{2\partial h_y^2} \right) \right],
\]
where
\[
g(\rho_y) = \frac{j}{k_{jm}\rho_y} \frac{e^{-j\rho_y}}{2\pi\rho_y} .
\]
It is worth noting that derivatives appearing in eqs. (11) and (13) can be easily evaluated numerically.

3. Results

To verify the accuracy of the asymptotic admittance expressions (11) and (13) we evaluated the mutual admittance of a slot pair versus relative distance and detailed comparisons were made of both approximate and exact admittance. As exact solution we have regarded the solution obtained with a very accurate numerical procedure integration. Figures 2a shows the relative error for an arbitrary orientation of the slot pair. In particular, the slots orientation, with regard to the slot center joining segment, is of \( \theta = 45^\circ \) and \( \theta^\prime = 60^\circ \). In all simulations we have considered a PPW height of \( h = 1.6mm \), a dielectric with a relative constant of \( \varepsilon_r = 3.38 \) and a tangent loss of \( \tan \delta = 2.5 \times 10^{-3} \). Figure 2a shows that, when the first order of the asymptotic expansion is also included, we can obtain a relative error less than 1% for a normalized distance of 1.5 in the PPW region (solid curve) and of 2.4 in the grounded half-space region (dashed line). Similar results are obtained for any slots orientations excepting a few cases. These occur when the radiation pattern of the basis and/or of the weighting functions presents a radiation null in the slot centre joining vector direction. It is worth noting that in these cases the zero order terms (i.e., terms behaving as \( 1/ \sqrt{\rho_y} \) for the PPW region and \( 1/ \rho_y \) for the grounded half-space) vanishes. However, the coupling between the two slots is so weak that a larger error in the mutual admittance estimation does not virtually affect the final linear system solution.
Figure 2 – (a) Relative error versus slot center normalized distance for an arbitrary orientation of the slot pair ($W = 0.4 \text{ mm, } L = 5.66 \text{ mm}$): (solid) PPW region, (dashed) grounded half-space region; (b) CP-RLSA typical design and slots inside a circle of one wavelength radius.

Table 1 shows the average number of slots that are outside a circle of radius $R$ for a typical circularly polarized (CP) RLSA having 1866 slots, whose layout is sketched in Fig. 2b. To calculate the mean the circle centre is moved along all the RLSA slots. It is worth noting that if we choose to apply the asymptotic expression for all those slots that are, for example, outside a circle of radius $R = 2\lambda_0$ in the PPW region (i.e., we accept an error of about 1% in the mutual admittance) these represent the 92% of the total number of slots. This means that we can avoid the cumbersome numerical integration in more than 92% of the slot pairs, with a save of CPU time of the same order.

Further results will be presented at conference time.

Table 1 – Number of slots outside a circle of radius $R$

<table>
<thead>
<tr>
<th>Circle radius $R$</th>
<th>$\lambda_0$</th>
<th>$2\lambda_0$</th>
<th>$3\lambda_0$</th>
<th>$4\lambda_0$</th>
<th>$5\lambda_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average number of slots outside the circle</td>
<td>1829</td>
<td>1729</td>
<td>1546</td>
<td>1348</td>
<td>1172</td>
</tr>
<tr>
<td>And its percentage with respect the total slot number</td>
<td>98.0%</td>
<td>92.7%</td>
<td>82.6%</td>
<td>72.1%</td>
<td>62.8%</td>
</tr>
</tbody>
</table>

4. References


