Feasibility Study of a Gradient Redirecting Algorithm Combined with Step Length Estimation for Including A Priori Data in the Image Reconstruction in Microwave Tomography

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Abstract

An iterative electromagnetic time-domain inversion algorithm for microwave tomography exploiting a priori data of the dielectric properties of the object being imaged is described. The algorithm is based on solving the regular and the adjoint Maxwell's equations in order to compute gradients, which are used to update the dielectric profile with the conjugate-gradient method. The conjugate-gradient method however requires a time consuming line search procedure to be used for minimizing a penalty function. In this paper we have also studied the possibility of replacing the line search procedure and instead using an estimated step length for the update of the reconstruction. It was found that these methods combined can be used to significantly decrease the computation time for the reconstruction process and improve the convergence.

1. Introduction

The inverse electromagnetic problem in the image reconstruction process is inherently ill-posed and nonlinear. An iterative reconstruction algorithm, which involves a cost function that is either maximized or minimized, is therefore necessary. However the nonlinearity of the problem often causes a local search algorithm to get trapped in a local minimum, leading to incorrect reconstructions, [6]. Possible ways to overcome this includes introducing a priori information about the object being imaged, [7]. It can also be beneficial to use a frequency-hopping technique where low frequency content of the electromagnetic pulse is used to reduce the nonlinearities. This is then followed by using successively higher frequency content to improve the resolution, [3], [4]. The problem of getting trapped in a local minimum can as an alternative often be helped with a clever initial guess or by initializing the optimisation with an ideal model of the targets under reconstruction. This is for instance viable in biomedical imaging where the organs and tissues can be assumed known, [2], or when testing an object for a defect when the unperturbed object otherwise is known, [1].

Another problem with the iterative reconstruction algorithms is that they in general are very computational intensive, both with respect to computer memory requirements and reconstruction time. The most time consuming part of the reconstruction process is the line search where several evaluations of the cost functional are required to find the minima. Often 5-10 evaluations of the functional are necessary before the minimum have been found. If this number could be reduced it would significantly speed up the reconstruction procedure.

In this paper we investigate a novel method for introducing a priori data in the reconstruction algorithm which we combine it with an estimation of the step length replacing a line search algorithm. The estimation is based on a polynomial approximation of second order of the cost functional and the corresponding step length is approximated with only one additional evaluation of the gradient, [8].

2. The Reconstruction Algorithm

The iterative electromagnetic time-domain inversion algorithm, modified to incorporate a priori knowledge of the shape and dielectric properties, is similar to what is used in our microwave tomography experiment, [4], [5]. The algorithm assumes that measurements of transient pulses have been made for several transmitter/receiver combinations surrounding the target region. In the reconstruction process an initial dielectric distribution is assumed, and if no a priori information of the targets is included in the algorithm it is set equal to the uniform background properties. It is further assumed that the material is non magnetic. The objective of the reconstruction procedure is to minimize the cost functional, F, defined as

$$F(\varepsilon,\sigma)=\int_0^T \sum_{m=1}^{M}\sum_{n=1}^{N}\left|E_m(\varepsilon,\sigma,R_n,t)-E_m^{meas}(R_n,t)\right|^2 dt, \qquad (1)$$

where $E_m(\varepsilon,\sigma,R_n,t)$ is the calculated field from the computational model in successive iterations of the inversion algorithm. $E_m^{meas}(R_n,t)$ is the measured data. In this work simulations are however used to generate also the measured data. Furthermore M is the number of transmitters and N is the number of receivers. To minimize the functional, gradients are computed and used in a conjugate-gradient algorithm. These gradients are derived by considering a small increment in the dielectric profile, and then the corresponding change in the functional is calculated by means of a perturbation analysis. The residuals between the measured and the simulated fields on the receivers are used as the driving sources. The Fréchet derivative of the functional can be derived as

$$
\begin{aligned}
G_\varepsilon(x) &= 2\sum_{m=1}^{M}\int_0^T \widetilde{E}_m(\varepsilon,\sigma,x,t)\cdot\partial_t E_m(\varepsilon,\sigma,x,t)dt \\
G_{\sigma/\langle\sigma\rangle}(x) &= 2\langle\sigma\rangle\sum_{m=1}^{M}\int_0^T \widetilde{E}_m(\varepsilon,\sigma,x,t)\cdot E_m(\varepsilon,\sigma,x,t)dt
\end{aligned}
\qquad (2)
$$

where $E_m(\varepsilon,\sigma,x,t)$ is the numerically computed E-field in the reconstruction domain and $\widetilde{E}_m(\varepsilon,\sigma,x,t)$ is the solution to the adjoint problem and $\langle\sigma\rangle$ is a scaling parameter compensating for the different scaling of the permittivity and the conductivity in the penalty functional. In the original version of the algorithm these gradients are updated and used with the conjugate-gradient update and a line search is performed to find the minimum of the objective functional. As the minimum is found the dielectric properties of the reconstructed object are updated accordingly.

2.1 Taking A Priori Data Into Account

The handling of a priori data is based on the gradients described above which are modified to improve the convergence towards the final reconstruction. The computed gradients consist of a direction in the permittivity and in the conductivity for each pixel in the reconstruction grid. Here a method to create a stronger correlation between the permittivity and the conductivity images is suggested and investigated. For each pixel in the reconstruction the dielectric values are represented as a position in a ε–σ-plane. In the line search process this point will move along a line in this plane and end up at new coordinates in the next iteration. If the electrical properties of the measured object are a priori known these properties can be defined as target points or target regions in this search space. The original gradients are now modified such that in each pixel of the image the gradient is pointing towards the closest target region, see figure 1. We have chosen to define the closest direction in terms of the least angle and the original search direction vector is projected onto this new direction. If the new direction has an angle of more than 90° to the original then the new direction vector is simply set to zero. A numerical test case has been designed in order to test the performance of this method of treating the a priori data. Different sized circular objects were spread out in the reconstruction domain according to figure 2.

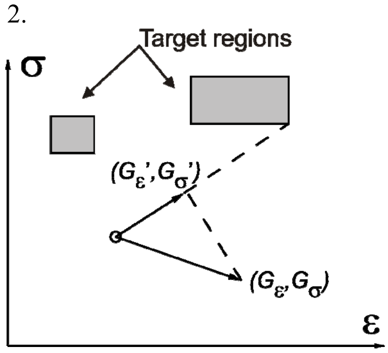

Figure 1. An example with two rectangular target regions in the plane. The direction *(G_ε,G_σ)* is projected onto the direction *(G_ε',G_σ')* towards the closest target region.

2.2 Step Length Estimation

In order to minimize the functional a line search method is often used. This however has the disadvantage of being quite time consuming. In this case, each line search requires about 5-10 evaluations of the functional and each evaluation of the functional requires one FDTD simulation for each transmitting antenna. In this paper we have implemented a method for estimation of the optimal step length, [8], based on the assumption that the penalty function is a second order polynomial function. The computational cost for this estimation is only one extra evaluation of the gradient.

3. Results

The step length estimation and the redirecting of the gradients towards a priori regions were used in a numerical study with reconstruction of simulated data. Furthermore the frequency hopping technique was used to resolve the eight circles with different sizes as shown in figure 2. The diameters of the circles are varying from 8 mm to 73 mm. The background had $\varepsilon_r = 12$ and $\sigma = 0$. The circles all had values $\varepsilon_r = 15$ and $\sigma = 0.2$ *S/m*. Measurement data pulses were numerically simulated at the center frequencies 500 MHz, 1.25 GHz and 3 GHz, with the same full width half maximum bandwidths as the center frequencies. The reconstruction was starting at 500 MHz and 15 iterations were made. This result was used as initialization to the 1.25 GHz iterations with 15 more iterations and finally 15 iterations were made with 3 GHz. The reconstructions shown for each intermediate step in the frequency hopping scheme is shown in figure 3 where the a priori data have not been taken into account and in figure 4 where it have been included. Here we have a priori assumed that the objects should have properties $\varepsilon_r = 15$ and $\sigma = 0.2$ *S/m.* Furthermore the relative logarithm of the relative error in the reconstruction of the permittivity and the conductivity have been plotted in figure 5 for these two cases and also for a line search algorithm and for the step length estimation method described above.

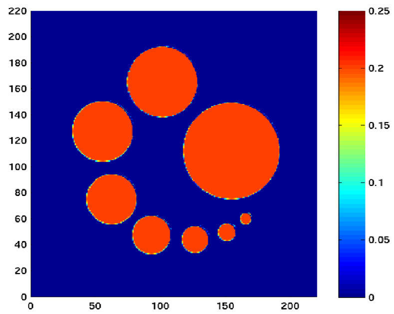

Figure 2. The original test object. The background has $\varepsilon_r = 12$ and $\sigma = 0$. The circles all have values of $\varepsilon_r = 15$ and $\sigma = 0.2$ *S/m* and the diameter vary between 8 mm to 73 mm.

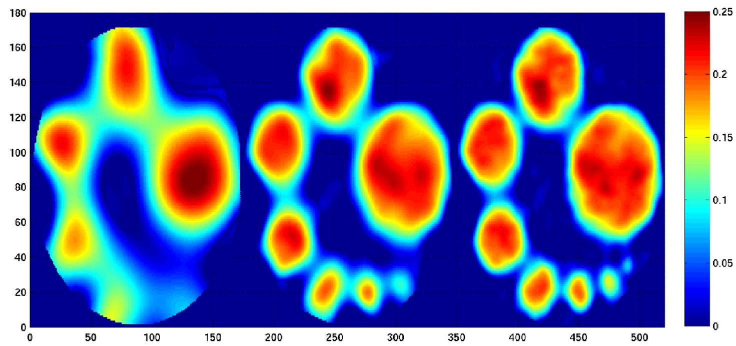

Figure 3. The figure shows the reconstruction of the conductivity without taking the a priori data into account. Also shown are the intermediate reconstructions resulting from each step in the frequency hopping scheme.

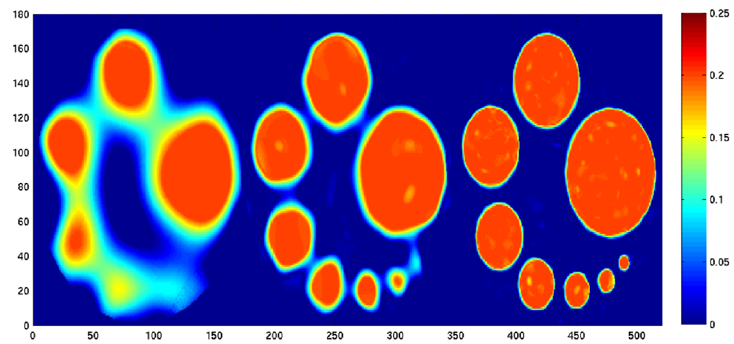

Figure 4. The figure shows the reconstruction of the conductivity when taking the a priori dielectric data of the objects into account. Also shown are the intermediate reconstructions resulting from each step in the frequency hopping scheme.

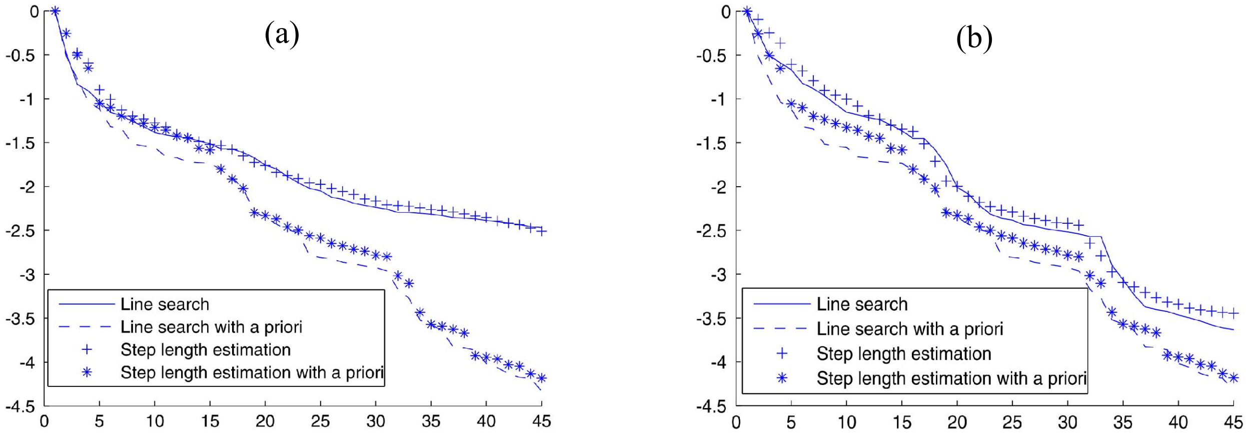

Figure 5. The figure shows plots of the normalized relative errors in the reconstructed images, in (a) for the conductivity and in (b) for the permittivity. Results are shown with and without considering a priori data and with line search or step length estimation. We see that the results when estimating the step length are almost identical to the results obtained when performing the line search. We also see a significant improvement when utilizing the a priori data.

4. Conclusion

The results show an improved convergence rate when including the a priori data. In particular this can be seen in the reconstruction of the conductivity in this example. The reason for the improvement in this case is that at the same time as the gradients are redirected towards the target region a coupling between the two properties is introduced. We also see no significant difference between the reconstructions using the line search algorithm and the step length estimation. This means that we have replaced the need for 5-10 evaluations of the cost functional with only one extra evaluation of the gradient in order to estimate the step length where one evaluation of the gradient requires two FDTD simulations. In conclusion these techniques appear to be feasible in improving the speed and convergence rate of the reconstruction problem.

5. References

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