An imaging tool for intra-wall investigations: a feasibility study

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Abstract

The problem of characterizing inhomogeneous targets buried into a multi-layered structure is dealt with. Such a problem is relevant in intra-wall imaging investigations, where one aims at detecting hidden objects, such as weapons and/or drugs, put into cavities or gaps embedded in a wall. In this framework, the paper presents a novel shape reconstruction approach based on the Linear Sampling Method (LSM), an imaging technique that provides accurate reconstructions by observing the behavior of an indicator function arising from the solution of a regularized linear problem. However, despite its effectiveness, the LSM cannot distinguish between homogeneous and inhomogeneous targets, so that it is not suitable for intra-wall imaging. To overcome this limitation, while preserving the valuable features of the LSM, we propose an imaging approach in which location and shape of the embedded cavity are first retrieved by using the standard method and then the inclusions possibly hidden in it are imaged by exploiting a new (LSM-like) indicator function. Preliminary results are provided in order to verify the reconstruction capabilities of the introduced approach in the framework of intra-wall inspections.

1. Introduction

The possibility of exploiting electromagnetic fields to reconstruct physically inaccessible objects is of huge interest in several applicative areas such as, f.i., civil engineering, homeland security, medical imaging, geophysical investigations and landmine detection [1]. However, the imaging of unknown targets from the measures of the fields they scatterer, after being illuminated by known primary sources, is a challenging issue, which involves the solution of a non-linear ill-posed inverse scattering problem. To this end, many approximate and full-wave reconstruction strategies have been proposed.

Among the other approaches proposed in literature, the Linear Sampling Method (LSM) [2] is worth to be considered, since it allows one to retrieve geometrical features of scattering systems by simply observing the behavior of an indicator function. This last is easily obtained by solving, in each point of the investigated domain, a linear system whose kernel is the measured scattered fields matrix and its right hand side is the Green's function pertaining the considered geometry [2]. Notably, LSM does not rely on an approximate scattering model, it can be applied to retrieve both dielectric as well as metallic objects and it allows quasi real-time data processing. Several tests against synthetic and experimental data have assessed its reconstruction capabilities against both full-aperture data collected in free space [1,3-5] and aspect-limited data measured at the interface between two different media [6,7]. On the other hand, LSM is not able to distinguish between homogeneous or inhomogeneous targets. As a matter of fact, the indicator functions attains high values in points wherein the permittivity contrast is zero and low values elsewhere, regardless of the specific value of the permittivity contrast. Therefore, the map of the indicator function does not provide any information to detect inclusions which are possibly embedded into an object. Consequently, the LSM is not suitable for intra-wall imaging applications.

In order to overcome this limitation, we take advantage of the physical interpretation of the LSM as an electromagnetic focusing problem [3] to propose a shape reconstruction approach which preserves the attractive features of the LSM and it is suitable for imaging piecewise inhomogeneous targets (i.e., to reconstruct an object and to detect its inclusions). This approach is then applicable in intra-wall imaging inspections, wherein one is both interested in retrieving hidden caches and characterizing their content. The proposed approach breaks the problem at hand into two parts: the first one deals with the morphological characterization of the "external" target (a cavity or a gap, in intra-wall inspections); while, the other one is devoted to estimate the features of the hidden inclusions. Hence, a two-steps procedure is introduced, wherein the standard implementation of the LSM is initially exploited and then a suitable reformulation of it is used to define a new indicator function. A preliminary result concerning two dimensional targets embedded into a three layers structure is proposed to verify the effectiveness of the suggested strategy in the framework of intra-wall prospecting.

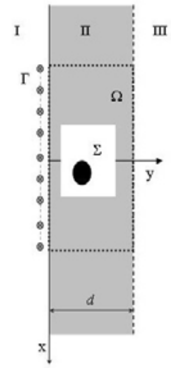


Figure 1: Reference geometry

2. A brief review of the Linear Sampling Method

Let us consider the reference geometry sketched in Fig.1, wherein the first and third layers are made of air; the second one schematizes a concrete wall whose thickness is d. For sake of simplicity, the canonical 2D scalar case is considered and the z-axis is assumed as the invariance direction. The investigated region is denoted by Ω and it is embedded in the second layer. The data are gathered by using N transmitting and receiving probes located on a line Γ, which is in the first medium. $\mathbf{E}_s$ denotes the $N\times N$ data matrix, whose generic element is the scattered field measured, at a fixed frequency, by the receiver j when illuminating by means of the transmitter i.

In the framework of LSM, the shape reconstruction is provided by plotting, in each sampling point $\mathbf{r}_s \in \Omega$, the L^2-norm of the indicator function [2]:

$$\mathbf{g}(\mathbf{r}_s) = \sum_{n=1}^{N} \frac{\lambda_n}{\left(\lambda_n^2 + \alpha^2\right)} \langle \mathbf{f}(\mathbf{r}_s), \mathbf{u}_n \rangle \mathbf{v}_n \tag{1}$$

where $\{\mathbf{v}_n, \mathbf{u}_n, \lambda_n\}$ is the Singular Value Decomposition of the matrix $\mathbf{E}_s$ and represent the n-th left singular vector, the n-th right singular vector and the n-th singular value (ordered for decreasing magnitude), respectively. The N-dimensional vector $\mathbf{f}$ contains the value of the field radiated by a line source located in $\mathbf{r}_s$ as computed at the N receiving positions on Γ and it corresponds to the Green's function [8] pertaining the considered reference geometry. The coefficient α is the Tikhonov regularization parameter, which in the following is fixed equal to $0.01\lambda_1$.

For each sampling point, the indicator (1) defines a linear combination of the measured scattered fields $\mathbf{E}_s$ such to approximately match the background Green's function $\mathbf{f}$ on Γ. Its norm allows to characterize the scatterers' shape, as it is claimed to assume low values inside the targets and large values elsewhere [2]. Even if a general theoretical proof is still an open issue, this behaviour can be explained from a physical point of view by relating the LSM to the possibility or impossibility of solving an electromagnetic focusing problem [3]. According to this interpretation, LSM aims at synthesizing, for each sampling point $\mathbf{r}_s$, a source of energy $\left\|\mathbf{g}(\mathbf{r}_s)\right\|$, which is able to induce in Ω a current focused in $\mathbf{r}_s$, i.e., the current that radiates the field $\mathbf{f}$ on Γ. Apart some special cases [3], such a current can be induced, by means of a finite energy primary source, in points belonging to the scatterers, whereas it cannot be achieved in external points. As a matter of fact, the induced current must be exactly null in points located outside of the targets [8], so that the energy of the source synthesized in these points becomes large in the impossible attempt of inducing therein a non zero and focused current [3].

It is worth to note that as far as aspect limited data are concerned, a modified indicator function, obtained by dividing that defined in eq.(1) by $\left\|\mathbf{f}(\mathbf{r}_s)\right\|$, i.e. $\hat{\mathbf{g}} = \mathbf{g}/\left\|\mathbf{f}(\mathbf{r}_s)\right\|$, is exploited [7,9,10]. By doing so, more accurate reconstructions of the deeper parts of the probed region can be achieved [9].

3. A distorted Linear Sampling Method for intra-wall investigations

As described in the previous Section, points having a zero or not permittivity contrast may be identified by simply observing the behavior of $\left\|\mathbf{g}\right\|$ or $\left\|\hat{\mathbf{g}}\right\|$. On the other hand, since the values of these indicators are not proportional to the value of the permittivity contrast, no information about electromagnetic parameters of the scatterers (i.e., their relative permittivity and/or conductivity) or about their homogeneous or inhomogeneous nature may be obtained. As a consequence, at least as far as its standard implementation is concerned, the LSM is unsuitable to image inclusions embedded into objects whose geometrical features may be unknown, as in material's defects identifications and/or in intra-wall investigations.

In intra-wall imaging inspections, it is realistic to assume that the "external" object is a cavity or an air gap, hence its electromagnetic features, being that of free space, are known. By exploiting such an information, one can derive an effective inversion strategy which overcomes the shortcoming of the LSM, while preserving its advantages.

The proposed approach splits the problem at hand into two parts: the first one deals with the shape reconstruction of the "external" object (the cache), the second one with the imaging of the inclusions. In the first step, location and shape of the "external" object are retrieved by means of the LSM indicator function, wherein the vector **f** is computed by exploiting the Green's function pertaining the reference geometry shown in fig.1 [8]. Let Σ denote the so retrieved shape. In the second step, by relying on the physical interpretation of the LSM, one pursues a focusing strategy explicitly aimed at imaging the inclusions possibly hidden in the cache. To this end, for each $\mathbf{r}_s \in \Sigma$, one should synthesize a source able to induce a current focused in the sampling point and such that its energy is low when $\mathbf{r}_s$ belongs to the inclusions and high elsewhere. By paralleling the reasoning done in Section 2 for the LSM, one notices that such a focusing is equivalent to combine the fields scattered by the only inclusions in order to approximately match on Γ the field corresponding to the perturbed Green's function $\mathbf{f}_\Sigma$, which takes into account the presence of Σ in the original reference scenario. Therefore, a new indicator function can be introduced by using the above mentioned knowledge on the retrieved shape. As a matter of fact, by computing the $N\times N$ data matrix $\mathbf{E}_\Sigma$, whose generic element is the field that the empty cavity would scatter at the j-th receiving point on Γ when the i-th primary source is considered, one obtains the scattered field data matrix $\Delta\mathbf{E}$ pertaining the sole inclusions as $\Delta\mathbf{E}=(\mathbf{E}_s- \mathbf{E}_\Sigma)$. Then, by denoting with $\{\mu_n, \gamma_n, \beta_n\}$ the SVD of $\Delta\mathbf{E}$, a morphological characterization of the inclusions is directly achieved by means of the new indicator function:

$$\mathbf{g}_\Sigma(\mathbf{r}_s) = \sum_{n=1}^{N} \frac{\beta_n}{(\beta_n^2 + \alpha^2)} \langle \mathbf{f}_\Sigma(\mathbf{r}_s), \gamma_n \rangle \mu_n \tag{2}$$

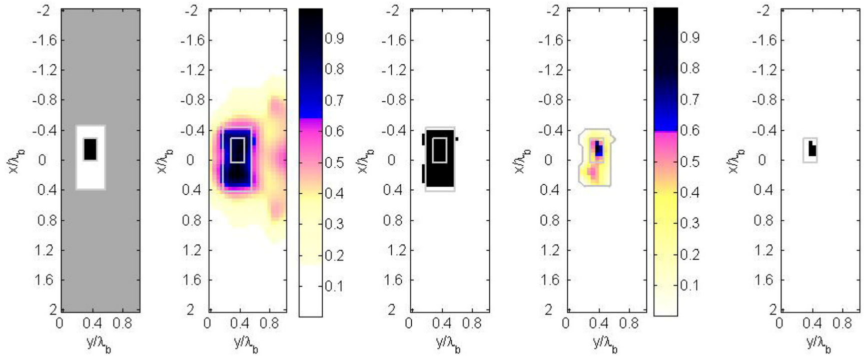
where the Tikhonov regularization parameter α is now fixed equal to $0.01\beta_I$. Of course, the expression of the indicator (2) is formally equivalent to that of LSM, as they rely on the same physical concept. It is also worth to observe that the vector $\mathbf{f}_\Sigma$, i.e., the perturbed Green's function, is provided when computing the field $\mathbf{E}_\Sigma$, as it corresponds, by virtue of reciprocity, to the total field computed inside Σ. Thus, as both $\mathbf{E}_\Sigma$ and $\mathbf{f}_\Sigma$ can be obtained by means of an effective forward solver, the computational burden of the proposed approach is still comparable with that of the LSM and thus much lower than that of the other linear and non linear inversion procedures.

4. Numerical Analysis

Aim of this section is to provide an example of the reconstruction capabilities of the introduced approach.

With respect to the three layers structure sketched in fig.1, N=21 line sources are uniformly spaced on the measurement line Γ, which is $4\lambda_b$ long (λ_b being the background wavelength) and it is closed to the air-wall interface. The scattered fields are measured by means of N=21 receivers located in the same points of the primary sources. The wall thickness is $d=1\lambda_b$, the relative permittivity is $\varepsilon_b=4$ and the conductivity is $\sigma_b= 0.01$S/m. The investigated domain has size $4\lambda_b$x$1\lambda_b$ and it is partitioned into 80x20 square cells, whose centers are assumed as sampling points. The buried target is a cavity whose size is $0.86\lambda_b$x$0.38\lambda_b$, while its hidden inclusion is a rectangular object having size $0.3\lambda_b$x$0.16\lambda_b$ and electromagnetic features $\varepsilon_x=10$ and $\sigma_x= 0.0$S/m. A sketch of the reference profile is given in fig.2.a. The data have been simulated by means of a full-wave forward solver based on the method of moments and have been corrupted by a 10% of additive noise to simulate the unavoidable measurement errors.

Figure 2.b shows the reconstruction achieved by applying the standard formulation of the LSM. In particular, the normalized map concerning the indicator function $\mathbf{h} = 1/\left\|\hat{\mathbf{g}}(\mathbf{r}_s)\right\|$ is plotted, where $\hat{\mathbf{g}}$ is the modified LSM support indicator, then high values are related to points which are inside the scattering system and low values identify external points. From this result, which also represents the first step of the proposed approach, it can be observed that LSM cannot provide information on the content of the cavity, but only on its shape. To apply the second step and overcome LSM drawbacks, we first build the binary map given in fig.2.c. Such a map is obtained by exploiting the standard MATLAB® ISOSURFACE routine, where the isosurface value **G** has been fixed equal to 0.65. This last has been heuristically chosen as the value corresponding to the maximum variation of the normalized map, which occurs on the boundary of the object. With respect to this target, black pixels correspond to the support Σ.



(a) (b) (c) (d) (e)

Figure 2: An intra-wall imaging example: (a) Reference profile; (b) standard LSM reconstruction; (c) binarized LSM shape; (d) Image of the inclusion supplied by the proposed indicator function; (e) binarized image of the inclusion.

Then, we compute the matrix $\mathbf{E}_\Sigma$ of fields scattered by the empty cache having shape Σ, the *N*-dimensional vector $\mathbf{f}_\Sigma$, the $N \times N$ dimensional matrix $\Delta\mathbf{E}$ and its SVD and we plot, in each point $\mathbf{r}_s \in \Sigma$, the indicator function $\mathbf{h}_\Sigma = 1/\left\|\mathbf{g}_\Sigma\left(\mathbf{r}_S\right)\right\|$, whose normalized map is given in fig.2.d. As it can be observed, the hidden target is now clearly detected and well located, moreover a satisfactory reconstruction of its shape is achieved even if its size is slightly under estimated. This is well shown in fig.2.e, where the binary map, which is given by the MATLAB® ISOSURFACE routine for the isosurface value $\mathbf{H}$ = 0.57, is plotted. Remarkably, the overall computational time is in the order of few seconds and it slightly larger than that necessary for the standard LSM implementation.

5. Conclusion

A new shape reconstruction approach, which takes advantage of the effectiveness and the simplicity of LSM, has been proposed to image piecewise inhomogeneous targets. The problem has been broken up in two parts. In the first one, LSM is used to retrieve the external shape of the object, then a new indicator function is defined to image the inclusions. A numerical example has been given to assess feasibility in intra-wall inspections. The achieved results are encouraging and suggest further investigations to establish limits and potentialities of the proposed approach.

6. References

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