Shape Reconstruction using Multi-static and Multi-spectrum Data

Songming Hou * Hongkai Zhao †

Abstract

Applications such as medical imaging, non-destructive testing, seismic imaging, and target detection/recognition utilize active arrays of transducers that emit signals and record reflected and/or transmitted signals. Recording the inter-element response forms the response matrix of an active array. This paper discusses reconstructing the shape of targets using the multi-static and multi-spectrum data with a direct imaging method that is efficient and robust.

1 Introduction

Reconstructing the shapes of obstacles or inhomogeneities of a medium, using near field or far field data, has important applications in radar, sonar, and geophysical exploration, in medical imaging, and in nondestructive testing. This is in general an ill-posed (non-linear) inverse problem. Imaging the whole medium using a general inverse problem approach may be too complicated and too expensive to be practical in many applications, for instance if the imaging domain is large compared to the wavelength. If the background medium is homogeneous and some simple boundary condition is satisfied at the boundary of the target, the inverse problem can be turned into a geometric problem, that is, the problem of determining the shape of the target from the scattered wave field pattern. Using non-linear optimization approach in this case is still difficult and computationally expensive.

Direct imaging methods, which are not based on non-linear optimization and hence do not require forward solver or iterations, have attracted a lot of attention recently. If the targets are small compared with the array resolution, the location information can be obtained while the shape information is not resolved. Several matched filter type of algorithms have been developed for imaging/locating point targets, for example the MUltiple SIgnal Classification (MUSIC) algorithm [8, 1, 6, 7, 3, 2]. However, with the point target assumption, physical properties and the geometry of the target are neglected. More importantly an extended target is not a superposition of point targets. For extended targets the response matrix has a more complicated structure. In [4, 5], the MUSIC algorithm is generalized to image the location and shape of extended targets. A crucial step is to use resolution and noise level based thresholding to determine how many singular vectors of the response matrix span the signal space.

Although the generalized MUSIC algorithm for a single frequency is capable of imaging different types of targets with efficiency, robustness and accuracy, provided full aperture data is given, for limited or synthetic aperture, the results are typically not very good. Multi-spectrum data should be used to complement the lack of spatial aperture.

The MUSIC algorithm is based on the singular value decomposition (SVD) of the response matrix which allows an arbitrary complex phase. In this paper, we propose a direct imaging algorithm that makes use of the phase information. In particular, we take advantage of phase coherence for multiple frequencies to improve both resolution of robustness of the imaging. The crucial points in our algorithm are (1) physically based factorization of the response matrix that transforms a passive target detection problem to an active source detection problem, (2) a phase coherent imaging function that can superpose multiple tones and multiple frequencies to take advantage of both spatial diversity (aperture) of the array and/or the bandwidth of the probing signal. Moreover, the superposition weights for different frequencies can be adaptively tuned according to the background medium and probing distance, etc, to achieve desirable resolution and/or robustness.

The outline of the paper is as follows. In Section 2 we describe the MUSIC algorithm for imaging the shape of extended targets using single frequency data. In Section 3, we propose a method that uses multi-spectrum

*Mathematics and Statistics, Louisiana Tech University, Ruston, LA, 71272, shou@latech.edu

†Deptartment of Mathematics, University of California at Irvine, Irivne CA 92697, zhao@math.uci.edu

data in a phase coherent way to image the shape of extended targets. Numerical experiments are presented in Section 4.

2 The MUSIC Algorithm for Extended Targets

In this section, we summarize the results from our previous work [4] and [5].

First consider sound-soft targets or penetrable targets.

Let $P = U\Sigma V^H$ be the singular value decomposition of the response matrix. For near field data, define the illumination vector

$$\mathbf{g}(\mathbf{x}) = [G(\mathbf{x}, \xi_1), \ldots, G(\mathbf{x}, \xi_m)]^T,$$

where $\xi_1, \ldots, \xi_m$ are the transducer locations, $\mathbf{x}$ is any point in space, G is the Green's function for the Helmholtz equation in free space, which has an analytic expression that depends on the dimension.

For far field data, define the illumination vector

$$\mathbf{g}(\mathbf{x}) = [e^{ik\mathbf{x}\cdot\mathbf{d}_1}, \ldots, e^{ik\mathbf{x}\cdot\mathbf{d}_m}]^T,$$

where $\mathbf{d}_j$ are the propagation directions of incident waves and $\mathbf{x}$ is any point in space.

For full aperture data, the MUSIC imaging function presented in [4] may be used to image the shape of the targets:

$$I(\mathbf{x}) = \frac{1}{\|\mathbf{g}(\mathbf{x})\|_2^2 - \sum_{\ell=1}^{s} |\mathbf{g}(\mathbf{x})^H \mathbf{u}_\ell|^2}, \tag{1}$$

where $\mathbf{u}_\ell$ is the ℓth column of the matrix U and the number of singular vectors s that spans the signal space is determined by the thresholding algorithm in [4] based on resolution analysis.

For sound-hard targets, the information about the normal direction of the boundary is needed but it is part of the unknown. We presented a method to solve this problem for near field data in [4] and far field data in [5].

3 Imaging using Multi-static and Multi-spectrum Data

First, let us assume a Dirichlet boundary condition for the target, i.e., a sound soft target.

The result of the MUSIC algorithm degenerates with limited or synthetic aperture data. The MUSIC algorithm, as a projection algorithm, does not utilize phase information. Here we propose a direct imaging method that uses coherent phase information.

For an active array, e.g., the transmitters and receivers coincide, the response matrix P is complex symmetric, which can be factored at $P = U\Sigma U^T$. This unique factorization (up to a sign) helps to eliminate the arbitrary phase generated by matlab when taking SVD. We propose the imaging function

$$I^M(\boldsymbol{x}) = \sum_{\omega} \alpha(\omega) \sum_{m=1}^{M^\omega} [\hat{\mathbf{g}}^H(\boldsymbol{x};\omega)\mathbf{u}_m^\omega]^2. \tag{2}$$

where $\hat{g}$ is the normalized illumination vector from the transducers to a search point x, u_m is the mth row of the matrix U, α is the weight for multi-spectrum, M^ω is a threshold determined by the algorithm in [4].

Unlike the MUSIC algorithm where phase information is lost when taking the norm square, the phase is just doubled when taking the square. By combining multi-spectrum information in a phase coherent way, better imaging result can be expected for limited or synthetic aperture data compare with the MUSIC algorithm.

If the transmitters and receivers do not coincide, e.g., there are s transmitters located at $\boldsymbol{\xi}_1, \ldots, \boldsymbol{\xi}_s$ and there are r receivers located at $\boldsymbol{\eta}_1, \ldots, \boldsymbol{\eta}_r$. The response matrix is $s \times r$ and its element P_{ij} records the response at j-th receiver corresponding the a signal sent out from i-th transmitter. Define the illuminations vector with respect to the receiver array and transmitter array respectively as

$$\mathbf{g}_r(\boldsymbol{x}) = [G^0(\boldsymbol{\eta}_1, \boldsymbol{x}), G^0(\boldsymbol{\eta}_2, \boldsymbol{x}), \ldots, G^0(\boldsymbol{\eta}_r, \boldsymbol{x})]^T,$$

and

$$\mathbf{g}_s(\boldsymbol{x}) = [G^0(\boldsymbol{\xi}_1, \boldsymbol{x}), G^0(\boldsymbol{\xi}_2, \boldsymbol{x}), \ldots, G^0(\boldsymbol{\xi}_s, \boldsymbol{x})]^T.$$

Then our imaging function is

$$I^M(\boldsymbol{x}) = \sum_{\omega} \alpha(\omega) \sum_{m=1}^{M^\omega} [\hat{\mathbf{g}}_r^H(\boldsymbol{x};\omega)\mathbf{u}_m^\omega][\hat{\mathbf{g}}_s^H(\boldsymbol{x};\omega)\overline{\mathbf{v}}_m^\omega]. \tag{3}$$

Here $\hat{\mathbf{g}}$ denotes the normalized illumination vector.

For a sound-hard target, with a Neumann boundary condition for the extended target, the source of the scattered wave field is a (unknown) weighted superposition of dipoles $\frac{\partial \hat{\mathbf{g}}}{\partial \nu}$ at the boundary. Therefore, the normal direction is part of the unknown in the imaging function. As is done in [4] we will incorporate a direction search in our imaging function, e.g., among a fixed collection of discretized directions, $\mu_j, j = 1, 2, \ldots$, we maximize the imaging function among these directions at a searching point $\boldsymbol{x}$. Our imaging function is then in the general case:

$$I^M(\boldsymbol{x}) = \max_j \,|\sum_{\omega} \alpha(\omega) \sum_{m=1}^{M^\omega} [\frac{\partial \hat{\mathbf{g}}_r^H(\boldsymbol{x};\omega)}{\partial \nu_j}\mathbf{u}_m^\omega][\frac{\partial \hat{\mathbf{g}}_s^H(\boldsymbol{x};\omega)}{\partial \nu_j}\overline{\mathbf{v}}_m^\omega]\,|\,. \tag{4}$$

4 Numerical Experiments

We illustrate how our imaging algorithm can be used to image the shape for extended targets using multi-static and multi-spectrum data. The experiments for using the MUSIC algorithm with single frequency data can be found in [4] for near field data and [5] for far field data.

Figure 1 (left) shows the imaging result for a sound-hard target with full aperture near field data. Figure 1 (right) shows the imaging result for two sound-soft targets with full aperture far field data.

Figure 2 (left) shows the imaging result with synthetic aperture data for a sound-soft target. Here we use 10% multiplicative noise. Figure 2 (right) shows the imaging result with synthetic aperture data for a sound-soft target in a 10% weakly inhomogeneous medium.

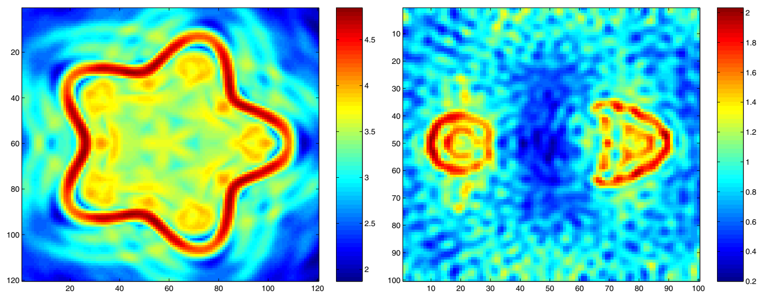

Figure 1: Left: imaging a sound-hard target with full aperture near field data; Right: imaging a sound-soft target with full aperture far field data

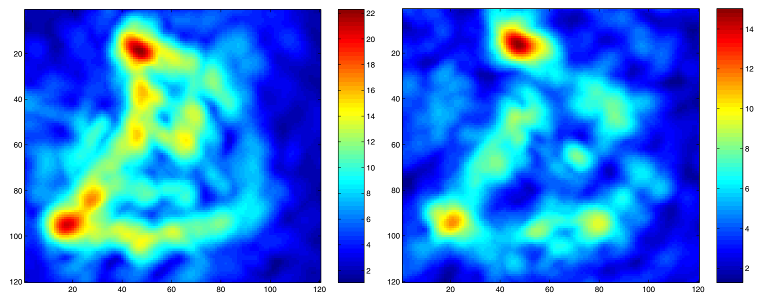

Figure 2: Left: Imaging using synthetic aperture data with 10% multiplicative noise; Right: Imaging using synthetic aperture data in a weakly inhomogeneous medium

5 Conclusions

We propose a direct imaging algorithm for imaging the shape of extended targets using multi-static and multi-spectrum data. The algorithm is simple and efficient because no forward solver or iteration is needed. The algorithm can also deal with different material properties and different type of illuminations and measurements. This method provides a framework for balancing spatial diversity via the singular value decomposition with frequency diversity. By taking advantage of multi-spectrum phase coherence, the imaging is enhanced and is robust with respect to noise.

References

[1] Devaney. Super-resolution processing of multi-static data using time-reversal and MUSIC. *to appear in Journal of the Acoustical Society of America.*

[2] A. J. Devaney F. K. Gruber, E. A. Marengo. Time-reversal imaging with multiple signal classification considering multiple scattering between the targets, 2004.

[3] F. K. Gruber, E. A. Marengo, and A. J. Devaney. Time-reversal imaging with multiple signal classification considering multiple scattering between the targets. *J. Acoust. Soc. Am.*, 115:3042–3047, 2004.

[4] Songming Hou, Knut Solna, and Hongkai Zhao. A direct imaging algorithm for extended targets. *Inverse Problems*, 22:1151–1178, 2006.

[5] Songming Hou, Knut Solna, and Hongkai Zhao. A direct imaging method using far field data. *Inverse Problems*, 23:1533–1546, 2007.

[6] E. Kerbrat, C. Prada, and M. Fink. Imaging in the presence of grain noise using the decomposition of the time reversal operator. *Journal of the Acoustical Society of America*, 113(3):1230–1240, March 2003.

[7] Claire Prada and Jean-Louis Thomas. Experimental subwavelength localization of scatterers by decomposition of the time reversal operator interpreted as a covariance matrix. *Journal of the Acoustical Society of America*, 114(1):235–243, 2003.

[8] R. O. Schmidt. Multiple emitter location and signal parameter estimation. *IEEE Trans. Antennas Propagation.*, 34(3):276–280, 1986.