Diffraction by a Half-Plane with Different Anisotropic Impedance Faces in a Bianisotropic Medium

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Abstract

A general theory to study the electromagnetic diffraction by a half-plane with two different anisotropic impedance faces immersed in a linear homogeneous bianisotropic medium is presented. The problem is formulated in terms of Wiener-Hopf equations that, in general, involve matrices of order four. In the simpler case of materials with constitutive tensors of special form, the Wiener-Hopf matrices reduce to order two and, for these special cases, the possibility to solve the factorization problem in closed form is discussed.

1. Introduction

We consider the frequency-domain diffraction problem of a plane wave impinging on a half-plane immersed in a homogeneous bianisotropic medium (Fig. 1). The *anisotropic* surface-impedance of the upper (a) and lower (b) face of the half-plane are $\boldsymbol{Z}_a$ and $\boldsymbol{Z}_b$, respectively. In the frequency domain, the constitutive relations of the medium in which the half-plane is immersed are [1]

$$\begin{aligned} \boldsymbol{D} &= \varepsilon\, \boldsymbol{E} + \boldsymbol{\xi}\, \boldsymbol{H} \\ \boldsymbol{B} &= \boldsymbol{\mu}\, \boldsymbol{H} + \boldsymbol{\zeta}\, \boldsymbol{E} \end{aligned} \tag{1}$$

where $\boldsymbol{E}$ is the electric field, $\boldsymbol{H}$ the magnetic field, $\boldsymbol{D}$ and $\boldsymbol{B}$ the electric and magnetic flux densities, respectively; $\boldsymbol{\xi}$ and $\boldsymbol{\zeta}$ are cross tensors that relate the magnetic and electric field to the electric and magnetic flux densities, respectively. The incident electromagnetic wave is

$$\begin{bmatrix} \boldsymbol{E}^i(z,x,y) \\ \boldsymbol{H}^i(z,x,y) \end{bmatrix} = E_o \exp(j\omega t)\ \exp\left[-j\left(\alpha_o z + \eta_o x\right) - j\kappa\left(\alpha_o,\eta_o\right) y\right] \begin{bmatrix} \boldsymbol{e}_o(\alpha_o,\eta_o) \\ \boldsymbol{h}_o(\alpha_o,\eta_o) \end{bmatrix} \tag{2}$$

where ω is the angular frequency, t the time, and $\boldsymbol{e}_o(\alpha_o,\eta_o)$, $\boldsymbol{h}_o(\alpha_o,\eta_o)$ the incidence polarization vectors, while α_o, η_o, and $\kappa\left(\alpha_o,\eta_o\right)$ are the vector wavenumber components. The time-dependence factor $\exp(j\omega t)$ is assumed and suppressed throughout the paper whereas, for phase continuity, the z-dependence factor $\exp(-j\alpha_o z)$ is common to all the incident and diffracted field components. The evaluation of $\kappa\left(\alpha_o,\eta_o\right)$ that satisfies the dispersion equation of the medium, of the polarization vectors $\boldsymbol{e}_o(\alpha_o,\eta_o)$ and $\boldsymbol{h}_o(\alpha_o,\eta_o)$, for any given value of α_o and η_o, is discussed in [1].

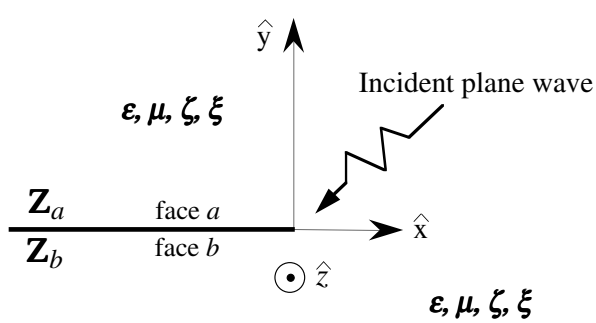

Figure 1: Geometry of the problem.

To formulate the Wiener-Hopf (WH) problem we consider the tangential field components $\boldsymbol{E}_t = \hat{\boldsymbol{z}} E_z + \hat{\boldsymbol{x}} E_x$, $\boldsymbol{H}_t = \hat{\boldsymbol{z}} H_z + \hat{\boldsymbol{x}} H_x$, and introduce the one-dimensional Fourier transforms

$$\begin{aligned} \boldsymbol{V}(\eta, y) &= \exp(j\alpha_0 z) \int_{-\infty}^{+\infty} \hat{\boldsymbol{y}} \times \boldsymbol{E}_t(x, y, z) \exp(j\eta x) \mathrm{d}x \\ \boldsymbol{I}(\eta, y) &= \exp(j\alpha_0 z) \int_{-\infty}^{+\infty} \boldsymbol{H}_t(x, y, z) \exp(j\eta x) \mathrm{d}x \end{aligned} \tag{3}$$

The transverse field equations obtained by using the Bresler-Marcuvitz formalism yield [1]

$$-\frac{\mathrm{d}}{\mathrm{d}y} \begin{bmatrix} \boldsymbol{V} \\ \boldsymbol{I} \end{bmatrix} = \begin{bmatrix} \boldsymbol{T}_e(\eta) & \boldsymbol{Z}(\eta) \\ \boldsymbol{Y}(\eta) & \boldsymbol{T}_h(\eta) \end{bmatrix} \begin{bmatrix} \boldsymbol{V} \\ \boldsymbol{I} \end{bmatrix} = \boldsymbol{P} \begin{bmatrix} \boldsymbol{V} \\ \boldsymbol{I} \end{bmatrix} \tag{4}$$

where $\boldsymbol{P}$ is a known (4×4) polynomial matrix partitioned into four (2×2) polynomial sub-matrices of order two, that is with coefficients given by second-degree polynomials of the variable η. On the upper $(y = 0_+)$ and lower $(y = 0_-)$ face, $\boldsymbol{V}$ and $\boldsymbol{I}$ are related to each other by the two characteristic impedance matrices $\overrightarrow{\boldsymbol{Z}}(\eta)$ and $\overleftarrow{\boldsymbol{Z}}(\eta)$ discussed and evaluated in [1]

$$\begin{aligned} \boldsymbol{V}_a(\eta) &= \boldsymbol{V}(\eta, 0_+) &&= \overrightarrow{\boldsymbol{Z}}(\eta)\, \boldsymbol{I}(\eta, 0_+) &&= \overrightarrow{\boldsymbol{Z}}(\eta)\, \boldsymbol{I}_a(\eta) \\ \boldsymbol{V}_b(\eta) &= \boldsymbol{V}(\eta, 0_-) &&= -\overleftarrow{\boldsymbol{Z}}(\eta)\, \boldsymbol{I}(\eta, 0_-) &&= -\overleftarrow{\boldsymbol{Z}}(\eta)\, \boldsymbol{I}_b(\eta) \end{aligned} \tag{5}$$

By noticing that the tangential components of the electromagnetic fields at $y = 0$ are continuous for all $x > 0$, one can introduce two *plus* functions $\boldsymbol{V}_+(\eta)$ and $\boldsymbol{I}_+(\eta)$ which are regular in the upper half-plane of the complex η variable, to further write

$$\begin{bmatrix} \boldsymbol{V}_a(\eta) \\ \boldsymbol{V}_b(\eta) \end{bmatrix} = \begin{bmatrix} \boldsymbol{V}_+(\eta) + \boldsymbol{V}_{a-}(\eta) \\ \boldsymbol{V}_+(\eta) + \boldsymbol{V}_{b-}(\eta) \end{bmatrix} , \quad \begin{bmatrix} \boldsymbol{I}_a(\eta) \\ \boldsymbol{I}_b(\eta) \end{bmatrix} = \begin{bmatrix} \boldsymbol{I}_+(\eta) + \boldsymbol{I}_{a-}(\eta) \\ \boldsymbol{I}_+(\eta) + \boldsymbol{I}_{b-}(\eta) \end{bmatrix} \tag{6}$$

where $\boldsymbol{V}_{a-}$, $\boldsymbol{V}_{b-}$, $\boldsymbol{I}_{a-}$, and $\boldsymbol{I}_{b-}$ are *minus* functions, regular in the lower half-plane of the complex η variable. The boundary conditions on the impedance half-plane

$$\begin{aligned} \boldsymbol{V}_{a-}(\eta) &= \boldsymbol{Z}_a\, \boldsymbol{I}_{a-}(\eta) \\ \boldsymbol{V}_{b-}(\eta) &= -\boldsymbol{Z}_b\, \boldsymbol{I}_{b-}(\eta) \end{aligned} \tag{7}$$

together with (5) yield to the following functional equations

$$\begin{aligned} \boldsymbol{V}_+(\eta) - \overrightarrow{\boldsymbol{Z}}(\eta)\, \boldsymbol{I}_+(\eta) &= \left(\overrightarrow{\boldsymbol{Z}}(\eta) + \boldsymbol{Z}_a \right) \boldsymbol{I}_{a-} \\ \boldsymbol{V}_+(\eta) + \overleftarrow{\boldsymbol{Z}}(\eta)\, \boldsymbol{I}_+(\eta) &= -\left(\overleftarrow{\boldsymbol{Z}}(\eta) + \boldsymbol{Z}_b \right) \boldsymbol{I}_{b-} \end{aligned} \tag{8}$$

2. The Wiener-Hopf equations of the half-plane problem

After some algebraic manipulation, eqs. (8) yield to the following WH system of order four

$$\boldsymbol{G}(\eta) \begin{bmatrix} \boldsymbol{V}_+(\eta) \\ \boldsymbol{I}_+(\eta) \end{bmatrix} = \begin{bmatrix} \boldsymbol{Z}_a\, \boldsymbol{I}_{a-}(\eta) \\ \boldsymbol{I}_{b-}(\eta) \end{bmatrix} \tag{9}$$

where

$$\boldsymbol{G}(\eta) = \begin{bmatrix} \mathbf{1} & -\overrightarrow{\boldsymbol{Z}}(\eta) \\ \overleftarrow{\boldsymbol{Y}}(\eta) & \mathbf{1} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{1} + \overrightarrow{\boldsymbol{Z}}(\eta)\, \boldsymbol{Y}_a & \mathbf{0} \\ \mathbf{0} & \mathbf{1} + \overleftarrow{\boldsymbol{Y}}(\eta)\, \boldsymbol{Z}_b \end{bmatrix} \tag{10}$$

is the kernel matrix, and with $\overleftarrow{\boldsymbol{Y}}(\eta) = \overleftarrow{\boldsymbol{Z}}^{-1}(\eta)$ and $\boldsymbol{Y}_a = \boldsymbol{Z}_a^{-1}$.

To obtain the non homogeneous form of (9) one has to consider the geometrical optics contributions of the incident field, which permit one to show that the arrays in (9) have a pole at $\eta = \eta_o$, associated with the following residues

$$\mathrm{Res}\left[\begin{array}{c}\boldsymbol{V}_+\\ \boldsymbol{I}_+\end{array}\right]_{\eta=\eta_o} = \boldsymbol{T}_o \;=\; jE_o\left[\begin{array}{c}\hat{\boldsymbol{y}}\times\boldsymbol{e}_o\\ \boldsymbol{h}_o\end{array}\right] \tag{11}$$

$$\mathrm{Res}\left[\begin{array}{c}\boldsymbol{Z}_a\,\boldsymbol{I}_{a-}\\ \boldsymbol{I}_{b-}\end{array}\right]_{\eta=\eta_o} = \boldsymbol{R}_o \;=\; \boldsymbol{G}(\eta_o)\,\boldsymbol{T}_o \tag{12}$$

Thus, the non-homogeneous form of the WH system reads

$$\left[\begin{array}{c}\boldsymbol{V}_+(\eta)\\ \boldsymbol{I}_+(\eta)\end{array}\right] = \boldsymbol{G}_+^{-1}(\eta)\,\boldsymbol{G}_-^{-1}(\eta_o)\,\frac{\boldsymbol{R}_o}{\eta-\eta_o} \tag{13}$$

$$\left[\begin{array}{c}\boldsymbol{Z}_a\,\boldsymbol{I}_{a-}(\eta)\\ \boldsymbol{I}_{b-}(\eta)\end{array}\right] = \boldsymbol{G}(\eta)\left[\begin{array}{c}\boldsymbol{V}_+(\eta)\\ \boldsymbol{I}_+(\eta)\end{array}\right] \tag{14}$$

where $\boldsymbol{G}_+(\eta)$ and $\boldsymbol{G}_-(\eta)$ are obtained by factorizing $\boldsymbol{G}(\eta) = \boldsymbol{G}_-(\eta)\,\boldsymbol{G}_+(\eta)$. An efficient technique to approximately factorize $\boldsymbol{G}(\eta)$ is described in [1, 2].

There are important special cases where $\boldsymbol{G}(\eta)$ simplifies and can be factorized in closed form. This could happen if the $\boldsymbol{T}_e(\eta)$ and $\boldsymbol{T}_h(\eta)$ sub-matrices appearing in (4) vanish [1]. In this case one gets $\overleftarrow{\boldsymbol{Z}}(\eta) = \overrightarrow{\boldsymbol{Z}}(\eta) = \boldsymbol{\gamma}_V\,\boldsymbol{Y}^{-1}(\eta) = \boldsymbol{\gamma}_V^{-1}\,\boldsymbol{Z}(\eta)$, with $\boldsymbol{\gamma}_V = \sqrt{\boldsymbol{Z}(\eta)\boldsymbol{Y}(\eta)}$ (see [1]). For example, for $\boldsymbol{\varepsilon} = \varepsilon_z\hat{\boldsymbol{z}}\hat{\boldsymbol{z}} + \varepsilon_x\hat{\boldsymbol{x}}\hat{\boldsymbol{x}} + \varepsilon_y\hat{\boldsymbol{y}}\hat{\boldsymbol{y}}$ and $\boldsymbol{\mu} = \mu_z\hat{\boldsymbol{z}}\hat{\boldsymbol{z}} + \mu_x\hat{\boldsymbol{x}}\hat{\boldsymbol{x}} + \mu_y\hat{\boldsymbol{y}}\hat{\boldsymbol{y}}$ diagonal (so that $\boldsymbol{T}_e(\eta)$ and $\boldsymbol{T}_h(\eta)$ vanish), the matrices $\boldsymbol{Z}(\eta)$, $\boldsymbol{Y}(\eta)$ and $\boldsymbol{\gamma}_V$ are diagonal at normal incidence ($\alpha = 0$); in this case, the factorization problem reduces to the factorization of two scalar quantities whenever the half-plane impedance matrices $\boldsymbol{Z}_a$ and $\boldsymbol{Z}_b$ are equal and diagonal. Exact factorization at skew incidence ($\alpha \neq 0$) is possible for isotropic (i.e., scalar) half-plane impedances $\boldsymbol{Z}_a = \boldsymbol{Z}_b$ in case of diagonal $\boldsymbol{\varepsilon}$ and $\boldsymbol{\mu}$, with $\varepsilon_z = \varepsilon_x$, $\mu_z = \mu_x$.

3. Acknowledgments

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4. References

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