A general theory to study the electromagnetic diffraction by a half-plane with two different anisotropic impedance faces immersed in a linear homogeneous bianisotropic medium is presented. The problem is formulated in terms of Wiener-Hopf equations that, in general, involve matrices of order four. In the simpler case of materials with constitutive tensors of special form, the Wiener-Hopf matrices reduce to order two and, for these special cases, the possibility to solve the factorization problem in closed form is discussed.

1. Introduction

We consider the frequency-domain diffraction problem of a plane wave impinging on a half-plane immersed in a homogeneous bianisotropic medium (Fig. 1). The anisotropic surface-impedance of the upper (a) and lower (b) face of the half-plane are \( Z_a \) and \( Z_b \), respectively. In the frequency domain, the constitutive relations of the medium in which the half-plane is immersed are 

\[
D = \varepsilon E + \xi H \\
B = \mu H + \zeta E
\]

where \( E \) is the electric field, \( H \) the magnetic field, \( D \) and \( B \) the electric and magnetic flux densities, respectively; \( \xi \) and \( \zeta \) are cross tensors that relate the magnetic and electric field to the electric and magnetic flux densities, respectively. The incident electromagnetic wave is

\[
\begin{bmatrix}
E'(z, x, y) \\
H'(z, x, y)
\end{bmatrix} = E_o \exp(j\omega t) \exp[-j(\alpha_o z + \eta_o x) - j\kappa(\alpha_o, \eta_o) y] \begin{bmatrix}
e_o(\alpha_o, \eta_o) \\
h_o(\alpha_o, \eta_o)
\end{bmatrix}
\]

where \( \omega \) is the angular frequency, \( t \) the time, and \( e_o(\alpha_o, \eta_o), h_o(\alpha_o, \eta_o) \) the incidence polarization vectors, while \( \alpha_o, \eta_o, \) and \( \kappa(\alpha_o, \eta_o) \) are the vector wavenumber components. The time-dependence factor \( \exp(j\omega t) \) is assumed and suppressed throughout the paper whereas, for phase continuity, the \( z \)-dependence factor \( \exp(-j\alpha_o z) \) is common to all the incident and diffracted field components. The evaluation of \( \kappa(\alpha_o, \eta_o) \) that satisfies the dispersion equation of the medium, of the polarization vectors \( e_o(\alpha_o, \eta_o) \) and \( h_o(\alpha_o, \eta_o) \), for any given value of \( \alpha_o \) and \( \eta_o \), is discussed in [1].
To formulate the Wiener-Hopf (WH) problem we consider the tangential field components \( \mathbf{E}_t = \hat{z} E_z + \hat{x} E_x, \mathbf{H}_t = \hat{z} H_z + \hat{x} H_x \), and introduce the one-dimensional Fourier transforms

\[
\mathbf{V}(\eta, y) = \exp(j\alpha_0 z) \int_{-\infty}^{+\infty} \mathbf{E}_t(x, y, z) \exp(j\eta x) dx
\]

\[
\mathbf{I}(\eta, y) = \exp(j\alpha_0 z) \int_{-\infty}^{+\infty} \mathbf{H}_t(x, y, z) \exp(j\eta x) dx
\]

The transverse field equations obtained by using the Bresler-Marcuvitz formalism yield [1]

\[
-\frac{d}{dy} \begin{bmatrix} \mathbf{V} \\ \mathbf{I} \end{bmatrix} = \begin{bmatrix} \mathbf{T}_e(\eta) & \mathbf{Z}(\eta) \\ \mathbf{Y}(\eta) & \mathbf{T}_h(\eta) \end{bmatrix} \begin{bmatrix} \mathbf{V} \\ \mathbf{I} \end{bmatrix} = \mathbf{P} \begin{bmatrix} \mathbf{V} \\ \mathbf{I} \end{bmatrix}
\]

where \( \mathbf{P} \) is a known \((4 \times 4)\) polynomial matrix partitioned into four \((2 \times 2)\) polynomial sub-matrices of order two, that is with coefficients given by second-degree polynomials of the variable \( \eta \). On the upper \((y = 0_+)\) and lower \((y = 0_-)\) face, \( \mathbf{V} \) and \( \mathbf{I} \) are related to each other by the two characteristic impedance matrices \( \mathbf{Z}(\eta) \) and \( \mathbf{Z}(\eta) \) discussed and evaluated in [1]

\[
\mathbf{V}_a(\eta) = \mathbf{V}(\eta, 0_+) = \mathbf{Z}(\eta) \mathbf{I}(\eta, 0_+) = \mathbf{Z}(\eta) \mathbf{I}_a(\eta)
\]

\[
\mathbf{V}_b(\eta) = \mathbf{V}(\eta, 0_-) = -\mathbf{Z}(\eta) \mathbf{I}(\eta, 0_-) = -\mathbf{Z}(\eta) \mathbf{I}_b(\eta)
\]

By noticing that the tangential components of the electromagnetic fields at \( y = 0 \) are continuous for all \( x > 0 \), one can introduce two plus functions \( \mathbf{V}_+ (\eta) \) and \( \mathbf{I}_+ (\eta) \) which are regular in the upper half-plane of the complex \( \eta \) variable, to further write

\[
\begin{bmatrix} \mathbf{V}_a(\eta) \\ \mathbf{V}_b(\eta) \end{bmatrix} = \begin{bmatrix} \mathbf{V}_+(\eta) + \mathbf{V}_-(\eta) \\ \mathbf{V}_+(\eta) + \mathbf{V}_b-(\eta) \end{bmatrix}, \quad \begin{bmatrix} \mathbf{I}_a(\eta) \\ \mathbf{I}_b(\eta) \end{bmatrix} = \begin{bmatrix} \mathbf{I}_+(\eta) + \mathbf{I}_a-(\eta) \\ \mathbf{I}_+(\eta) + \mathbf{I}_b-(\eta) \end{bmatrix}
\]

where \( \mathbf{V}_a-, \mathbf{V}_b-, \mathbf{I}_a-, \) and \( \mathbf{I}_b- \) are minus functions, regular in the lower half-plane of the complex \( \eta \) variable. The boundary conditions on the impedance half-plane

\[
\mathbf{V}_a-(\eta) = \mathbf{Z}_a \mathbf{I}_a-(\eta)
\]

\[
\mathbf{V}_b-(\eta) = -\mathbf{Z}_b \mathbf{I}_b-(\eta)
\]

Together with (5) yield to the following functional equations

\[
\mathbf{V}_+(\eta) - \mathbf{Z}(\eta) \mathbf{I}_+(\eta) = \left( \mathbf{Z}(\eta) + \mathbf{Z}_a \right) \mathbf{I}_a-
\]

\[
\mathbf{V}_+(\eta) + \mathbf{Z}(\eta) \mathbf{I}_+(\eta) = -\left( \mathbf{Z}(\eta) + \mathbf{Z}_b \right) \mathbf{I}_b-
\]

### 2. The Wiener-Hopf equations of the half-plane problem

After some algebraic manipulation, eqs. (8) yield to the following WH system of order four

\[
\mathbf{G}(\eta) \begin{bmatrix} \mathbf{V}_+(\eta) \\ \mathbf{I}_+(\eta) \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_a \mathbf{I}_a-(\eta) \\ \mathbf{I}_b-(\eta) \end{bmatrix}
\]

where

\[
\mathbf{G}(\eta) = \begin{bmatrix} 1 & -\mathbf{Z}(\eta) \\ \mathbf{Y}(\eta) & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 + \mathbf{Z}(\eta) \mathbf{Y}_a & 0 \\ 0 & 1 + \mathbf{Y}(\eta) \mathbf{Z}_b \end{bmatrix}
\]

is the kernel matrix, and with \( \mathbf{Y}(\eta) = \mathbf{Z}^{-1}(\eta) \) and \( \mathbf{Y}_a = \mathbf{Z}_a^{-1} \).
To obtain the non homogeneous form of (9) one has to consider the geometrical optics contributions of the incident field, which permit one to show that the arrays in (9) have a pole at $\eta = \eta_0$, associated with the following residues

$$\text{Res} \begin{bmatrix} V_+ \\ I_+ \end{bmatrix}_{\eta = \eta_0} = T_o = jE_o \begin{bmatrix} \hat{y} \times e_o \\ h_o \end{bmatrix}$$ (11)

$$\text{Res} \begin{bmatrix} Z_a I_a- \\ I_b- \end{bmatrix}_{\eta = \eta_0} = R_o = G(\eta_0) T_o$$ (12)

Thus, the non-homogeneous form of the WH system reads

$$\begin{bmatrix} V_+(\eta) \\ I_+(\eta) \end{bmatrix} = G_+^{-1}(\eta) G_-^{-1}(\eta_0) \frac{R_o}{\eta - \eta_0}$$ (13)

$$\begin{bmatrix} Z_a I_a-(\eta) \\ I_b-(\eta) \end{bmatrix} = G(\eta) \begin{bmatrix} V_+(\eta) \\ I_+(\eta) \end{bmatrix}$$ (14)

where $G_+(\eta)$ and $G_-(\eta)$ are obtained by factorizing $G(\eta) = G_-(\eta) G_+(\eta)$. An efficient technique to approximately factorize $G(\eta)$ is described in [1, 2].

There are important special cases where $G(\eta)$ simplifies and can be factorized in closed form. This could happen if the $T_e(\eta)$ and $T_h(\eta)$ sub-matrices appearing in (4) vanish [1]. In this case one gets $\tilde{Z}(\eta) = \tilde{Z}(\eta) = \gamma_V Y^{-1}(\eta) = \gamma_V^{-1} Z(\eta)$, with $\gamma_V = \sqrt{Z(\eta) Y(\eta)}$ (see [1]). For example, for $\varepsilon = \varepsilon_z \hat{z} \hat{z} + \varepsilon_x \hat{x} \hat{x} + \varepsilon_y \hat{y} \hat{y}$ and $\mu = \mu_z \hat{z} \hat{z} + \mu_x \hat{x} \hat{x} + \mu_y \hat{y} \hat{y}$ diagonal (so that $T_e(\eta)$ and $T_h(\eta)$ vanish), the matrices $Z(\eta), Y(\eta)$ and $\gamma_V$ are diagonal at normal incidence ($\alpha = 0$); in this case, the factorization problem reduces to the factorization of two scalar quantities whenever the half-plane impedance matrices $Z_a$ and $Z_b$ are equal and diagonal. Exact factorization at skew incidence ($\alpha \neq 0$) is possible for isotropic (i.e., scalar) half-plane impedances $Z_a = Z_b$ in case of diagonal $\varepsilon$ and $\mu$, with $\varepsilon_z = \varepsilon_x, \mu_z = \mu_x$.

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4. References
