A Ray Tracing Algorithm for Multiple Straight Wedge Diffraction

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Abstract

An efficient algorithm for the tracing of multiply edge diffracted rays is presented. The algorithm assumes a given sequence of infinite edges and complete visibility among them; then, the ray tracing problem is formulated as the minimization of the ray total path length. Since such a cost function is strictly convex, except for coplanar edges in the plane-wave far-field regime, then the problem admits a unique global minimum and allows the use of the Newton (or Quasi Newton/Trust Region) search algorithm, that exhibits a very high converging rate. We also propose an *ad-hoc* modified Newton's method for the specific problem and a convenient starting point to effectively initialize the minimization algorithm.

1. Introduction

Ray theories, like the Geometrical Theory of Diffraction (GTD) and its uniform version (UTD) [1], are a very useful approach to characterize the scattering from objects and to estimate the electromagnetic field in arbitrary complex environments. They permit to model propagation and antenna performances or coupling on complex platforms like vehicles, airplanes, ships or satellites [2]. They have been also applied to characterize urban propagation [3], [4], indoor wireless network channels and Multiple Input Multiple Output (MIMO) communication channels [5]. In such theories, the electromagnetic field is described in terms of rays arising from a source, propagating through the scenario and interacting with it. These rays can be reflected, refracted (Geometrical Optics (GO) rays) or diffracted by the objects around the source. When the environment is complex, considering only the (singly) wedge diffracted rays is usually inadequate to reach the desired accuracy, especially when observing in low-field shadow regions. Therefore it is necessary to introduce higher order diffraction mechanisms that consist in vertex and multiple edge diffracted rays. The latter are the topic of the present paper. A crucial step in the application of ray theories, is the ray tracing operation. When the scenario is complex, the ray tracing can become computationally very heavy, especially if high interaction orders are considered. Conversely, once the rays are traced, the field estimation is computationally very fast, because UTD formulas involve closed form expressions. Once that the geometry of the scenario and the location of the source and observation points are known, the ray tracing operation for multiple diffracted rays consists in finding hit point(s) on the edges of the scatterers, where the scattering phenomenon is localized at high frequency. The hit point(s) are found by imposing the Keller's diffraction law at each edge. The ray path is then constructed by joining the source, the hit point(s), and finally the observation point. Indeed, in a homogenous background medium, like air, the propagation occurs along straight rays. The ray tracing on arbitrary geometries often requires a numerical hit point searching [6], which is much more time consuming. In this paper, the ray tracing problem for multiple diffraction by N edges is formulated as an N-dimensional minimization problem, whose solution is strictly related to the behavior of the cost function to be minimized, i.e., the total path length, according to the generalized Fermat's principle. Since this functional is strictly convex, except for consecutive coplanar edges in the plane-wave far-field regime, where a caustic occurs, the problem admits a unique global minimum and allows the use of the Newton (or Quasi-Newton/Trust Region) solution algorithm [7], that exhibits a very high convergence rate.

2. Formulation

Let us consider N straight edges in arbitrary configuration, tagged by $i = 1,...,N$ (Fig. 1). Some edges may coincide, except for consecutive edges. The N-ple of edges is therefore ordered. A multiple diffraction among the same edges but in a different order must be considered by re-tagging the edges as a separate problem. Therefore the complete description of the N edges scenario involves the statement of the same problem for all the possible edges sequences. We suppose infinite-length edges and a complete visibility between any pair of edges as in [6]. An a posteriori check will verify whether the founded ray is present or not because shadowed or because its diffraction points lie outside the actual finite edges. A point source at P' radiates in this environment.

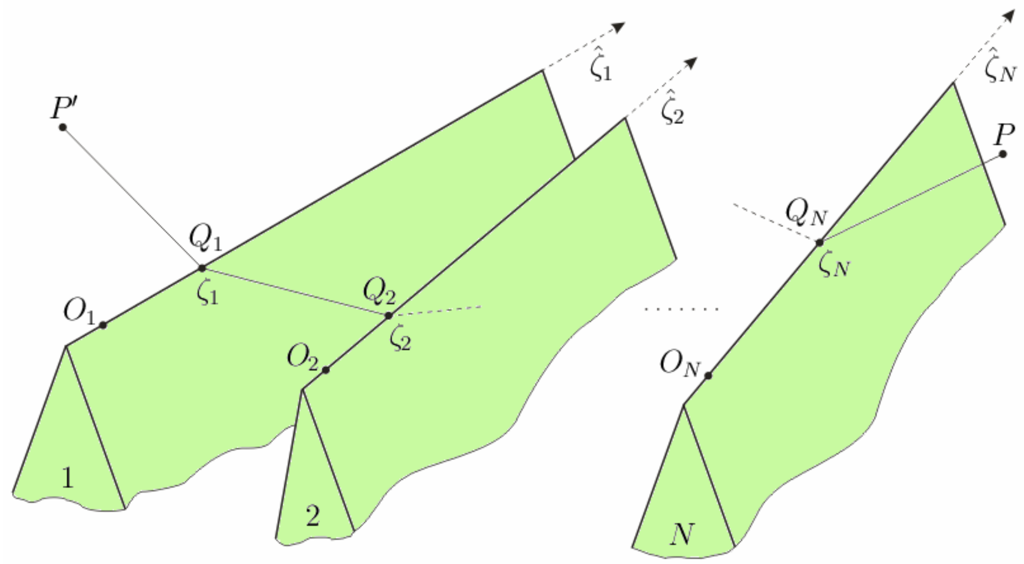

Figure 1. N -ple wedge diffraction geometry. P' and P are the source, where the ray arises, and the observation point, where it arrives, respectively.

Our purpose is to find the ray reaching the observation point P after experiencing N consecutive diffractions by the edges, in the prescribed order. Each i -th edge position and direction are specified by the location of a point O_i on it and by a unit vector $\hat{\zeta}_i$, respectively. A reference axis ζ_i is defined along the edge with origin at O_i, oriented as $\hat{\zeta}_i$. Any point Q_i, on the i -th edge, is parameterized by $Q_i = O_i + \zeta_i \hat{\zeta}_i$. Let us consider the distance between the point source P' and a generic point Q_1 belonging to the first edge, the distance between the latter point Q_1 and a generic point Q_2 on the second edge, and so on the distance between the generic point Q_i belonging to the i -th edge and the generic point Q_{i+1} belonging to the $i+1$ -th edge, and finally the distance between the last point Q_N on the N -th edge and the observation point P. If we introduce the vector $\underline{\zeta} = [\zeta_1, \zeta_2, \ldots, \zeta_N] \in \mathbb{R}^N$, the sum of all the above distances, i.e. the total length of the path, defines the cost function $L(\underline{\zeta}) : \mathbb{R}^N \to \mathbb{R}$,

$$L(\underline{\zeta}) = |Q_1 - P'| + \sum_{i=1}^{N-1} |Q_{i+1} - Q_i| + |P - Q_N|. \tag{1}$$

The stationary points of the cost function $L(\underline{\zeta})$ individuate the diffraction points along the edges, and the value of the cost function at those points correspond to the length of multiple diffracted ray paths, as dictated by the generalized Fermat's principle. In [6], the cost function (1) is demonstrated to be strictly convex, except for the case of plane wave illumination and far field observation. Since the cost function is strictly convex, then it admits a unique stationary point that is a global minimum, corresponding to the presence of only one multiple diffracted ray. This property greatly helps its searching strategy. The gradient and the Hessian matrix of $L(\underline{\zeta})$ can be analytically calculated from (1). They are always continuous in $\mathbb{R}^N$, except when two consecutive edges intersect; i.e., when $Q_i \equiv Q_{i+1}$. In this situation, the gradient components are discontinuous and the relevant Hessian terms are singular. For nonintersecting consecutive edges, $L(\underline{\zeta})$ exhibits a continuous gradient and a positive definite nonsingular Hessian matrix. These hypotheses permit to state that finding the global minimum point $\underline{\zeta}^*$ is equivalent to satisfy the condition $\nabla L(\underline{\zeta}^*) = 0$. This relation leads to solve a nonlinear system of equation, that in general does not admit a closed form solution. However, if some consecutive edges are intersecting, the global minimum point may coincide with an intersection point where the gradient and the Hessian matrix can not be defined. In this case it is not possible to use the relation $\nabla L(\underline{\zeta}^*) = 0$. Therefore, it is preferable to develop a general algorithm that can solve any arbitrary edge arrangement with the same strategy. This is achieved by minimizing the cost function with a numerical method.

3. Minimization algorithm

To minimize the cost function of our problem we implement a modified Newton's algorithm. Newton's methods are based on the minimization of a quadratic approximation of the function that have to be minimized; i.e., at the k-th iteration the descent direction is evaluated minimizing the quadratic approximation of the function. This result in solving a linear system whose matrix is the Hessian matrix of the function. These methods present a high convergence rate. It is necessary to pay attention to the cases when they are not applicable; i.e., when the Hessian matrix is singular or ill conditioned and the Newton's direction is not definable. In our algorithm we use the steepest descent (Gradient Method) direction when this situation is encountered. To enforce global convergence, the step size may be reduced by a parameter $\alpha \in [0,1]$ on the basis of a backtracking line search approach in order to satisfy the first Wolfe (or Armijo) condition [7]. The algorithm stops when the norm of the step between the k-th and $k-1$-th iterations is less than a tolerance ε_ζ; i.e., when $\left|\underline{\zeta}_k - \underline{\zeta}_{k-1}\right| < \varepsilon_\zeta$ and $\underline{\zeta}_k$ becomes the numerical estimate of the minimum point $\underline{\zeta}^*$.

4. Initial guess choice

Any iterative method needs a starting point. A good starting point is crucial to solve the problem by few iterations, especially when a fast local convergence is available. Therefore we developed the following strategy for the initial guess choice. We consider the new cost function

$$\tilde{L}(\underline{\zeta}) = |Q_1 - P'|^2 + \sum_{i=1}^{N-1} |Q_{i+1} - Q_i|^2 + |P - Q_N|^2, \tag{2}$$

i.e., the sum of the squares of the ray section lengths. By comparing (1) to (2), it is apparent that $\tilde{L}(\underline{\zeta})$ is strictly related to $L(\underline{\zeta})$ but, unlike $L(\underline{\zeta})$, it admits a minimum point $\underline{\zeta}_0$ that can be evaluated analytically in closed form. In most cases, $\underline{\zeta}_0$ is close to $\underline{\zeta}^*$; i.e., the minimum point of the functional $L(\underline{\zeta})$, because of the similarity between $L(\underline{\zeta})$ and $\tilde{L}(\underline{\zeta})$. Therefore we found very convenient to assume $\underline{\zeta}_0$ as starting point to initialize the iterative method for the minimization of $L(\underline{\zeta})$. The advantages given by the definition of the new cost function $\tilde{L}(\underline{\zeta})$ come from the fact that it is exactly a convex quadric in $\mathbb{R}^N$, whose unique global minimum point $\underline{\zeta}_0$ can be evaluated by solving the linear equation system that results from the condition $\nabla\tilde{L}(\underline{\zeta}_0) = 0$; i.e., $\underline{\zeta}_0 = -H_{\tilde{L}}^{-1}\nabla\tilde{L}(0)$, where $H_{\tilde{L}}$ is the Hessian matrix of $\tilde{L}(\underline{\zeta})$. For the quadric function $\tilde{L}(\underline{\zeta})$, the solution $\underline{\zeta}_0$ is found exactly in one Newton step.

5. Numerical Example

In this section, to test the accuracy and the efficiency of our formulation, we present an example inspired by an urban scenario. Other examples will be shown during the presentation. We denote with A_i, B_i, $i = 1,\ldots,N$, the edge end-points, whose coordinates are listed in Table 1. Edge origins O_i are chosen at the first points A_i and edge direction unit vectors are $\hat{\zeta}_i = (B_i - A_i)/|B_i - A_i|$. The source and the observation points are located at $P' = (0.5, 5, 4.5)$ and $P = (3, 2, 0.5)$, respectively. Note that the units of all coordinates and lengths are arbitrary (meters, wavelengths, or others). The test scenario is also drawn in Fig. 2 where edges belong to plates whose exact position and dimensions do not affect the ray tracing procedure. The edges involved in the ray tracing are drawn in thick line style to distinguish them from all other plate edges. The diffraction points Q_i^* are calculated by using the presented algorithm. If we assume that the arbitrary unit in the above examples was of the same order of the wavelength, we may set $\varepsilon_\zeta = 10^{-4}$ to achieve a very good accuracy. Such a choice requires only 3 iterations. The incidence and diffraction angles β_i' and β_i [1] at each i-th edge are also reported to check the angular accuracy with which the Keller's diffraction law is satisfied. The values coincides for the first 6 significant digits and therefore they are reported only once in Table 1.

This correspond to an angular error lower than 10^{-4} degrees. To show how close the initial guess is to the final solution, both the respective rays are drawn in the scenario pictures in Fig. 2 as a thin dash dotted and a thick dashed line, respectively.

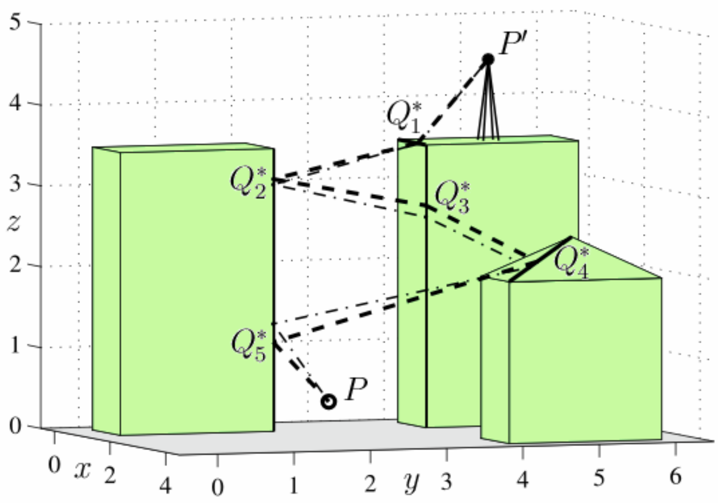

Figure 2. Example geometry: 5-ple edge diffraction in an urban scenario. Initial guess ray path (dash dotted thin line) and multiply diffracted ray path (dashed thick line).

Table 1

i	A_i	B_i	Q_i^*	$\beta_i' = \beta_i$ (rad)
1	$(0,4,3.5)$	$(1,4,3.5)$	$(0.70499,4.00000,3.50000)$	1.42685
2	$(1,2,3.5)$	$(1,2,0)$	$(1.00000,2.00000,3.12277)$	1.38632
3	$(1,4,3.5)$	$(1,4,0)$	$(1.00000,4.00000,2.74958)$	1.38632
4	$(3.5,5,2.5)$	$(4,4,2)$	$(3.77498,4.45003,2.22502)$	1.22118
5	$(1,2,3.5)$	$(1,2,0)$	$(1.00000,2.00000,1.10508)$	1.27701

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