Overview of Theory and Applications of Epsilon-Near-Zero Materials

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Abstract

Materials with unconventional values of permittivity and or permeability have been the object of intense research in the last few years. For example, artificial materials with simultaneously negative permittivity and permeability have received significant attention due to their potential breakthroughs in subwavelength imaging and miniaturization of several devices. Recently, our research group has been particularly interested in the study of a different class of materials characterized by a 'low' permittivity as compared to the permittivity of free-space. Our studies reveal that these epsilon-near-zero (ENZ) materials may enable remarkable phenomena and effects. Additionally, they may have interesting applications in a variety of seemingly diverse problems, like squeezing electromagnetic energy through very narrow channels, design of matched zero-index materials, and shaping the radiation pattern of a source. In this talk, we will describe the results of our on-going research in these fields and forecast some exciting future research directions.

1. Introduction

The advent of metamaterials created the opportunity for designing new materials with a tailored electromagnetic response and unusual values of permittivity and permeability. Recently, the class of materials with effective permittivity and/or permeability near zero has received significant attention. Materials with epsilon near zero (ENZ) may be directly found in nature. A well-known example is an electron gas, which, due to the conduction current created by the drift of free electric charges, may effectively interact with radiation as continuous medium characterized by a Drude-type dispersion model, which near its plasma frequency has ε near zero. As is well documented, the ionosphere layer of the Earth atmosphere exhibits such properties around 30MHz-40MHz (HF). Similarly, at infrared and optical frequencies some low loss noble metals, like Ag and Au [1], some semiconductors, e.g. indium antimonide [2], and some polar dielectrics like silicon carbide (SiC) [3] may behave as ENZ materials near their plasmas frequencies. It is also expected that such ENZ materials may be fabricated at a desired frequency as metamaterials by embedding suitable inclusions in a host medium.

Our recent studies have revealed that the ENZ materials may enable some counterintuitive phenomena and effects, and have interesting potentials in several, seemingly uncorrelated, problems. In this work we will describe an overview of the theory and applications of this class of materials.

2. Supercoupling Effect

Some of the interesting applications of ENZ materials are related to the tunneling effect originally investigated in [4]-[5]. In these works we have analyzed the general problem in which two parallel-plate waveguides are connected using an intermediate arbitrary channel of arbitrary shape and length, as illustrated in Fig. 1. Intuitively, one expects that if this intermediate channel has either a very irregular geometry (e.g. if it has many bends) or if its cross-section is very narrow and tight, it will not be possible to couple a significant amount of electromagnetic energy from the input waveguide to the output waveguide. This is indeed what happens when the intermediate channel is filled with the same dielectric material as the other two waveguides. However, something unexpected happens if the narrow channel is filled with an ENZ material. In a counterintuitive manner, and despite the huge wave impedance mismatch between the ENZ material and the dielectrics, it turns out that in such scenario the wave may tunnel through the narrow channel and there may be an almost perfect coupling between two waveguides. Moreover, surprisingly, in the case of a low loss ENZ material, it is possible to squeeze increasing energy through the narrow channel by decreasing its transverse cross section! We have named this phenomenon as "supercoupling" effect in [5]. Following the exact theory described in [4], it is possible to quantify the exact amount of reflected energy in the ENZ regime. Under TEM wave incidence, the reflection coefficient is given by the formula,

$$\rho = \frac{(a_1 - a_2) + i\,k_0 \mu_{r,p}\, A_p}{(a_1 + a_2) - i\,k_0 \mu_{r,p}\, A_p} \tag{1}$$

where $k_0 = \omega / c$ is the free-space wave number, $\mu_{r,p}$ is the relative permeability of the ENZ material, and A_p is the total area of the transverse cross-section of channel. The above formula is exact in the ε=0 limit and for perfectly conducting plates, and applies independent of the geometry of the channel. It shows that when the two waveguides have the same cross-section, $a \equiv a_1 = a_2$, the reflection coefficient can be made arbitrarily small by making the channel more and more tight. It was analytically demonstrated in [5] that the effect of metallic losses on the waveguide walls is typically negligible, and that the dielectric loss in the ENZ material is in general the dominant loss mechanism. The described theory was generalized in [6] to fully three-dimensional coaxial metallic channels filled with an ENZ material.

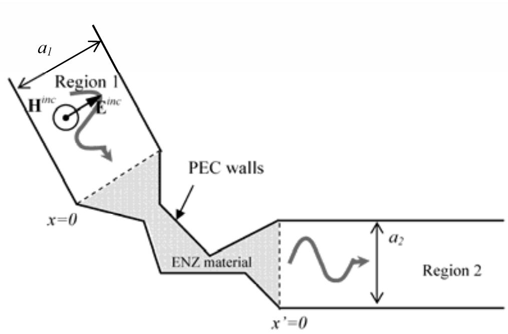

Fig. 1. (From reference [4]) Two parallel-plate waveguides are connected by an intermediate narrow channel filled with an ENZ material. The incoming wave is the fundamental TEM waveguide mode.

An important consequence of the described tunneling effect follows from the conservation of power flow and implies that the electric field inside the ENZ material may become several folds larger than the electric field associated with the incoming wave. As demonstrated in [5], this may enable the possibility of concentrating and confining the electromagnetic fields in a very subwavelength air cavity inside the ENZ material with enormous electric field enhancement. Other potential applications that we are currently studying explore the use of these enhanced fields in sensing and switching.

The reported supercoupling and tunnelling effect has been recently experimentally demonstrated by our group at microwaves using a metallic waveguide setup, in which two wide waveguides are connected through an ultranarrow channel [7]. The ultranarrow channel is operated at the cut-off frequency in order to mimic the zero-permittivity properties. It was shown that, consistent with the theory of [4]-[5], the tunneling effect depends very weakly on the specific geometry of the narrow channel and it enables an almost perfect coupling between the two wide waveguides, with almost uniform phase along the narrow channel. In [8] we used a similar experimental setup to demonstrate that sharp bends may be inserted within the propagation channel without causing any sensible reflection or loss. Analogous results have been reported independently in [9]-[10], using a screen patterned with complementary split-ring resonators to create the ENZ material properties.

3. Design of Matched Zero-Index Metamaterials

Another potential application of ENZ materials is the design of structured matched zero-index media (ZIM), with both ε and μ near zero at the frequency of interest. Besides of being of conceptual and academic interest [11], such materials may have key applications in the realization of delay lines, in increasing the directivity of an antenna, in transforming curved wavefronts into planar ones (as described in the following section), and in enhancing the efficiency of some waveguides with abrupt junctions or bends.

Matched ZIM are not directly found in nature, and thus they have to be synthesized as metamaterials. Since such materials evidently have both an electric and magnetic response it could be thought, in analogy with media with simultaneously negative permittivity and permeability, that their realization would require very complex resonant inclusions suitably arranged in a regular lattice. In a recent work [12], we have theoretically demonstrated that the design of such materials is in principle much simpler than that. It was rigorously proven, that the response of a matched ZIM can be perfectly mimicked by a continuous ENZ material loaded with suitable non-magnetic inclusions.

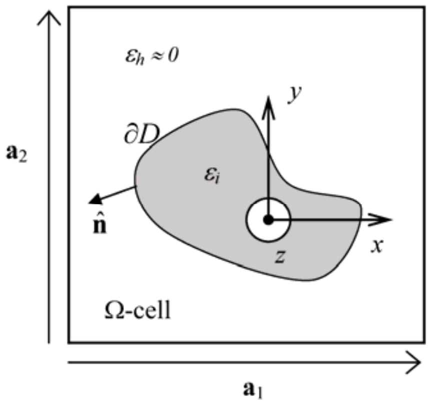

Fig. 2. (From reference [12])Geometry of the unit cell-Ω of a periodic non-magnetic crystal in which the host medium has permittivity ε_h near zero. The inclusion in the unit cell is a regular dielectric with permittivity ε_i.

Even though such composite ZIM does not need to be periodic, it is convenient to think that it is formed by a regular arrangement of dielectric particles with unit cell as depicted in Fig. 2. Notice that the dielectric particles are embedded in an ENZ material. Our theoretical analysis shows that the considered composite medium has two key characteristics at the frequency where the permittivity of the host material vanishes: *(i)* regardless of the geometry and of the permittivity of the inclusions, the effective permittivity of the composite structure remains equal to zero. *(ii)* due to the extremely long wavelength in the ENZ material there is no coupling or interaction between the different dielectric particles; furthermore, each inclusion behaves as lumped circuit element characterized by a certain internal impedance $\overline{Z}_{\text{int}}$, which only depends on its specific geometry and on its material parameters. The effective permeability of the composite structure is given by the exact result,

$$\frac{\mu_{eff}}{\mu_0} = 1 - f_V + \frac{\overline{Z}_{\text{int}}}{-i\omega\mu_0} \tag{2}$$

where f_V is the volume fraction of the inclusions. Thus, to obtain a matched ZIM all that is required is to design the inclusions with internal impedance $\overline{Z}_{\text{int}}$ such that μ_{eff} given by (2) vanishes.

Such homogenization results are not restricted to the case in which the inclusions are electrically small in terms of free-space wavelengths. In fact, independent of their actual physical size, the inclusions always behave as lumped discrete elements. This happens because the wavelength in the host medium is very large and thus, regardless of the physical size of the particles, their dimensions are always much smaller than the wavelength in the host medium.

A remarkable result obtained in [12] is that in some complex waveguide scenarios the response of a finite sized block of the ZIM metamaterial cannot be distinguished from the response of the equivalent continuous material characterized by $\varepsilon_{eff} = 0$ and μ_{eff} given by (2). This means that an incoming wave cannot detect by any means the actual granularity of the composite ZIM metamaterial. Furthermore, the response of the composite material is insensitive to disorder effects, i.e. the inclusions do not need to be arranged in a regular lattice.

3. Shaping the Radiation Pattern of a Source

Perhaps the pioneering application of low permittivity materials was the attempt to use such materials to enhance the radiation directivity. This possibility stems from the low wave number propagation characteristics of such materials, which implies that the phase variation of the electromagnetic fields may be very small over a physically long distance. In particular such important property suggests that the wavefronts in ENZ materials, and more generally in ZIM, are parallel to the interfaces, providing the possibility for directive radiation towards the broadside to a planar interface, as proposed for several antenna applications [13]. In a recent work [14], we investigated in detail the behavior of ENZ materials in the presence of electromagnetic sources, and the possibility of manipulating the phase fronts of such sources for obtaining anomalous imaging, lensing and radiative effects. We have demonstrated that by controlling the shape of the interfaces of the ENZ material it is possible to engineer the phase radiation pattern at will, to transform an incoming planar wavefront into a convergent cylindrical wavefront and vice-versa, or to transform and concave wavefront into a convex wavefront. These effects may have interesting applications in several fields.

4. References

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