Tensor Decomposition of MIMO Channel Sounding Measurements and its Applications

(*Invited Paper*)

Andreas Richter, Jussi Salmi, and Visa Koivunen
TKK Helsinki University of Technology, SMARAD Centre of Excellence
Signal Processing Laboratory
PO.Box 3000, FI-02015 TKK, Finland
E-mail: givename.surename@tkk.fi

Abstract

This paper links double-directional (MIMO) measurements to tensor analysis techniques. Tensor decompositions of three- and higher way arrays like PARAFAC (Parallel Factor Model), HOSVD (Higher Order SVD), or PACA (PARAFAC/CANDECOMP) are employed in psychometrics and chemometrics for a long time. These techniques have been developed mainly for analyzing the first moment of a tensor valued random process. In this paper, proper models and related decompositions for the first and second moment of MIMO channel sounding measurements are proposed. The proposed techniques can be employed for, e.g, filtering of MIMO channel sounding data to reduce measurement noise, which is of interest in realistic link-level simulations of (MIMO) transceiver structures using measured radio channels.

Keywords: Multi-way Arrays, Tensor, MIMO, Channel Sounding, Tensor Filtering, Link-Level Simulations

1 Introduction

About ten years ago, the concept of double-directional (MIMO) wideband radio channel measurements (see [1] and the references therein) was introduced. A wideband MIMO channel measurement can be interpreted as an observation of a 3-way tensor-valued random process. The three dimensions of the tensor correspond to the transmit antenna array ports, frequency-domain or time-delay-domain samples, and the receive antenna array ports. Although this fact is generally accepted in the research community, algorithms developed for tensor-analysis have, to the best of knowledge of the authors, not been employed for the analysis of such radio channel measurements. This is mainly due to the fact that most of the effort in wideband MIMO radio channel analysis was focused on the estimation of (concentrated) propagation path based radio channel models (PPCM). It is interesting to note, that the PPCM can be interpreted as a PARAFAC [2] model, with constraint manifolds (e.g. antenna array manifolds) for the factors in the three tensor dimensions.

Now suppose one is interested in filtering double-directional (MIMO) wideband radio channel measurements as a preprocessing step for, e.g. realistic link-level simulations. Then a restriction to this constraint manifolds may not be of interest, since it requires the use of special antenna arrays at the transmitter and the receiver, to ensure identifiability. It will be shown in this paper that employing a general PARAFAC model with, e.g., a simple orthogonality constraint allows filtering MIMO wideband channel measurements with relaxed requirements on the antenna arrays used in channel sounder measurements.

Furthermore, it has been shown in [3, 4] that the PPCM model is not entirely appropriate for wideband MIMO channel analysis. The main problem is that a MIMO radio channel observation doesn't lie entirely in the manifold spanned by the PPCM. Therefore, it has been proposed in [3] to extend the PPCM by an additional component, namely the dense-multipath (DMC). The model proposed for DMC in [3] is very restrictive and may not fit to a large variety of radio propagation scenarios. It will be shown in a second example that DMC can be modelled as a realization of a separable tensor-valued stochastic process. This leads to a DMC model that can be identified from the measurements, while being much more flexible than the model introduced in [3].

A variety of techniques for tensor-valued signal analysis using tensor decompositions, such as PARAFAC (Parallel Factor Model) [5], HOSVD (Higher Order SVD) [6], or PACA (PARAFAC/CANDECOMP) [2] have been developed [7, 8]. However, one should note that there are many open problems in the analysis of tensor valued processes. Just recently it has been shown that the well known Eckard-Young theorem, which is important for low-rank least-squares approximation of matrices (2-way tensors), cannot be extended to N-way tensors with $N > 2$. The analysis of tensor valued processes has been mainly focused on the estimation of first order moments of tensor-valued signals.

In this paper, it is shown that using a PARAFAC model as a model for a broadband MIMO channel observation the SNR of channel sounding measurements can be improved via a projection operation to signal subspace. Furthermore, a generalized model for DMC is proposed, modelling the DMC as a separable tensor-valued stochastic process. The new model allows for arbitrary spatial correlation of the DMC at the transmitter and the receiver. The paper is structured as follows. In Section 2, the Tucker- and PARAFAC-model for tensor-valued data are summarized. In Section 3, the application of a PARAFAC model for channel measurement filtering is proposed. In Section 4, a model for DMC which is based on separable tensor-valued processes, is introduced. Section 5 summarizes the contributions of this work.

2 Tensor Models

There are two models, which are mainly used in tensor analysis. The PARAFAC/CANDECOMP model for an L-way tensor $\mathcal{X} \in \mathbb{C}^{M_1 \times \cdots \times M_L}$ is given by

$$\mathrm{vec}\left\{\mathcal{X}\right\} = (\mathbf{U}_L \Diamond \cdots \Diamond \mathbf{U}_1) \cdot \mathbf{s}, \tag{1}$$

where $\mathbf{U}_l \in \mathbb{C}^{M_l \times K}, l = 1 \cdots L$ are the factors in the L dimensions of the tensor, $\mathbf{s} \in \mathbb{R}^{K \times 1}$ is a vector containing the weights, $\Diamond$ denotes the Khatri-Rao-product and $\mathrm{vec}\{\bullet\}$ denotes the vector operator, which stacks the elements of a tensor into a vector beginning with the lowest dimension of the tensor, see e.g. [9]. Depending on the application certain constraints may be imposed on the factor matrices. If all factor matrices are orthonormal $\mathbf{U}_l^H \mathbf{U}_l = \mathbf{I}, \forall l = 1 \cdots L$ the

model is called an all-orthogonal model. If the factors fullfill the condition $\left(\mathbf{U_1}^H\mathbf{U_1}\right)\odot\cdots\odot\left(\mathbf{U_L}^H\mathbf{U_L}\right)=\mathbf{I}$ [1] the model is called an orthogonal model.

The second model often used for tensor analysis is the Tucker-model. The Tucker-model for an L-way tensor $\mathcal{X}\in\mathbb{C}^{M_1\times\cdots\times M_L}$ is given by

$$\mathbf{x}=\mathrm{vec}\left\{\mathcal{X}\right\}=\left(\mathbf{U}_L\otimes\cdots\otimes\mathbf{U}_1\right)\cdot\mathbf{s}, \tag{2}$$

where $\mathbf{U}_l\in\mathbb{C}^{M_l\times K}, l=1\cdots L$ are the factors in the L dimensions of the tensor, $\mathbf{s}\in\mathbb{R}^{K\times 1}$ is a vector containing the weights, and $\otimes$ denotes the Kronecker-product. Algorithms to decompose a tensor into the factors of a PARAFAC- or a Tucker-model are based on alternating least squares or Gauss-Newton techniques.

3 Radio Channel Measurement Filtering

In this section the application of a tensor-decomposition algorithm to noise reduction is described. Filtering of MIMO radio channel observations is of interest for example in realistic link-level simulations using measured channels obtained by channel sounding. Assume the MIMO radio channel has been measured on M_f points in the frequency domain using M_T transmit antennas and M_R receive antennas. Then the observation tensor $\mathcal{X}$ is a 3-way tensor with dimensions $M_f\times M_R\times M_T$. The observation contains contributions of the measured MIMO channel $\mathcal{H}\in\mathbb{C}^{M_f\times M_R\times M_T}$ and i.i.d. complex Gaussian measurement noise $\mathcal{W}\in\mathbb{C}^{M_f\times M_R\times M_T}$.

$$\mathcal{X}=\mathcal{H}+\mathcal{W} \tag{3}$$

The MIMO radio channel can be interpreted as a superpostion of a large number P_g of propagation paths

$$\mathrm{vec}\left\{\mathcal{H}\right\}=\sum_{p=1}^{P_g}\mathbf{s}\left(\boldsymbol{\theta}_p\right). \tag{4}$$

The concentrated propagation paths are parametrized by a time-delay, a transmit angle (azimuth and elevation), a receive angle (azimuth and elevation), and the polarimetric path weights. For a discussion of the parametrization $\boldsymbol{\theta}_p$ of the propagation paths and the mapping of the parameters $\boldsymbol{\theta}_p$ to $\mathbf{s}\left(\boldsymbol{\theta}_p\right)$ see [3]. The mapping of the structural parameters (angles and delays) to the MIMO channel is unimportant for the proposed technique. The important fact for tensor-decomposition based MIMO channel measurement filtering is that the contribution of a propagation path can be expressed as a Khatri-Rao product

$$\mathrm{vec}\left\{\mathcal{H}\right\}=\sum_{p=1}^{P_g}\left(\mathbf{B}_{T,p}\Diamond\mathbf{B}_{R,p}\Diamond\mathbf{B}_{f,p}\right)\boldsymbol{\gamma}_p. \tag{5}$$

Consequently, the channel $\mathcal{H}$ can be interpreted in terms of the PARAFAC model (1). One should observe that (5) holds only if $\frac{l_a}{c_0}<<\frac{1}{B_m}$, where l_a is the largest dimension of the used antenna-arrays, c_0 is the speed of light and B_m is the measurement bandwidth. Using this observation one can apply the idea of principal component analysis and separate signal and noise subspace using the orthogonal constraint PARAFAC model. Having determined the factors of the signal-subspace $\mathbf{U}_{f,s}$, $\mathbf{U}_{R,s}$, $\mathbf{U}_{T,s}$ the filtered channel (estimate) is given by

$$\mathrm{vec}\left\{\hat{\mathcal{X}}\right\}=\boldsymbol{\Pi}_s\mathrm{vec}\left\{\mathcal{X}\right\}, \tag{6}$$

where $\boldsymbol{\Pi}_s$ is the projector to the signal-subspace, given by

$$\boldsymbol{\Pi}_s=\left(\mathbf{U}_{T,s}\Diamond\mathbf{U}_{R,s}\Diamond\mathbf{U}_{f,s}\right)\left(\mathbf{U}_{T,s}\Diamond\mathbf{U}_{R,s}\Diamond\mathbf{U}_{f,s}\right)^{\mathrm{H}}. \tag{7}$$

The proposed technique has been applied to a sequence of MIMO channel measurements with $M_f=193$, $M_T=31$, and $M_R=32$. To show the gain in SNR, the power delay profile (PDP) before and after filtering has been computed. The PDPs are shown in Fig. 1. The gain in SNR achieved in this example is close to 10 dB. This is a significant gain in SNR considering that the only constraint imposed on the structure of the data is the Khatri-Rao product in (5). Therefore, the proposed technique can also be applied if, e.g., the MIMO channel measurements have been conducted using antenna arrays, for which the estimation of directional channel parameters like transmit or receive angles is not applicable.

4 A Generalized DMC Model

It has been shown in, e.g. [3] that only part of a radio channel observation can be estimated as concentrated propagation paths, and the remaining component of the channel has been classified as dense multipath. Dense multipath can be modelled as a realization of a zero-mean circular complex Gaussian process. This process is fully characterized by its second order statistics. It has been proposed in [3] to model the process as spatially i.i.d., having only correlation in the frequency domain. The correlation in the frequency domain is modelled by a Toeplitz-matrix (WSSUS assumption), which is related to an exponentially decaying PDP of the dense multipath in the time-delay domain. This model fits well to some scenarios, but it has also been observed in [10] that the assumption of spatial independence is not always valid. In the following, a more general model is introduced, which is still identifiable from measurement data. Suppose a channel observation contains only contribution from DMC and measurement noise. I.e., assume that no concentrated propagation paths are present in the channel observation, or that they have been estimated and removed from the observation.

The basic model for an observation is the same as in the previous section. Therefore, in the following $\mathcal{H}_d$ will be used to indicate that the discussion is focused on the DMC model only

$$\mathcal{X}_d=\mathcal{X}-\mathcal{H}=\mathcal{H}_d+\mathcal{W}. \tag{8}$$

[1] $\odot$ denotes the Schur/Hadamard-Product.

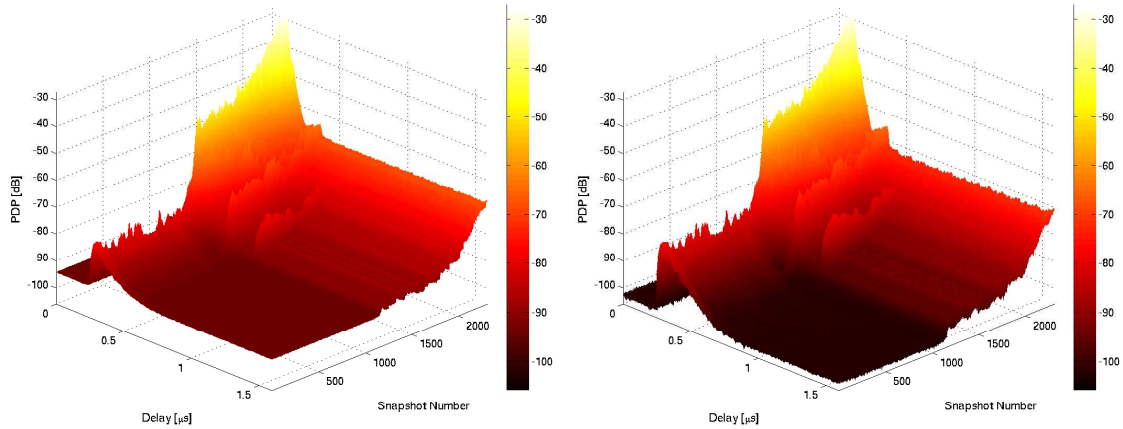

Fig. 1. Example for a sequence of measured MIMO PDPs before and after filtering. The SNR has been improved by $\approx 10dB$, what can be easily observed by the noise level before and after filtering at the beginning and the end of the measurements.

As already stated, the goal is to generalize the DMC model, such that it holds for a wider class of propagation scenarios. The idea is to model the DMC as a separable circular-complex Gaussian process, having distribution

$$\mathrm{vec}\left\{\mathcal{H}_{\mathrm{d}}\right\} \sim \mathcal{CN}\left(\mathbf{0}, \mathbf{R}_{\mathrm{T}}\left(\boldsymbol{\theta}\right) \otimes \mathbf{R}_{\mathrm{R}}\left(\boldsymbol{\theta}\right) \otimes \mathbf{R}_{\mathrm{f}}\left(\boldsymbol{\theta}\right)\right). \tag{9}$$

This model allows for an arbitrary spatial transmit-, receive-, and frequency-correlation of the DMC. However, this model is not unique, since

$$\begin{aligned}
\mathbf{R}_d\left(\boldsymbol{\theta}\right) &= \mathbf{R}_T\left(\boldsymbol{\theta}\right) \otimes \mathbf{R}_R\left(\boldsymbol{\theta}\right) \otimes \mathbf{R}_f\left(\boldsymbol{\theta}\right) \\
&= \left(a\mathbf{R}_T\left(\boldsymbol{\theta}\right)\right) \otimes \left(b\mathbf{R}_R\left(\boldsymbol{\theta}\right)\right) \otimes \left(\frac{1}{ab}\mathbf{R}_f\left(\boldsymbol{\theta}\right)\right), \forall a, b \in \mathbb{C} \\
&= \mathbf{R}'_T\left(\boldsymbol{\theta}\right) \otimes \mathbf{R}'_R\left(\boldsymbol{\theta}\right) \otimes \mathbf{R}'_f\left(\boldsymbol{\theta}\right),
\end{aligned}$$

where both sets of covariance matrices $\mathbf{R}_f\left(\boldsymbol{\theta}\right)$, $\mathbf{R}_R\left(\boldsymbol{\theta}\right)$, $\mathbf{R}_T\left(\boldsymbol{\theta}\right)$ and $\mathbf{R}'_f\left(\boldsymbol{\theta}\right)$, $\mathbf{R}'_R\left(\boldsymbol{\theta}\right)$, $\mathbf{R}'_T\left(\boldsymbol{\theta}\right)$ are valid Kronecker-factorizations of the covariance matrix $\mathbf{R}_d\left(\boldsymbol{\theta}\right)$. Therefore, a constraint has to be imposed on two of the three covariance matrices $\mathbf{R}_f\left(\boldsymbol{\theta}\right)$, $\mathbf{R}_R\left(\boldsymbol{\theta}\right)$, and $\mathbf{R}_T\left(\boldsymbol{\theta}\right)$ in order to ensure identifiability. A reasonable constraint is the trace of the covariance matrices. A natural choice for the constraints is $\mathrm{trace}\left(\mathbf{R}_{\mathrm{R}}\left(\boldsymbol{\theta}\right)\right) = \mathrm{M_R}$ and $\mathrm{trace}\left(\mathbf{R}_{\mathrm{T}}\left(\boldsymbol{\theta}\right)\right) = \mathrm{M_T}$.

As in the previous section, it is assumed that there is also additive i.i.d. circular-complex second order ergodic Gaussian observation noise $\mathcal{W} \in \mathbb{C}^{M_f \times M_R \times M_T}$ with variance σ^2 present (10). It is assumed that the noise is independent of $\mathcal{H}_d$.

$$\mathrm{vec}\left\{\mathcal{W}\right\} \sim \mathcal{CN}\left(\mathbf{0}, \sigma^2\mathbf{I}\right). \tag{10}$$

Consequently an observation is distributed as

$$\mathrm{vec}\left\{\mathcal{X}_{\mathrm{d}}\right\} = \mathbf{x}_d \sim \mathcal{CN}\left(\mathbf{0}, \mathbf{R}\left(\boldsymbol{\theta}\right)\right), \text{with} \tag{11}$$

$$\mathbf{R}\left(\boldsymbol{\theta}\right) = \mathbf{R}_T\left(\boldsymbol{\theta}\right) \otimes \mathbf{R}_R\left(\boldsymbol{\theta}\right) \otimes \mathbf{R}_f\left(\boldsymbol{\theta}\right) + \sigma^2\mathbf{I}. \tag{12}$$

In Fig. 2 the generating tensor signal model for the DMC is visualized.

Unfortunately, since a MIMO radio channel observation is a superposition of two tensor valued stochastic processes the separation property of the DMC process (9) is lost. Consequently, simple algorithms developed for the estimation of separable stochastic processes cannot be applied to estimate the covariance matrices of the proposed DMC model. Therefore, a new maximum-likelihood based algorithm has been developed to estimate them under the presence of noise [11].

The proposed algorithm has been applied to a MIMO radio channel measurement. For a description of the measurement setup as well as the measured scenario see [12]. As already stated above, a radio channel observation contains three components: contributions from dominant specular like propagation paths ($\mathcal{H}$), dense multipath ($\mathcal{H}_d$), and measurement noise ($\mathcal{W}$). Therefore, the Extended Kalman filter based estimator described in [13] has been applied in an alternating manner with the algorithm [11] in order to estimate these components. Note that the estimators depend on each other. Having estimates of the concentrated propagation paths and the DMC, one can first remove the contribution of the estimated propagation paths from the observations, and then use $\hat{\mathbf{R}}^{\frac{1}{2}}$ to whiten the remaining signal. Figure 3.(a), and 3.(b) show the transmit-angle-delay-power spectrum and the receive-angle-delay-power spectrum before and after whitening.

5 Summary

In this paper, it is shown in two examples that the interpretation of MIMO radio channel sounding observations as tensor-valued signals can provide new insight into the estimation problem. In the first example it has been shown that using a PARAFAC model as a model for a broadband MIMO channel observation the SNR of channel measurements can be improved via a projection operation to signal subspace. This is beneficial for example in applications using raw measurements in simulations, such as link-level simulations. In a second example a generalized model for DMC has been proposed, modelling the DMC as a separable tensor-valued stochastic process. The new model allows for arbitrary spatial correlation of the DMC at the transmitter and the receiver.

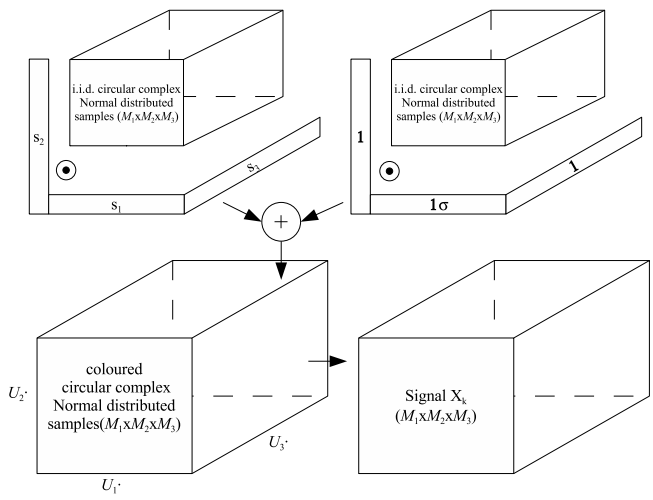

Fig. 2. Tensor Signal Model: The observed signal is a superposition of two tensor valued signals. The observed signal (lhs) is first coloured in the three dimensions by the singular values of the related covariance matrices $\mathbf{R}_i$. In the next step this signal is combined with the i.i.d. measurement noise (rhs) scaled by the standard deviation σ, and in the last step a correlation is introduced in the three dimensions by the eigenvectors $\mathbf{U}_i$ of the related covariance matrices.

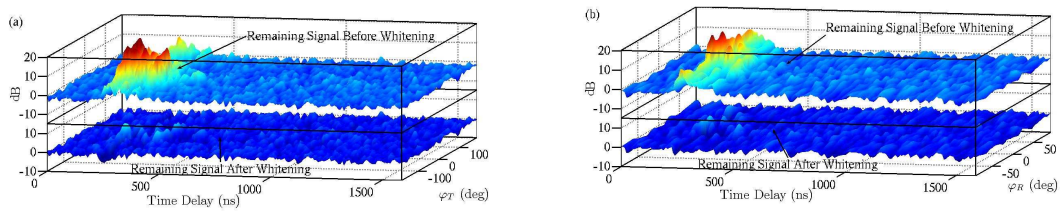

Fig. 3. Power-delay-transmit-azimuth (a) and Power-delay-receive-azimuth (b) of the remaining signal after removing estimated dominant propagation paths from a wideband MIMO radio channel measurement. The remaining signal consists of dense multipath and observation noise. In each plot the remaining signal before and after whitening with $\hat{\mathbf{R}}^{\frac{1}{2}}$ is shown.

Acknowledgments

The research was partially funded by the WILATI project that is a joint effort between three scandinavian universities and is part of the NORDITE research programme, which is funded by Finnish, Swedish and Norwegian national research institutes Tekes, Vinnova and RCN, respectively.

References

[1] M. Steinbauer, "The radio propagation channel - a non-directional, directional and double-directional point-of-view," Ph. D. Dissertation, Technische Universität Wien, Vienna, Austria, Nov. 2001.

[2] J. B. Kruskal, "Three-way arrays: rank and uniqueness of trilinear decomposition, with applications to arithmetic complexity and statistics," *Ann. Statistics*, vol. 18, pp. 95–138, 1977.

[3] A. Richter, "Estimation of radio channel parameters: Models and algorithms," Ph.D. Dissertation, Technische Universität Ilmenau, Ilmenau, Germany, 2005, urn:nbn:de:gbv:ilm1-2005000111. [Online]. Available: http://www.db-thueringen.de/servlets/DerivateServlet/Derivate-7407/ilm1-2005000111.pdf

[4] A. Richter, J. Salmi, and V. Koivunen, "On distributed scattering in radio channels and its contribution to mimo channel capacity," in *in Proc 1st European Conference on Antennas and Propagation (EuCAP 2006)*, Nice, France, November 2006, best Paper Award.

[5] L. R. Tucker, "Some mathematical notes on three-mode factor analysis," *Psychometrica*, vol. 31, pp. 279–311, 1966.

[6] L. D. Lathauwer, B. D. Moor, and J. Vandewalle, "A multilinear singular value decomposition," *SIAM J. Matrix Anal. Appl.*, vol. 21, pp. 1253–1278, April 2000.

[7] J. Ruiz-Alzola and C.-F. Westin, Eds., *Special Issue: Tensor Signal Processing*, ser. Signal Processing. ELSEVIER, February 2007, vol. 87, no. 2, issn 0165-1684.

[8] N. D. Sidropoulus, R. Bro, and G. B. Giannakis, "Parallel factor analysis in sensor array processing," *IEEE Transactions on Signal Processing*, vol. 48, no. 8, pp. 2377–2387, August 2000.

[9] R. A. Horn and C. R. Johnson, *Topics in Matrix Analysis*. Cambridge University Press, 1991.

[10] A. Richter, J. Salmi, and V. Koivunen, "Signal processing perspectives to radio channel modelling," in *Proceedings of the 2nd European Conference on Antennas and Propagation*, November 2007.

[11] ——, "Ml estimation of covariance matrix for tensor valued signals in noise," in *Proceedings IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP)*, Las Vegas, NV, USA, April 2008.

[12] U. Trautwein, M. Landmann, G. Sommerkorn, and R. Thomä, "System-oriented measurement and analysis of mimo channels," in *COST 273 TD(05) 063*. Bologna, Italy: COST 273, Jan. 2005.

[13] J. Salmi, A. Richter, and V. Koivunen, "Enhanced tracking of radio propagation path parameters using state-space modeling," in *14th European Signal Processing Conference*, Florence, Italy, July 2006.