A Correlation-Based Wideband MIMO Channel Model

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Abstract

In the following we review the basis for correlative MIMO channel modeling, which attempts to model the correlation between paths in a given channel. We present a wideband MIMO channel model called *the structured model*. Here, we focus on the parameters of the structured model. These include the third-order *H-tensor*, the *wideband correlation matrix,* the *one-sided correlation matrices* for wideband channels, and the *wideband coupling coefficients.* We then present the structured model and briefly discuss experimental results.

1. Introduction

Multiple-Input, Multiple-Output (MIMO) systems offer the promise of increased spectral efficiency by benefiting from multipath diversity [1]. Since multipath diversity results from the presence of scatterers in the channel, the performance increase over Single-Input, Single-Output (SISO) channels is dependent on the orientation of scatterers, and the correlation between resolvable paths in the channel. We refer to this as the *spatial structure* of the channel. Correlative channel models attempt to model the behavior of the MIMO channel by approximating the spatial correlation of a given channel.

The *Kronecker model* [2] assumes the channel correlation matrix can be approximated as the Kronecker product of the transmit and receive correlation matrices. The transmit and receive correlation matrices are collectively termed the *one-sided correlation matrices.* Together, the one-sided correlation matrices have fewer parameters than the channel covariance matrix.

The *structured model* is a correlative wideband MIMO channel model. It is unique because it uses tensor algebra to represent the correlation across receive-, transmit-, and delay-space. The structured model was first presented in [3]. Extensive analysis with real-life data shows that the structured model far outperforms the Kronecker model [4, 5]. Here, we focus on the parameters of the structured model and their meaning. We discuss the concept of an *H-tensor*, and define the *wideband correlation matrix*. We use tensor algebra [6] to extend the concept of one-sided correlation to include the wideband case. We then define the structured model synthesis equation.

The rest of the paper is organized as follows. Section 2 contains a brief introduction to the tensor operations used in the rest of the paper. Section 3 introduces the concept of an H-tensor. Section 4 describes the method by which we can use the full correlation to generate an ensemble of exemplar channels with the same spatial structure as a given channel. Section 5 focuses on the concept of one-sided correlation, and how it relates to the parameters of the narrowband and wideband Kronecker model. Section 6 introduces the structured model, which extends the concept of one-sided correlation to three dimensions, namely receive, transmit, and delay space. We conclude in Section 7.

2. Relevant Tensor Algebra

We denote tensors by calligraphic upper case letters $(\mathcal{A},\mathcal{B},\mathcal{C},\ldots)$, matrices by bold upper case $(\mathbf{A},\mathbf{B},\mathbf{C},\ldots)$, and vectors by bold lower case $(\mathbf{a},\mathbf{b},\mathbf{c},\ldots)$. We denote a single element of a tensor using an indexed lower case letter. For example, given a tensor $\mathcal{A}\in\mathbb{R}^{I_1\times I_2\times\ldots\times I_N}$, we denote the $i_1,\ldots,i_n,\ldots,i_N$ th element by $(\mathcal{A})_{i_1,\ldots,i_n,\ldots,i_N} = a_{i_1,\ldots,i_n,\ldots,i_N}$, where $1\le i_n \le I_n$. The order of a tensor is the number of dimensions needed to address a single element of a tensor, eg. $\mathcal{A}\in\mathbb{R}^{I_1\times I_2\times\ldots\times I_N}$ is an Nth order tensor.

The n^{th} *matrix unfolding* $\mathbf{A}_{(n)} \in \mathbb{R}^{I_n \times (I_{n+1} \ldots I_N I_1 \ldots I_{n-1})}$ is formed by stacking the columns formed by the nth dimension, one after the other. Using this definition, we can define the multiplication between a tensor and a matrix. Consider a tensor $\mathcal{A} \in \mathbb{R}^{I_1 \times I_2 \times \ldots \times I_n \times \ldots \times I_N}$ and a matrix $\mathbf{M} \in \mathbb{R}^{J_n \times I_n}$. The *n-mode product* between them is defined as $\mathcal{B} = \mathcal{A} \times_n \mathbf{M}$ where the product tensor $\mathcal{B} \in \mathbb{R}^{I_1 \times \ldots \times I_{n-1} \times J_n \times I_{n+1} \times \ldots \times I_N}$. Element-wise, the n-mode product is

$$b_{i_1 \ldots i_{n-1} j_n i_{n+1} \ldots i_N} = \sum_{i_n} a_{i_1 \ldots i_n \ldots i_N} m_{j_n i_n} \tag{1}$$

Using the nth unfolding of $\mathcal{A}$, we can also express the n-mode product as a matrix multiplication, $\mathbf{B}_{(n)} = \mathbf{M}\mathbf{A}_{(n)}$. We define the *tensor outer product* between two tensors $\mathcal{A} \in \mathbb{R}^{I_1 \times I_2 \times \ldots \times I_N}$ $\mathcal{B} \in \mathbb{R}^{J_1 \times J_2 \times \ldots \times J_Q}$ to be the tensor $\mathcal{C} \in \mathbb{R}^{I_1 \times I_2 \times \ldots \times I_N J_1 \times J_2 \times \ldots \times J_Q}$, that is $\mathcal{C} = \mathcal{A} \circ \mathcal{B}$. Element-wise, this can be expressed as

$$c_{i_1 i_2 \ldots i_N j_1 j_2 \ldots j_Q} \triangleq a_{i_1 i_2 \ldots i_N} b_{j_1 j_2 \ldots j_Q} . \tag{2}$$

3. The H-Tensor

Consider the linear time-invariant wideband MIMO channel. A discrete entity describing the complex gains of this type of channel is necessarily three dimensional, with dimensions M_{Rx} receivers, M_{Tx} transmitters, and D delay bins. In the past, the wideband MIMO channel gains have been quantified using an indexed H-matrix, ie. $\mathbf{H}[d]$. The entries of $\mathbf{H}[d]$ are the wideband SISO channels $h_{mn}[d]$ for $m = 1 \ldots M_{Rx}$, $n = 1 \ldots M_{Tx}$, $d = 1, \ldots, D$. Recasting the wideband channel gains as a third order tensor allows us to use tensor decomposition to develop a wideband channel model. Thus, we define the *wideband H-tensor* $\mathcal{H} \in \mathbb{C}^{M_{Rx} \times M_{Tx} \times D}$ to be a tensor whose element h_{mnd} is the complex gain of the channel between receiver m, transmitter n at delay d. In this way, $\mathbf{H}[d]$ consists of the subset of elements of $\mathcal{H}$ corresponding to delay tap d.

4. Channel Correlation and Synthesis

The goal of correlative MIMO channel modeling is to generate H-matrices with approximately the same spatial structure as a given channel, with as few parameters as possible. In the narrowband case, correlative channel models generate an ensemble of exemplar H-matrices by *spatially filtering* a statistically white H-matrix. The *narrowband correlation matrix* $\mathbf{R}_H$ is defined as

$$\mathbf{R}_H = E\left\{ \text{vec}(\mathbf{H}) \text{vec}^H(\mathbf{H}) \right\}, \tag{3}$$

where $\text{vec}(\mathbf{H})$ is the mapping of all elements in the H-matrix $\mathbf{H}$ such that all columns in $\mathbf{H}$ are stacked on top of each other to form a vector. We can generate an ensemble of H-matrices with the same spatial structure as a given channel using the synthesis equation [7],

$$\mathbf{H}_{synth} = \text{unvec}\left(\mathbf{R}_H^{1/2} \mathbf{g}_H \right), \tag{4}$$

where $\mathbf{g}_H \in \mathbb{C}^{M_{Rx} M_{Tx} \times 1}$ is a complex-Gaussian IID vector, and $\mathbf{R}_H^{1/2}$ is the matrix square root of $\mathbf{R}_H$. We refer to $\mathbf{H}_{synth}$ as the *synthetic H-matrix*.

We can extend the concept of channel correlation to the wideband MIMO channel. First, consider $\mathcal{H} \in \mathbb{C}^{M_{Rx} \times M_{Tx} \times D}$. The vectorization of $\mathcal{H}$, denoted $\text{vec}(\mathcal{H})$, is the mapping of all h_{mnd} to a vector [5]. The wideband correlation matrix $\mathbf{R}_{WB,H}$ is defined as

$$\mathbf{R}_{WB,H} = E\left\{ \text{vec}(\mathcal{H}) \text{vec}^H(\mathcal{H}) \right\}. \tag{5}$$

Following (4), we propose that the synthesis equation for the H-tensor to be of the form

$$\mathcal{H}_{\text{synth}} = \operatorname{unvec}\left(\mathbf{R}_{\mathrm{WB,H}}^{\frac{1}{2}}\mathbf{g}_{\mathrm{H}}\right), \tag{6}$$

where $\mathbf{g}_{\mathcal{H}} \in \mathbb{C}^{M_{Rx}M_{Tx}D\times 1}$ is a complex Gaussian vector and $\operatorname{unvec}(\bullet)$ is the reverse mapping of $\operatorname{vec}(\bullet)$.

5. One-Sided Correlation and the Kronecker Model

We can reduce the number of parameters used to describe $\mathbf{R}_{\mathrm{H}}$ by assuming that scatterers local to the transmitter fade independently from those at the receiver. We define the *one-sided correlation matrices* $\mathbf{R}_{\mathrm{Rx}}$ and $\mathbf{R}_{\mathrm{Tx}}$ to be

$$\mathbf{R}_{\mathrm{Rx}} = \frac{1}{\beta}\mathrm{E}\left\{\mathbf{H}\mathbf{H}^{\mathrm{H}}\right\} \quad \mathbf{R}_{\mathrm{Tx}} = \frac{1}{\alpha}\mathrm{E}\left\{\mathbf{H}^{\mathrm{H}}\mathbf{H}\right\}^{\mathrm{T}}, \tag{7}$$

where $(\bullet)^{\mathrm{H}}$ is the Hermitian transpose, and $(\bullet)^{\mathrm{T}}$ is the ordinary transpose. Each one-sided correlation matrix is normalized such that $\alpha\beta = \operatorname{Tr}(\mathbf{R}_{\mathrm{H}}) = \mathrm{E}\left\{\|\mathbf{H}\|_{\mathrm{F}}^{2}\right\}$, where $\operatorname{Tr}(\bullet)$ is the trace operator, and $\|\bullet\|_{\mathrm{F}}$ is the Frobenius norm. $\mathbf{R}_{\mathrm{Rx}}$ can be viewed as the correlation across all receivers for a fixed transmitter. Conversely, the transmit correlation matrix $\mathbf{R}_{\mathrm{Tx}}$ can interpreted as the correlation across all transmitters for a fixed receiver. From [2], the Kronecker model approximates $\mathbf{R}_{\mathrm{H}}$ as $\mathbf{R}_{\mathrm{H}} \approx \mathbf{R}_{\mathrm{H,Kron}} = \mathbf{R}_{\mathrm{Tx}} \otimes \mathbf{R}_{\mathrm{Rx}}$, where $\otimes$ denotes the Kronecker product. This approximation reduces the number of parameters needed to describe the correlation in the channel from $\left(M_{Rx}M_{Rx}\right)^{2}$ to $\left(M_{Rx}^{2} + M_{Tx}^{2}\right)$. The narrowband Kronecker synthesis equation is thus $\mathbf{H}_{\mathrm{Kron}} = \mathbf{R}_{\mathrm{Rx}}^{1/2}\mathbf{G}\left(\mathbf{R}_{\mathrm{Tx}}^{1/2}\right)^{\mathrm{T}}$ where $\mathbf{G}$ is a spatially white matrix with complex Gaussian entries. Yu et al. [2] extend the narrowband model by applying the narrowband synthesis equation to each delay tap in the PDP. The resulting model is

$$\mathbf{H}_{\mathrm{Kron}}[d] = \mathbf{R}_{\mathrm{Rx}}^{1/2}[d]\,\mathbf{G}[d]\left(\mathbf{R}_{\mathrm{Tx}}^{1/2}[d]\right)^{\mathrm{T}}, \tag{8}$$

where $\mathbf{R}_{\mathrm{Rx}}^{1/2}[d]$ and $\mathbf{R}_{\mathrm{Tx}}^{1/2}[d]$ are computed using the measured $\mathbf{H}[d]$ for $d = 1,\ldots,D$. Detailed analysis of the Kronecker model using real data have shown that it performs poorly, especially for systems in which the number of antennas is large (>3) [4, 7, 8]. In his thesis, Yu [9] quotes model errors as high as 49% and 60% for 3×2 and 2×3 arrays, respectively. Despite this, the Kronecker model remains popular, mostly due to its simplicity.

6. The Structured Model

In this section, we review a new wideband MIMO channel model called the structured model. In order to reveal more about the structure of the channel, and to develop the structured model, we extend the concept of one-sided correlation to three dimensions. We define the *receive, transmit, and delay correlation matrices* $\mathbf{R}_{\mathrm{Rx}} \in \mathbb{R}^{M_{Rx}\times M_{Rx}}$, $\mathbf{R}_{\mathrm{Tx}} \in \mathbb{R}^{M_{Tx}\times M_{Tx}}$, and $\mathbf{R}_{\mathrm{Del}} \in \mathbb{R}^{D\times D}$ to be

$$\mathbf{R}_{\mathrm{Rx}} = \mathrm{E}\left\{\mathbf{H}_{(1)}\mathbf{H}_{(1)}^{\mathrm{H}}\right\},\ \mathbf{R}_{\mathrm{Tx}} = \mathrm{E}\left\{\mathbf{H}_{(2)}\mathbf{H}_{(2)}^{\mathrm{H}}\right\},\ \mathbf{R}_{\mathrm{Del}} = \mathrm{E}\left\{\mathbf{H}_{(3)}\mathbf{H}_{(3)}^{\mathrm{H}}\right\}, \tag{9}$$

where $\mathbf{H}_{(n)}$ is the nth matrix unfolding of $\mathcal{H}$. We can apply the EVD to each one-sided correlation matrix to obtain an eigenbasis for the receive, transmit, and delay space,

$$\begin{aligned}\mathbf{R}_{X} &= \sum_{i=1}^{\mathrm{Limit}_X} \lambda_{X,i}\mathbf{u}_{X,i}\mathbf{u}_{X,i}^{\mathrm{H}} \\ &= \mathbf{U}_X\mathbf{\Lambda}_X\mathbf{U}_X^{\mathrm{H}},\end{aligned} \tag{10}$$

where $X \in \{Rx, Tx, Del\}$, and $Limit_X \in \{M_{Rx}, M_{Tx}, D\}$ respectively. We refer to $\mathbf{U}_{Rx}$, $\mathbf{U}_{Tx}$, and $\mathbf{U}_{Del}$ as the *one-sided eigenbases*.

We define a *wideband coupling coefficient* as the average power coupled between three given one-sided eigenvectors, i.e. we define the ijkth wideband coupling coefficient ω_{ijk} as

$$\omega_{ijk} = \left(\mathbf{u}_{Del,k} \otimes \mathbf{u}_{Tx,j} \otimes \mathbf{u}_{Rx,i}\right)^H \mathbf{R}_{WB,H} \left(\mathbf{u}_{Del,k} \otimes \mathbf{u}_{Tx,j} \otimes \mathbf{u}_{Rx,i}\right), \tag{11}$$

where $u_{Rx,mi}$ is the mth element of the ith receive eigenvector, $u_{Tx,nj}$ is the nth element of the jth transmit eigenvector, and $u_{Del,dk}$ is the dth element of the kth delay eigenvector, for $i = 1, \ldots, M_{Rx}$, $j = 1, \ldots, M_{Tx}$, and $k = 1, \ldots, D$. Let $\mathcal{W} \in \mathbb{C}^{M_{Rx} \times M_{Tx} \times D}$ be a tensor whose elements are computed as $w_{mnd} = g_{mnd}\sqrt{\omega_{mnd}}$, where g_{mnd} is a complex Gaussian random variable, and ω_{mnd} is the wideband coupling coefficient. After some manipulation, we can show that the synthesis equation for the structured model is

$$\mathcal{H}_{struct} = \mathcal{W} \times_1 \mathbf{U}_{Rx} \times_2 \mathbf{U}_{Tx} \times_3 \mathbf{U}_{Del}. \tag{12}$$

The structured model enables us to compute an ensemble of H-tensors with approximately the same spatial characteristics as the original H-tensor $\mathcal{H}$, but using far fewer parameters than (6). Equation (12) implies that we need $M_{Rx} M_{Tx} D + \left(M_{Rx}^2 + M_{Tx}^2 + D^2\right)$ parameters to generate an ensemble of H-tensors.

Experimental results using both outdoor and indoor wideband data, collected using two different apparatuses, show that the structured model outperforms the Kronecker model, especially with respect to predicting the capacity of real channels [4]. For example, in [5], we show that the structured model error, averaged over all data sets, was 4.1%, while the Kronecker model error was 46.1%. In addition, for all the cases considered, the structured model *used fewer parameters* than did the Kronecker model.

7. Conclusion

In this paper, we presented a few fundamentals of correlative MIMO channel modelling. We began with a narrowband and wideband synthesis equation that used the full channel correlation, and thus the most parameters, to generate an ensemble of exemplar channels with the same spatial structure as a given channel. We then defined the concept of one-sided correlation, which is used to reduce the number of model parameters. The Kronecker model uses the receive and transmit one-sided correlation matrices as parameters. In deriving the structured model, we extended the idea of one-sided correlation to three dimensions. The structured model uses the EVD of the one-sided correlation matrices, as well as the power coupled between eigenvectors, as parameters to generate an ensemble of H-tensors. The structured model has been previously shown to outperform the Kronecker model using real data.

8. References

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