Characterization of Materials and Mode Structure for High-Q Resonators using Bragg Confined Modes

***Jean-Michel LE FLOCH*[1], *Michael TOBAR*[1], *David MOUNEYRAC*[1,2], *Dominique CROS*[2], *Jerzy KRUPKA*[3]**

[1]School of Physics, University of Western Australia, 35 Stirling Hwy, Crawley 6009, Western Australia, lefloch@cyllene.uwa.edu.au

[2]XLIM -UMR CNRS n°6172 - 123, avenue A. Thomas, 87060 Limoges Cedex, France, david.mouneyrac@xlim.fr

[3]Institute of Microelectronics and Optoelectronics, University of Technology, Koszykowa 75, Warsaw, Poland

Abstract

This paper describes the design and mode structure of resonators using Bragg confined modes. We investigate a range of low-loss dielectric materials with aim to maximize the Q-factor. The structure is composed of a hollow dielectric cylinder with the mode confined in the central low-loss region. We illustrate the importance of material permittivity and show it is possible to obtain a better Q-factor using higher permittivity materials with larger intrinsic dielectric losses than single crystal sapphire. Also we illustrate the discovery of a new type of Bragg confined mode in a dielectric loaded cavity. The dielectric is placed in a silver platted copper cavity. A resonance was observed at 13.4 GHz with an unloaded Q-factor of order 2×10^5, which is more than a factor of six above the dielectric loss limit. Usually a Bragg structure requires a pure Transverse Electric mode with no azimuthal variations and only an electric field component, E_θ, while this mode possesses a number of azimuthal variations greater than zero.

1.Introduction

Bragg reflector resonators obtain high Q-factors by confining most of the energy in a central low-loss region (usually vacuum) using outer-layered dielectric materials loaded in a metallic cavity. The dielectric and resistive losses are decreased due to the reduced electromagnetic energy in the dielectric layers and at the cavity edge. So far the highest Q-factors have been obtained using single crystal sapphire due to its extremely low loss at microwave frequencies [1 - 2]. We investigated Non-Maxwellian simple models to obtain dimensions of a Bragg structure by solving simultaneous equations which allow a more compact design. The results obtained with this type of resonator have exceeded the best performance of commonly used dielectric resonators at room temperature [3 - 4]. The Non-Maxwellian models give solutions of frequency typically within 1% of the actual solution. For better accuracy, numerical techniques are required. In our case we use the method of lines software [5].

In this work we also investigated a range of alternative low-loss crystalline and non-crystalline dielectric materials [6] with the aim to obtain a high Q-factor like sapphire using cheaper materials with higher permittivity. Moreover we also discovered a new mode, which can be considered as a hybrid Bragg confined mode (all field components). However, the dominant fields for these modes are E_r , E_ϕ and H_z and Bragg reflection occurs as the radial component of the Electric field exists near the central region of the resonator and supplies a tangential boundary condition to the axial Bragg reflectors. In contrast the azimuthal electric field exists mainly at all other boundaries and Bragg confinement of the mode is also achieved in the all directions.

2.Bragg effect on material permittivity

To compare the efficiency of various materials we investigated a single Bragg reflector in the radial and axial regions as shown in figure 1. It has been shown that such structures that use single crystal sapphire may obtain Q-factors of order $3.0\text{x}10^5$ at room temperature [3-4]. It has also been shown that a close to optimum solution can be obtained when we use simple models to calculate the dimensions of the resonator [3-4]. Thus, the simple model technique was used to calculate the dimensions of a 10 GHz dielectric resonator loaded in a silver platted cavity for permittivities varying between 10 to 200, and for aspect ratios (AR) between 1.0 and 1.75 (ARs of these values were shown in [4] to be near optimum), where AR = L/2R (L is the height and R the radius of the cavity as shown in figure 1). The dimensions of the cavity were then entered in the Method of Lines [5] mesh and the Geometric-factor (*G*), electric energy filling factor (*Pe*) in the dielectric and frequency were calculated. The frequency calculated using the rigorous Method of Lines technique [5] gives the same frequency of 10 GHz to within 0.02 to 0.3 %. The Q-factor was assumed to be $Q = 3.0\text{x}10^5$ and the loss-tangent was calculated from

(1) at each value of permittivity assuming the surface resistance (Rs) to be 26 mΩ (the value of good silver plated copper [7]).

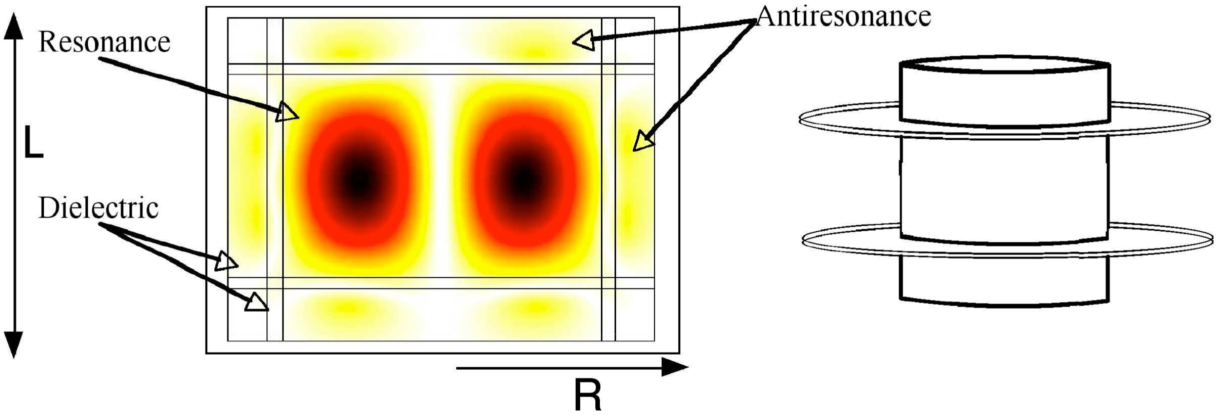

Figure 1: Right, illustration of the dielectric layers that form the cylindrical Bragg reflectors. Left, Electric field density plot (E_0) of the fundamental TE mode as calculated using the Method of Lines, showing the internal resonant region (free space), and outer antiresonant layered region. For a 10 GHz resonator the size ranges between L= 60 – 100 mm, with R ~ 30 mm for AR ranging between 1.0 – 1.75.

$$\tan\delta = \frac{Q^{-1} - R_S/G}{Pe} \tag{1}$$

A range of materials at 10 GHz from the database given in [6] were compared with the calculated $3.0\text{x}10^5$ Q-factor locus shown in figure 2 and tabulated in table 1. Low-loss dielectric materials and crystals with permittivity 50 and above (labeled 1-3 and 9) lie above the $3.0\text{x}10^5$ Q-factor locus, and have worse Q-factors. Also, the thickness of the dielectric Bragg reflectors need to be smaller than 1 mm, and thus will be mostly impractical to build. Crystalline sapphire (No. 14) and rutile doped alumina (No. 7) are the lowest loss tangent materials and lie close to the curve as expected, (Q-factors ~ $2.8\text{x}10^5$). However, there is a range of crystalline and non-crystalline materials with higher loss and permittivity, which give higher Q-factors above $3.0\text{x}10^5$, with only a small reduction in the thickness of the reflectors when compared to sapphire. This includes dielectric materials 4, 5 and 6 and the crystalline materials 10, 11 and 12 (see table 1). Thus, we have shown that an improved Q-factor may be obtained with a Bragg resonator by constructing the resonator from higher permittivity materials even if they have higher losses than sapphire. This occurs because of the smaller electric filling factor in the dielectric as the permittivity increases. For multi-layered Bragg reflector resonators, (not presented here), the results give the same trends, except Q-factors of one million may be obtained.

TABLE I: Low-loss dielectric materials plotted in figure 2, with measured permittivities and loss-tangents referred to 10 GHz [6]. The Q-factor of the structure in figure 1 is calculated using Method of Lines [5]. For anisotropic materials we only give the perpendicular values, which is the only one relevant for the TE Bragg confined mode. No. Material Permittivity Tanδ [10 GHz] Q-factor

No.	Material	Permittivity	Tanδ [10 GHz]	Q-factor
1	$CaTiO_3$	162	7.716×10^{-4}	4.655×10^{4}
2	TiO_2 +0.05 mol% Fe	104	1.988×10^{-4}	1.343×10^{5}
3	$Pb_{0.7}Ca_{0.3}La_{0.5}(Mg_{1/2}Nb_{1/2})O_3$	50	1.163×10^{-4}	1.493×10^{5}
4	$Ba[(Zn_{0.6}Co_{0.4})_{1/3}Nb_{2/3}]O_3$	35.6	2.841×10^{-5}	3.895×10^{5}
5	$Ba(Mg_{1/2}Ta_{2/3})O_3$: 0.5mol% $Ba(Mg_{1/2}W_{1/2})O_3$	24.2	2.500×10^{-5}	3.371×10^{5}
5	$Ba(Mg_{1/3}Ta_{2/3})O_3$: $BaSnO_3$, $BaWO_4$	24	2.326×10^{-4}	3.502×10^{5}
6	$0.92(Mg_{0.95}Co_{0.05})TiO_3$-$0.08CaTiO_3$	22.1	1.157×10^{-5}	4.717×10^{5}
7	Al_2O_3 (TiO_2 doped)	10.5	8.392×10^{-6}	2.862×10^{5}
8	Al_2O_3 Alumina	10.05	1.471×10^{-5}	2.267×10^{5}
9	TiO_2 Rutile	85.7	1.548×10^{-4}	1.490×10^{5}
10	$LaGaO_3$	26	1.667×10^{-5}	4.483×10^{5}
11	$LaAlO_3$	24	1.965×10^{-5}	3.867×10^{5}
12	$SrLaAlO_4$	16.85	1.592×10^{-5}	3.345×10^{5}
13	YAG	10.6	1.153×10^{-5}	2.599×10^{5}
14	AL_2O_3 Sapphire	9.935	8.547×10^{-6}	2.750×10^{5}

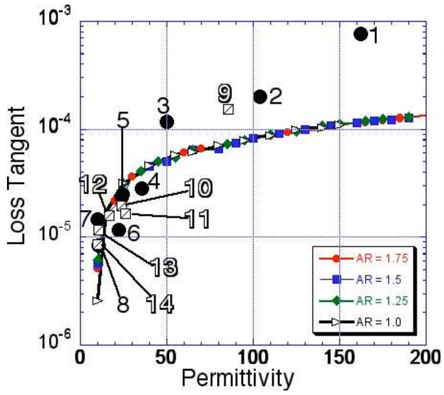

Figure 2 : Loss tangent versus permittivity showing the locus of Q = 300,000 at 10 GHz for the resonator design in figure 1, for aspect ratios (AR) between 1.0 and 1.75. Fourteen low loss materials of varying permittivity are also plotted (see table I). The bold circles (1-8) represent non-crystalline material, while the white squares represent crystalline material. The materials below the locus have a Q-factor greater than 300,000, while the ones above have a lower Q-factor.

3.New high confined mode in Alumina Bragg resonator

Bragg confined modes in a hollow alumina was investigated, the dielectric was manufactured with the following dimensions, the height is 49.94mm, the diameter 65.6mm and the radial and axial dielectric thicknesses are respectively 3.47mm and 3.42mm and supported by Teflon spaces in a cylindrical metallic silver platted cavity of 74.7mm diameter and 66.2mm height.

To characterize the dielectric properties of the alumina we use the whispering gallery mode method [8-9]. Modes are simulated using Method of Lines software [5] to predict the frequency, geometric factor (*G*) and electric filling factor (*pe*) of the fundamental WGH mode family. Simulations are compared with measurements in order to estimate the loss-tangent and the permittivity of the alumina sample. We calculated the parameters from equation (1), the loss-tangent as a function of frequency and the results are given in Table I. It's important to notice that the filling factor in Teflon is too small to be considered. The permittivity is also estimated to be 9.73 according the agreement with our measurements and the simulations. While we were investigating the alumina properties, Bragg confined resonances were also observed.

Table I. Measurements were done for a $WGH_{m,0,0}$ mode family, where m is the azimuthal mode number and also computations were done with the method of lines. The frequencies *f* are in GHz, *G* geometric factor and *Pe* the electric filling factor.

m	f meas	Q meas	f MoL	Pe MoL	G
9	7.881	4.33×10^4	7.877	0.880	1.31×10^3
10	8.384	4.81×10^4	8.376	0.889	1.76×10^3
11	8.877	5.02×10^4	8.873	0.898	2.38×10^3
12	9.379	4.94×10^4	9.368	0.906	3.23×10^3
13	9.860	4.32×10^4	9.861	0.913	4.40×10^3
14	10.357	3.60×10^4	10.352	0.919	6.00×10^3
15	10.839	3.60×10^4	10.840	0.925	8.20×10^3
16	11.328	3.40×10^4	11.325	0.931	1.12×10^4
17	11.801	3.00×10^4	11.808	0.936	1.54×10^4
18	12.320	2.80×10^4	12.289	0.940	2.12×10^4
19	---	---	12.767	0.945	2.91×10^4
20	13.248	3.00×10^4	13.244	0.949	4.01×10^4
21	13.715	2.70×10^4	13.718	0.952	5.52×10^4

We measured the loss-tangent of the alumina to be $tan\delta_{Alumina}=2.4\times10^{-6}f$ [GHz].

Most of these modes possess a number of azimuthal variations greater than zero. In contrast, previously Bragg resonances were only observed as pure Transverse Electric modes with no azimuthal variations and only an electric field component, E_θ. The new modes are both Bragg-like, (E_r is tangential to all dielectric boundaries) and also Hybrid-like, (possesses azimuthal variations), thus we call the modes Hybrid Bragg confined Modes (*HBM*). It possesses three main field components E_r, E_θ, H_z (see figure 2). The confinement in the inner region is around 90% of the total energy, which has never been reached before in a single layer structure.

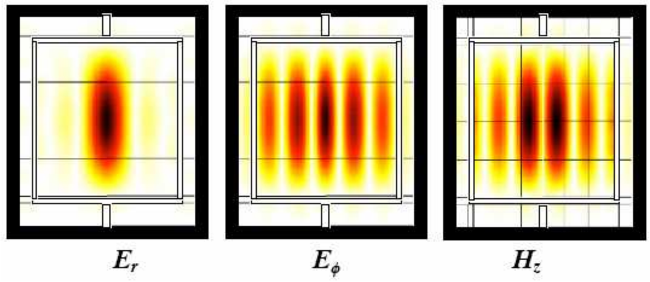

Figure 2: Density plot (modulus squared) of the dominant electric and magnetic field components for the 13.4 GHz mode of $m = 1$ as calculated using the Method of Lines [5].

4.High Order Bragg modes

Several confined higher order Bragg modes were measured in an alumina structure with Q-factors of order 10^5 or larger. Modes were identified after the successful permittivity and loss-tangent characterization and results are shown in figure 3. In previous work only Bragg confined modes of $m = 0$ have been characterized, and here we unequivocally identify Bragg confined modes of azimuthal mode number $m > 0$. The highest Q-factors are measured for the 11.34 ($Q = 2.25\times10^5$) and 13.40 ($Q = 1.91\times10^5$) GHz modes. Given that we measured the loss-tangent of the alumina to be $\tan\delta_{Alumina}=2.4\times10^{-6}f$ [GHz] both modes are a factor of 6.1 above the dielectric loss limit. We may also compare the results to a sapphire whispering gallery WGH mode. Given that the loss-tangent parallel to the c-axis is $\tan\delta_{Sapphire} = 4.2\times10^{-7} f^{\,1.09}$ [GHz] the 11.34 and 13.4 GHz modes have Q-factors of 1.3 and 1.4 times greater than sapphire WGH modes respectively.

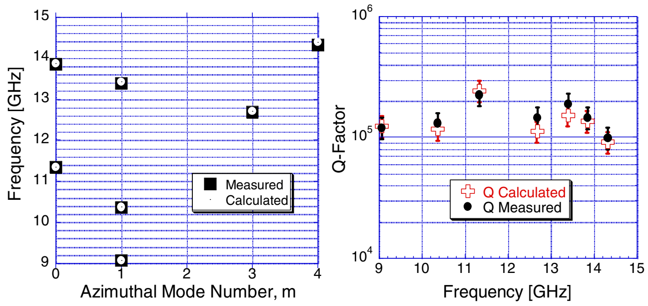

Figure 3: Left: Calculated and measured frequencies of some higher order confined Bragg modes. Right: Measured and calculated unloaded Q-factors of the Bragg modes.

5.References

1. Flory, C A and R C Taber, "High performance distributed Bragg reflector microwave resonator," *IEEE Transactions on Ultrasonics Ferroelectrics and Frequency Control (UFFC)*, **44**, 1997, pp. 486-495.
2. Flory, C A and H L Ko, "Microwave oscillators incorporating high performance distributed Bragg reflector microwave resonators," *IEEE UFFC*, **45**, 1998, pp. 824-829.
3. Tobar, M E, J M le Floch, D Cros, and J G Hartnett, "Distributed Bragg reflector resonators with cylindrical symmetry and extremely high Q-factors", *IEEE UFFC*, **52**, 2005, pp. 17-26.
4. JM Le Floch, M. E. Tobar, D. Cros, J. Krupka, "High Q-factor Distributed Bragg Reflector Resonators with Reflectors of Arbitrary Thickness," *IEEE UFFC, approved for publication*, October 2007.
5. O. Piquet, D. Cros, S. Verdeyme, P. Guillon, M.E. Tobar, W. Pascher, "Application of the method of lines for high-Q resonant structures", *EuMC,* 2002, Milan
6. Sebastian, M T, A-K Axelsson, and N Alford, *List of microwave dielectric resonator materials and their properties.* http://www.lsbu.ac.uk/dielectric-materials/.
7. Tobar, M E et al. *High-Q whispering modes in spherical cavity resonators, International Frequency Control Symposium (IFCS),* 2002, New Orleans, USA.
8. M.E. Tobar, et al., *Anisotropic Complex Permittivity Measurements of Mono-Crystalline Rutile Between 10-300 Kelvin*, *Journal of Applied Physics*, **83**, 1998, pp. 1604-1609.
9. J Krupka et al., "Complex Permittivity of some ultra low loss dielectric crystals at cryogenic temperatures," *Measurement Science and Technology (MST),* **10**, 1999, pp. 387-392.