Nonlinearity Estimation of ADCS

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Abstract : **In this paper the author comment on the time field test method –sinewave parameter estimation (FPE) method for ADCS' nonlinearity estimation. Through discussing sinewave parameter estimation method in details, four parameters sinewave fitting steps are given, and accuracy of estimation is analyzed, simulated. FFT method and phase difference method are used for comparing the FPE estimation suggested in the paper. Two ADCS, 16bits resolution are selected, and their effective number of bit (ENOB), as characterization of ADCS' nonlinearity, is computed with sinewave FPE method. Analyzing method suggested in the paper supplies significant estimation way to improve ADCS application.**

Key words: estimation, nonlinearity, ADCS, sinewave, fitting.

1. Introduction

With the increase in the last 10-15 years of applications requiring data processing based computer and data acquisition, analog-to-digital converters (ADC) and ADC system (ADCS), are widely applied in all fields. Characterization and modeling of ADCS (including digitizer, ADC device and other ADC application) are the research and development domain of past, present, and surely future activity.

The nonlinearity of ADCS, as the most important parameter evaluating ADCS, is developed by a few methods. The estimation of transition levels characterizing ADCS is commonly achieved using sinewave histogram test. The method requires the ADC to be stimulated using a sinewave generator exhibiting high spectral purity and possibly employing coherent sampling in order to maximize the efficiency of the estimation algorithm [1]. For evaluating INL and DNL within a given level of uncertainty, the number of samples and records, amount of overdriving voltage, and ratio between sampling rate and sinewave frequency may be selected in accordance to some test standards recognized internationally. In the situation of testing high-resolution ADCS (16bits or more), it is usually convenient the FFT test, which can be satisfactorily performed using several thousands samples. But the drawback of FFT test is that it yields a frequency domain description of the integral nonlinearity, which can not be directly employed for assessing the uncertainty of static measurement [3]. In addition to that, FFT test requires test system parameters to be configured into coherent sample mode, and high frequency spectral purity.

Estimating sinewave parameters is a basic function in many test and data processing systems. Based on accurate knowledge of sinewave parameters (amplitude, dc offset, frequency, and phase), the original sinewave is reconstructed. Parameter estimation is usually applied in circuit identification. A linear circuit can be completely characterized by inputting a sinewave and measuring the parameters of output signal. Differences between the actual parameters of input sinewave and the measured output, reflects the circuit's impact on the data and help identify its unique transfer function. Relying on the parameter estimation theory, the nonlinearity and effective bits of ADCS can be estimated. The purpose of this paper is to describe parameter estimation method which used to estimate the nonlinearity of ADCS, and discuss the details of the associated calculation procedures.

2. Nonlinearity estimation by sinewave

A sinewave can be completely characterized by its four parameters: amplitude, dc offset, frequency, and phase. Mathematically, the analog signal is represented as a continuous function of time by:

$$x(t) = A \cdot \sin(2\pi \cdot ft + \theta) + C_0 \quad (2\text{-}1)$$

Where A = amplitude, D = dc offset, f = frequency, θ = phase angle. Another representation is:

$$x(t) = A_0 \cdot \cos(\omega_0 t) + B_0 \cdot \sin(\omega_0 t) + C_0 \quad (2\text{-}2)$$

Where $A_0 = A \cdot \sin(\theta)$, $B_0 = A \cdot \cos(\theta)$, $\omega_0 = 2\pi \cdot ft$. If the sinewave, represented by (2-2), is sampled in the t_k, $(k = 0,1,2,\cdots,N-1)$, the value sampled is then given by:

$$x(t_k) = A_0 \cdot \cos(\omega_0 t_k) + B_0 \cdot \sin(\omega_0 t_k) + C_0 \quad (2\text{-}3)$$

The error between original signal and signal sampled will be represented by :

$$E(A_0, B_0, C_0) = \sum_{k=0}^{N-1} [x(t_k) - A_0 \cdot \cos(\omega_0 t_k) - B_0 \cdot \sin(\omega_0 t_k) - C_0]^2 \quad (2\text{-}4)$$

With the sinewave parameters estimation procedure, a sinewave with specified parameters is applied to the input of the ADC(or ADCS), and a record of data is taken. By varying the phase, amplitude, dc value, and (if necessary) frequency of the fit function, a sinewave function is fitted to the record taken to minimize the sum of the squared difference between the function and the data. In the situation which sample clock and the input frequency are known and stable, three parameters (A_0 , B_0 , C_0) least squared fitting method is used to estimate parameter matrix. Defining D , X , C as following

$$D = \begin{bmatrix} \cos(\omega_0 t_0) & \sin(\omega_0 t_0) & 1 \\ \cos(\omega_0 t_1) & \sin(\omega_0 t_1) & 1 \\ \vdots & \vdots & \vdots \\ \cos(\omega_0 t_{N-1}) & \sin(\omega_0 t_{N-1}) & 1 \end{bmatrix}$$

$$X = \begin{bmatrix} x(t_0) \\ x(t_1) \\ \vdots \\ x(t_{N-1}) \end{bmatrix}, \quad C = \begin{bmatrix} A_0 \\ B_0 \\ C_0 \end{bmatrix}$$

parameters matrix C can be computed by (2-5).

$$C = (D^T D)^{-1}(D^T X) \qquad (2\text{-}5)$$

For general application, input signal frequency needs to be estimated. the estimation method given above transfers to four parameters estimation. The basic thoughtway, used in four parameters estimating of sinewave, is iteration. The iterative procedure is completed by varying frequency value of fitting function, and locates the minimum error value among $E_i(A_0, B_0, C_0)$, $i = 0,1,2,\cdots$.

The parameters group (ω_0, A_0, B_0, C_0), corresponding the minimum error, is parameter estimation result. The four parameters estimation procedure is as following:

(1) from test configuration, a approximate range of frequency is scaled. Giving the initial frequency value, a data record is taken.

(2) using DFT (or FFT), input signal frequency, represented ω_F , is computed, and its up-limit and down-limit is respectively $\omega_0^L = \omega_F - \omega_C / N$, and $\omega_0^H = \omega_F + \omega_C / N$. Where ω_C is the sampling clock frequency, N is the size of DFT (or FFT). In addition to the DFT for finding out ω_0^L , ω_0^H , step (3) gives another way.

(3) by locating the position of cross zero point among data record, the ω_0^L , ω_0^H can be obtained. Assuming the cross zero point sequence

$$(p_0, p_1, p_2, \cdots, p_m, \cdots, p_n, \cdots),$$

and corresponding sample range:

$$[(t_k^0, t_{k+1}^0), (t_k^1, t_{k+1}^1), \cdots, (t_k^m, t_{k+1}^m), \cdots, (t_k^n, t_{k+1}^n), \cdots] \quad ,$$

frequency range can be computed as following:

$$\omega_0^L = 2\pi[(n-m)/2]/(t_{k+1}^n - t_k^m)$$

$$\omega_0^H = 2\pi[(n-m)/2]/(t_k^n - t_{k+1}^m) .$$

(4) setting $\Delta\omega_0 = \omega_0^H - \omega_0^L$, and taking $2M+1$ points from range $[\omega_0^m, \omega_0^n)$ with equal interval, 2M+1 points can be represented:

$$\omega_0^{mn}, \omega_1^{mn}, \cdots \omega_{2M+1}^{mn} ,$$

the matrix G_0 is formed:

$$G_0 = \begin{bmatrix} A_0(\omega_0^{mn}) & B_0(\omega_0^{mn}) & C_0(\omega_0^{mn}) & E_0(\omega_0^{mn}) \\ A_1(\omega_1^{mn}) & B_1(\omega_1^{mn}) & C_1(\omega_1^{mn}) & E_1(\omega_1^{mn}) \\ \vdots & \vdots & \vdots & \vdots \\ A_{2M+1}(\omega_{2M+1}^{mn}) & B_{2M+1}(\omega_{2M+1}^{mn}) & C_{2M+1}(\omega_{2M+1}^{mn}) & E_{2M+1}(\omega_{2M+1}^{mn}) \end{bmatrix}$$

Where $[A_i(\omega_i^{mn}), B_i(\omega_i^{mn}), C_i(\omega_i^{mn}), E_i(\omega_i^{mn})]$ is the parameter group at ω_i^{mn} .

(4) getting the minimum value:

$$E_r = \min(E_0(\omega_0^{mn}), E_1(\omega_1^{mn}), \cdots, E_{2M+1}(\omega_{2M+1}^{mn})) ,$$

correspondingly, $(A_r(\omega_r^{mn}), B_r(\omega_r^{mn}), \cdots, E_r(\omega_r^{mn}))$ group is formed and the maximum bias of frequency among the group can be gained by: $\Delta\omega = \Delta\omega_0 / M$.

(5) changing the frequency range $[\omega_j^L, \ \omega_j^H]$ (after the (j-1)th computing),

$$\omega_i^L = \omega_{j-1}^{mn} - \Delta\omega / M^{j-1} ,$$

$$\omega_i^H = \omega_{j-1}^{mn} - \Delta\omega / M^{j-1} .$$

(6) by using the step of (4) and (5), the matrix G_j , and E_r^j is computed, and maximum frequency bias is given by: $\Delta\omega = \Delta\omega_0 / M^j$.

(7) repeating the step (5) and (6) until the frequency accuracy required can be obtained. the frequency estimation maximum bias is $\Delta\omega_{max} = \Delta\omega_0 / M^p$. Where p is the iterative number when the accuracy required is acquired.

3. Four parameter estimation (FPE) accuracy of sinewave

Giving the ideal sinewave parameters: ω_0, A_0, B_0, C_0 , the estimation parameters are $\hat{\omega}_0$, $\hat{A}_0, \hat{B}_0, \hat{C}_0$, from section 2 and 3 above, the difference between ideal and estimation sinewave at sample position t_k is:

$$\begin{aligned} e_k &= \hat{A}_0 \cos(\hat{\omega}_0 t_k) + \hat{B}_0 \sin(\hat{\omega}_0 t_k) + \hat{C}_0 \\ &\quad - A_0 \cos(\omega_0 t_k) - B \sin(\omega_0 t_k - C_0 \end{aligned}$$

That is

$$e_k = [A_0 \sin(\omega_0 t_k) - B_0 \cos(\omega_0 t_k)] \cdot t_k \cdot \Delta\omega + \Delta A_0 \sin(\omega_0 t_k) - \Delta B_0 \cos(\omega_0 t_k) - \Delta C \tag{3-1}$$

Removing 2^{nd} small terms, e_k can be represented approximately to:

$$e_k \approx [A_0 \sin(\omega_0 t_k) - B_0 \cos(\omega_0 t_k)] \cdot t_k \cdot \Delta\omega + \Delta A_0 \sin(\omega_0 t_k) - \Delta B_0 \cos(\omega_0 t_k) - \Delta C \tag{3-2}$$

Where $\Delta\omega_0 = \hat{\omega}_0 - \omega_0$, $\Delta A_0 = \hat{A}_0 - A_0$, $\Delta B_0 = \hat{B}_0 - B_0$, $\Delta C_0 = \hat{C}_0 - C_0$.

To least square solution, under the condition $\omega = \omega_0$, is $\hat{A}_0 = A_0$, $\hat{B}_0 = B_0$, $\hat{C}_0 = C_0$, from (3-2), following approximate equalities can be obtained:

$$\begin{cases} \Delta A_0 \approx B_0 \Delta\omega \cdot (t_{N-1}/2) \\ \Delta B_0 \approx -A_0 \Delta\omega \cdot (t_{N-1}/2) \\ \Delta C_0 = 0 \end{cases} \tag{3-3}$$

replacing (3-2) with (3-3), the estimation error can be represented to:

$$e_k \approx [A_0 \sin(\omega_0 t_k) - B_0 \cos(\omega_0 t_k)] \cdot (t_k - t_{N-1}/2) \cdot \Delta\omega \tag{3-4}$$

4. Algorithm simulation

Using Matlab tool implements algorithm, and compares that with FFT method, phase difference (PD for short) method. Simulation condition is as following:

(1) sequence size: N=1024, cosine amplitude A=3.0V, sine amplitude B=4.0V, frequency $\omega = \pi$,C=0.2V;

(2) using equal interval sampling, $t_k = kT$, $(k = 0,1,2,\cdots,N-1)$, $T = 0.01s$;

(3) noise characteristics: Gauss noise model, $E(V_n) = 0$, $Var(V_n) = 0.125 \times 10^{-4}$;

(4) spurious frequency:
f_1=1.0(Hz), f_2=2 π (Hz), A_1=A_2=$0.005/2^{0.5}$V

(5) FFT window function is hanning window;

(6) M=5 for four parameters estimation procedure;

(7) simulation number is 100, and E_{rms} is computed to each simulation; The simulation results show:

(1) under equal interval sampling mode, PD method is good in parameter estimation, compared to FPE;

(2) under unequal interval sampling mode, FFT and PD method is unavailable. However, FPE procedure can accurately estimates four parameters of sinewave;

(3) under noise and non-noise circumstance, compared to PD method, FPE method can increase estimation accuracy by increasing iteration number, and is of more flexible and stronger noise immunity ability.

Accuracy of estimation algorithm is simulated by setting noise level and spurious harmonic terms level that is expressed to SNR(dB). By varying SNR, the results, derived from PD method and FPE method, can be described respectively by each parameter versus SNR. As shown by the figure (4.4), with decrease of SNR, estimation error of C_0 increases when using PD method and FPE method respectively. Between the PD method and FPE, the later has much higher estimation accuracy to DC parameter. However, within smaller SNR value range (less than 40dB), FPE method is less estimation error than that of PD method. For parameters (ω_0, A_0, B_0), PD and FPE have approximately equal accuracy.

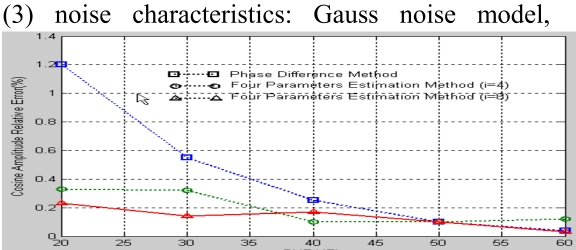

Figure (4.1) Cosine Amplitude Relative Error vs. SNR

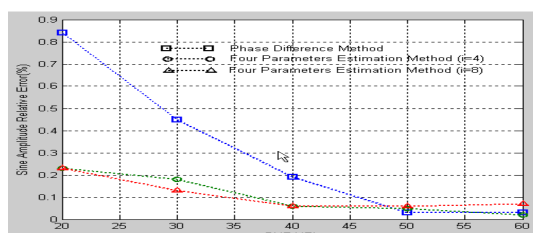

Figure (4.2) Sine Amplitude Relative Error vs. SNR

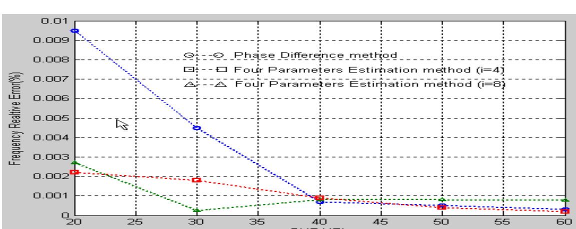

Figure (4.3) Frequency Relative Error vs. SNR

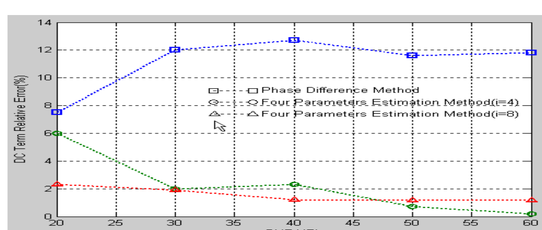

Figure (4.4) DC Term Relative Error vs. SNR

5. ADCS nonlinearity test using FPE

Nonlinearity of ADCs (or ADCS) is characterized by loss of their efficient number of bits (ENOB). For an input sinewave of specified frequency and amplitude, after correction for gain and offset, the ENOB is

$$\mathrm{ENOB} = \mathrm{B} - \log_2\left(\frac{V_{rms}(\text{actual} - \text{noise})}{V_{rms}(\text{ideal - quantization - error})}\right) \qquad (5\text{-}1)$$

Where B is the number of ADCS, $V_{rms}(\text{ideal} - \text{error})$ come from the quantization noise, represented as $\mathbf{e_q}$ (uniformity distribution), its statistics characteristic is:

$$\begin{cases} E(e_q) = 0 \\ Var(e_q) = \dfrac{q^2}{12} \end{cases} \qquad (5\text{-}2)$$

In (5-2), q, the ideal quantization interval, equal to $FSR/2^N$. $V_{rms}(\text{ideal} - \text{error})$ is correspondingly $q/\sqrt{12}$, that is

$$V_{rms}(\text{ideal} - \text{error}) = \frac{FSR}{2^N\sqrt{12}} \qquad (5\text{-}3)$$

Replacing $V_{rms}(\text{ideal} - \text{error})$ with (5-3), (5-1) is

$$\mathrm{ENOB} = \log_2\left(\frac{FSR}{\sqrt{12}\cdot V_{rms}(\text{actual - noise})}\right) \qquad (5\text{-}4)$$

Using the sinewave FPE method given in the paper, $V_{rms}(\text{actual} - \text{noise})$ can be obtained by fit residuals, that is

$$r_m = x_m - \hat{A}_0\cos(\hat{\omega}_0 t_m) - \hat{B}_0\sin(\hat{\omega}_0 t_m) - \hat{C}_0 \qquad (5\text{-}5)$$

And the rms error is given as (5-6).

$$e_{rms} = \sqrt{\frac{1}{N}\sum_{m=1}^{N} r_m^2} = V_{rms}(\text{actual} - \text{noise}) \qquad (5\text{-}6)$$

Consequently, (5-4) is

$$\mathrm{ENOB} = \log_2(FSR) - \frac{1}{2}\log_2\left(\frac{12}{N}\sum_{m=1}^{N} r_m^2\right) \qquad (5\text{-}7)$$

In (5-7), FSR is full scale range of ADCS. N is the number of sample points.

With the test profile described above, using sinewave FPE procedure and algorithm, two different ADCS: NI PCI-6251 (National Instrument Co.), PCI-1716 (Advantech Co.) are tested, and the test result are shown in figure 5.1.

From figure 5.1, it is shown that the ENOB is the function of frequency. For validating of ENOB, Agilent3458A is used to test input signal rms value. ENOB test results shown that FEP has enough accuracy in estimating ADCS' ENOB.

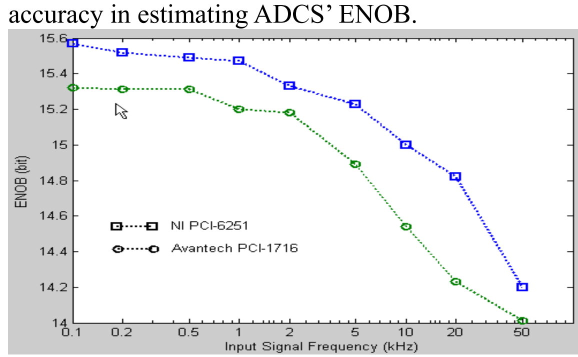

Figure 5.1 ENOB vs. Input Signal Frequency

6. Conclusion

Because of not restrict limit to sampling interval, FPE method has good estimation to ADCS. Consequently, test configuration cost will be decreased, especially in sample clock jitter situation. To rms test, after ENOB is estimated, the measurement error can be estimated with enough accuracy.

7. References

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