Analytical Solution to the Fourth Moment Equation for Radio Wave Propagating in the Turbulent Ionosphere

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It is well known that many natural and artificial media vary randomly with time and space, and therefore waves propagating through these media will suffer distortions. The phases and amplitudes of waves will be fluctuated, i.e., scintillation. The influences from the turbulent ionosphere will bring radio wave with the fluctuation of arrival time, broadening pulse width, altering energy distribution, changing kurtosis, phase and amplitude scintillation. Additionally, these problems are of importance to design, operation and mitigation for various kinds of systems, such as communications, remote sensing systems, radar and sonar

The topic of this paper is focused on the fourth order moment of radio wave propagation through the turbulent ionosphere. The method developed here is also suitable for other similar fields. There are many techniques and methods for solving the fourth moment of wave propagation through random media developed by worldwide researchers for a few decades, such as the diagram method, phase screen theory, multiple-phase-screen model, Born and Rytov solution, path integral method, and parabolic equation method (PEM). Among the different theories developed, an important place is occupied by PEM or moment equations. For weak fluctuation regimes, solutions obtained by phase screen method and Rytov approximation are satisfied and agree well with experimental data, while for strong fluctuation regimes, solutions are obtained only for some special cases. An analytic solution to the fourth moment of a plane wave in the general scattering regimes has been found by means of a modification of the Gaussian solution $\Gamma_{2,2}^{G}$ for the full saturation case by adding a correction term r, which is obtained by Rytov method for its own equation.

$$\Gamma_{2,2} = \Gamma_{2,2}^{G} + r \tag{1}$$

$$\Gamma_{2,2}^{G} = \left\langle u_1 u_1'^{*} \right\rangle \left\langle u_2 u_2'^{*} \right\rangle + \left\langle u_1 u_2'^{*} \right\rangle \left\langle u_2 u_1'^{*} \right\rangle \tag{2}$$

The term of Gaussian solution can be obtained from the well-known solutions to the second moment. The term of correction r is obtained by Rytov approximation method. Afterwards, the solution $\Gamma_{2,2}$ is used to investigate the intensity coherence function and statistical properties with simulations. Also, the solution (1) is used to derive the amplitude scintillation index S4, which is validated by analyzing the ionospheric scintillation with experimental data of satellite beacon observations.