Quasi-linear and Nonlinear Wave-Particle Interactions in the Outer Electron Radiation Belt

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Recent work has reemphasized the importance of cyclotron-resonant wave particle interactions for radiation belt electrons, being at least partly responsible for both acceleration and loss. In particular, plasmaspheric hiss, electromagnetic ion cyclotron (EMIC), and whistler-mode chorus waves in the low density outer zone have been implicated. Quasi-linear diffusion coefficients, which have recently been calculated efficiently, provide reasonable estimates for the timescales, given empirical, statistical models of the wave populations [1]. For detailed comparison with particle observations it is necessary to evolve the particle distribution function according to the corresponding diffusion equation. However, this is numerically challenging because the diffusion is fully two dimensional, i.e., pitch angle, energy, and cross diffusion coefficients can be comparable in magnitude, and rapidly varying, leading to unphysical numerical results.

A promising approach to dealing with this is to transform to new variables in which the cross diffusion term vanishes. This transformation involves a set of ordinary differential equations for level curves of the new variables, in the plane of the first two adiabatic invariants J_1 and J_2 (both of which are broken by the wave particle interactions). Choosing the level curves to be orthogonal leads to the eigenvectors of the diffusion matrix, but this is not necessary. One of the new variables may be specified as an arbitrary, convenient function of (J_1, J_2), while still eliminating the cross diffusion term from the transformed equation. Numerical simulation in the new variables, after transforming back to energy and pitch angle, gives well-behaved and realistic heating and loss of outer zone electrons by chorus waves, with time scales on the order of 1 day following magnetic storms [2].

The quasi-linear description becomes invalid for sufficiently strong waves, and nonlinear effects have been identified for both EMIC and chorus waves. In this case a fully nonlinear treatment is called for; this is conveniently done in a Hamiltonian framework [3]. A monochromatic analysis recovers diffusion in one limit and phase bunching and phase trapping effects arise in the opposite limit, leading to advection rather than diffusion in particle pitch angle and energy. The calculation of phase trapping effects can be generalized to incorporate time-varying frequency as well as relativistic particle energy. The corresponding rates have been evaluated for wave parameters appropriate to the outer electron radiation belts, and can give very rapid energization for phase trapped electrons. Note that neither the quasi-linear nor nonlinear approaches discussed here address the self-consistent generation of waves. However, this must also play an important role, which is still not well understood.

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[3] J.M. Albert, “Gyroresonant interactions of radiation belt particles with a monochromatic electromagnetic wave,” J. Geophys. Res., vol. 105, pp. 21,191-21,209, 2000.