HUYGEN'S PRINCIPLE APPLIED TO RADIO WAVE PROPAGATION IN ANISOTROPIC MEDIA

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Abstract
Huygens principle describes radio wave propagation in terms of the development of the electromagnetic field from one wavefront to another. This approach finds its mathematical expression in terms of Kirchhoff integrals. For isotropic propagation media, the necessary integrals can be derived from reciprocity relations. In the current paper, it is shown that it is possible to extend the approach to anistropic media and hence produce effective propagation algorithms, in the spirit of Huygen's principle, for such media. In particular, it is shown that these ideas can be useful in describing propagation through the ionosphere.

The Basic Method
According to Huygen's principle, each point of a wavefront is the source of a secondary spherical wave and the envelope of these secondary waves forms the new wavefront. This principle finds its mathematical expression in the integral equations that can be derived from the reciprocity principle. In his classic monograph on the reciprocity principle, Monteath [1] establishes that, for an arbitrary time harmonic electromagnetic field $(\underline{E},\underline{H})$,

$$\underline{E}(\underline{r}_0)\cdot\underline{J}_0 = \int_S (\underline{E}\times\underline{H}_0 - \underline{E}_0\times\underline{H})\cdot d\underline{S} \tag{1}$$

where $(\underline{E}_0,\underline{H}_0)$ is the electromagnetic field of a Hertzian dipole with current vector $\underline{J}_0$ and located at point $\underline{r}_0$. Similarly, it can be shown that

$$\underline{H}(\underline{r}_0)\cdot\underline{M}_0 = -\int_S (\underline{E}\times\underline{H}_0 - \underline{E}_0\times\underline{H})\cdot d\underline{S} \tag{2}$$

where $(\underline{E}_0,\underline{H}_0)$ is the electromagnetic field that results from magnetic dipole with current vector $\underline{M}_0$ and located at point $\underline{r}_0$. These integral relations are valid for an isotropic medium having arbitrary spatial variations in both permittivity and permeability and S can be any surface that separates the sources of $(\underline{E},\underline{H})$ from the point $\underline{r}_0$. Given suitable dipole solutions, equations (1) and (2) can be used to develop the field $(\underline{E},\underline{H})$ away from surface S. In particular, by dividing space by a sequence of surfaces, electromagnetic field $(\underline{E},\underline{H})$ can be advanced from surface to surface and hence throughout space. If the surfaces are sufficiently close, the dipole fields need only be approximate (geometric optics solutions for example) and it is possible to advance the fields through space for quite general spatial variations in both permittivity and permeability. In the case that geometric optics solutions are used, the surfaces will need to be a distance L apart, where $L << D^2/\lambda$ for D a typical length scale in the propagation medium. This procedure can form the basis of a quite general radio wave propagation algorithm, as shown in [2] and [3].

Although reciprocity results do not exist for completely general anisotropic media, Budden [4] has shown that a reciprocity style result can still exist for magneto-ionic media. For this generalization to hold, however, the direction of the background magnetic field must be reversed between the 'reciprocal' fields. It is also possible to show that equations (1) and (2) will still hold for magneto-ionic media providing the background magnetic field is reversed between the fields $(\underline{E},\underline{H})$ and $(\underline{E}_0,\underline{H}_0)$. Indeed, from the standard derivation of (1) and (2), it can be seen that they will hold for general anisotropic media providing the permittivity and permeability tensors are both transposed between the fields $(\underline{E},\underline{H})$ and $(\underline{E}_0,\underline{H}_0)$ (that is $\varepsilon_{0ij} = \varepsilon_{ji}$ and $\mu_{0ij} = \mu_{ji}$). Based upon this extension to (1) and (2), the current paper seeks to extend the propagation techniques developed in [2] and [3] so that they apply to anisotropic propagation media. In particular, the paper develops the approach for ionospheric radio wave propagation applications.

The Geometric Optics Solution

In order to apply the above procedure, we need to develop suitable dipole solution. To achieve this, we assume that the standard geometric optics (GO) ansatz $\underline{E} = \underline{\hat{E}} \exp(-j\beta\phi)$ to be valid. Furthermore, we take the anisotropy to be weak with $\varepsilon_{ij} = \varepsilon_0 \left(N^2 \delta_{ij} + \frac{j}{\beta} \tau_{ij} \right)$ and $\tau_{ij} = -\tau_{ij}$. In the magneto-ionic case, $N^2 = 1 - X$, $\tau_{12} = -Y_3 X$, $\tau_{13} = Y_2 X$, and $\tau_{23} = -Y_1 X$ where $X = \omega_c{}^2 / \omega^2$, $\underline{Y} = -(\omega_B / \omega)(\underline{B}_0 / B_0)$, ω_c is the plasma frequency, ω_B is the gyro frequency, ω is the wave frequency and $\underline{B}_0$ is the background magnetic field. For weak anisotropy, we can load all of its effect into the polarization vector $\underline{P} = \underline{\hat{E}} / \hat{E}$ and the field can be derived from the ray tracing equations in which the rays satisfy the equation

$$\frac{d^2 x_i}{dg^2} = N \frac{\partial N}{\partial x_i} \tag{3}$$

and the polarization vectors satisfy

$$\frac{dP_i}{dg} + \frac{dx_i}{dg} \frac{P_j}{N} \frac{\partial N}{\partial x_j} + \frac{\beta}{2} \left(\tau_{ij} P_j \right) - \frac{\beta}{2N^2} \frac{dx_i}{dg} \left(\frac{dx_j}{dg} \tau_{jk} P_k \right) = 0 \tag{4}$$

where g is the group distance along the ray path (we have assumed the Einstein summation convention). The change in amplitude $\hat{E}$ can be found by calculating the ray the deviations X_i^I of nearby rays from the main ray path (note that $x_i + X_i^I$ are the coordinates of the nearby ray that is labeled by I). The deviations satisfy

$$\frac{d^2 X_i^I}{dg} = \frac{1}{N^2} \frac{\partial N^2}{\partial x_i} \frac{dx_j}{dg} \frac{dX_j^I}{dg} + \frac{1}{2} \left(\frac{\partial N^2}{\partial x_i \partial x_j} - \frac{1}{N^2} \frac{\partial^2 N^2}{\partial x_i} \frac{\partial N^2}{\partial x_j} \right) X_j^I \tag{5}$$

and we calculate deviations for two independent nearby rays (I from 1 to 2). We then calculate the ray orthogonal deviations according to $\overline{X}_i^I = X_i^I - (dx_j / dg) X_j^I (dx_i / dg) / N^2$ and from them the effective spreading distance according to $s = \sqrt{N(\underline{x}) A(\underline{x}) / N(\underline{x}_0) A(\underline{x}_0)}$ where $\underline{x}_0$ is the coordinate on the main ray at unit distance from the source. We will have $\hat{E} = (\omega\mu I h / 4\pi s) \exp(-\beta\phi)$ and $\hat{E}_i = \hat{E} P_i$ where Ih is the dipole moment. It should be noted that $\underline{H} = (d\underline{x}/dg) \times \underline{E} / \eta_0$ and phase distance is calculated from $\phi = \int N^2 dg$ where the integral is evaluated along the ray path from the source to the field observation point.

The Practical Implementation

The integral equations (1) to (2) and the above GO solutions provide, in principle, a means of developing the electromagnetic field out from a given source. In practice, however, this can be computationally expensive since there is a need to solve a large system of ray tracing equations for a large number of ray paths between intermediate surfaces. Fortunately, there are a large number of simplifications that can be applied. For propagation that does not deviate too greatly from that of the GO approximation, the integral equations (1) to (2) reduce to

$$\underline{E}(\underline{r}_0) \cdot \underline{J}_0 = -\int_S \frac{2N}{\eta_0} \underline{E} \cdot \underline{E}_0 \, dS \tag{7}$$

where the surface of integration is tangent to the wave front (we have assumed that the propagation directions of fields $\underline{E}$ and $\underline{E}_0$ are approximately parallel). Using kernel $\underline{E}_0$, this equation allows us to propagate the electric filed $\underline{E}$ forward from the surface S. In practice, we will use a GO approximation to the electric field for kernel $\underline{E}_0$ and will need some simplification to the ray tracing procedure in order to make this practical. Since most propagation of interest

will have approximately planar ray paths, the change in polarization can be described by the angle γ between the polarization vector and the normal to the plane and, from (4), this angle satisfies

$$\frac{d\gamma}{dg} = \frac{\beta_H X}{2N} \frac{\underline{B}_0}{B_0} \cdot \frac{d\underline{r}}{dg} \tag{8}$$

(see [5] for a similar expression). Furthermore, over short distances (i.e. the distance between intermediate surfaces), we can use the approximation $\hat{E}_0 \approx (\omega\mu I h / 4\pi d)\exp(-\beta\phi)$ where d is the geometric distance and ϕ is evaluated along a linear path between end points.

Based upon the above simplifications, a propagation algorithm that goes one stage beyond the GO approximation could consist of the following steps:

1) Calculate the ray path between receiver and transmitter.
2) Divide the ray path into suitably short segments and consider a sequence of surfaces that are orthogonal to the path and that pass through the end points of these segments.
3) Propagate the electric field from one of the above surfaces to the next by means of (7) and the simplified GO solutions above.

Clearly, a major part of the computational effort in the above algorithm will consist of the evaluation of the integrals over the intermediate surfaces. These integrals can rarely be evaluated analytically and so a numerical procedure is required. In the current work, a simple 2D Simpson rule has been used with the integrals truncated at three times the Fresnel scale $D = \sqrt{\lambda L}$ from the ray path (L is the total propagation distance). The discretisation interval is given by $\Delta = 0.01H^2 / D$ where H is the scale of variations in the refractivity. The integrals themselves are very slowly convergent in terms of distance from the ray path and a means of accelerating their convergence is required. The nature of the integral, however, means that it will oscillate about its final value as the integration distance from the ray path increases. By tracking these oscillations, it is therefore possible to estimate the converged value and this approached has been used in the current work to great effect. The above simplifications are only feasible when the ray paths between representative surfaces are mildly nonlinear. When the ray paths between surfaces are strongly nonlinear, however, full ray tracing will need to be used in calculating the GO solution. Such a situation could occur when, for example, calculating ionospheric HF propagation between terrestrial stations with some irregular structure present in the ionosphere. If the structure scale is of only a few kilometers, the GO solution will break down. For a single intermediate surface through the ray path apogee, the above approach can be applied since the GO solutions from either side will be valid up to this surface. The paths to the surface, however, will be highly nonlinear.

A Simple Example

We consider the case of a transmitter 500km above the ground and a receiver directly below (a satellite communication link scenario). The link is assumed to be at a latitude of $65^o N$ with an ionosphere consisting of a single F2 layer with $foF2 = 10MHz$, $hmF2 = 250km$ and $ymF2 = 50km$. Consider the situation where a 10% depletion moves across the line of sight (the depletion is assumed to be infinite in vertical extent and have a Gaussian distribution laterally with a width of 10km). Fig.2 shows the signal strength at the receiver (scaled on the value for free propagation) as a function of depletion displacement from line of sight. (In the simulations, both receiver and transmitter have linear polarized antennas that are aligned). The figure shows signal strengths for the GO approximation and the current approach (labeled Huygen) with, and without, the effect of Earths magnetic field. It will be noted that, as the depletion moves across the link line of sight, there is considerable variation in signal strength. The effect of Earths magnetic is considerable, mainly through Faraday rotation (as indicated by the GO solution). From the Huygen solutions, however, it will be noted that diffraction effects are very strong when the depletion is crossing the line of sight. Furthermore there is considerable difference between the effect with and without Earths magnetic field. It will be noted that once the depletion has passed, both the GO and Huygen approaches give much the same result.

References

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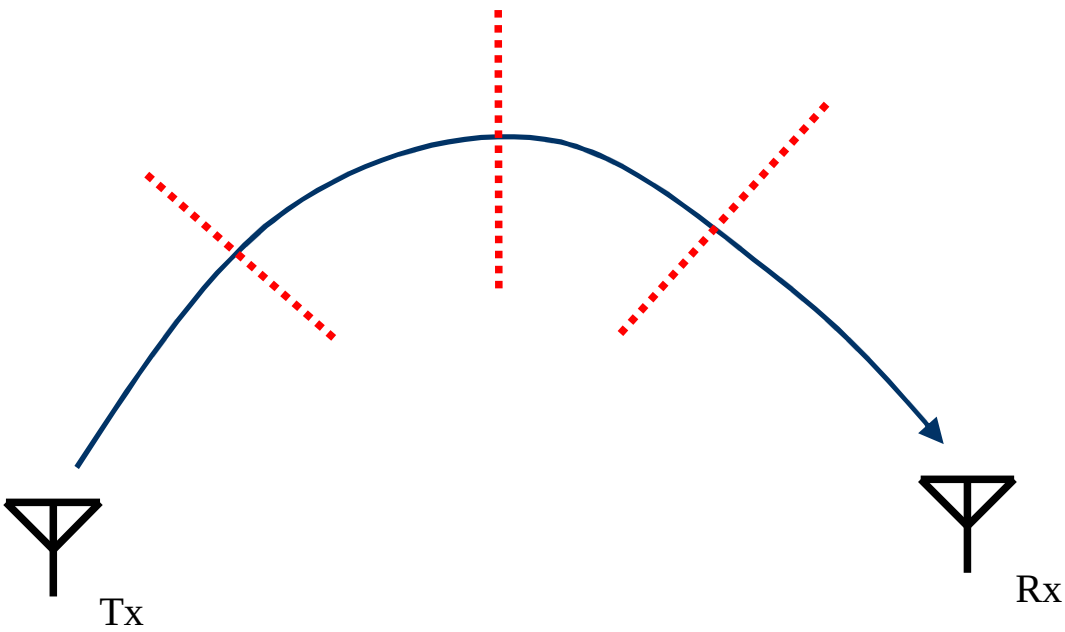

Figure 1. GO propagation path with intermediate surfaces for evaluating integrals.

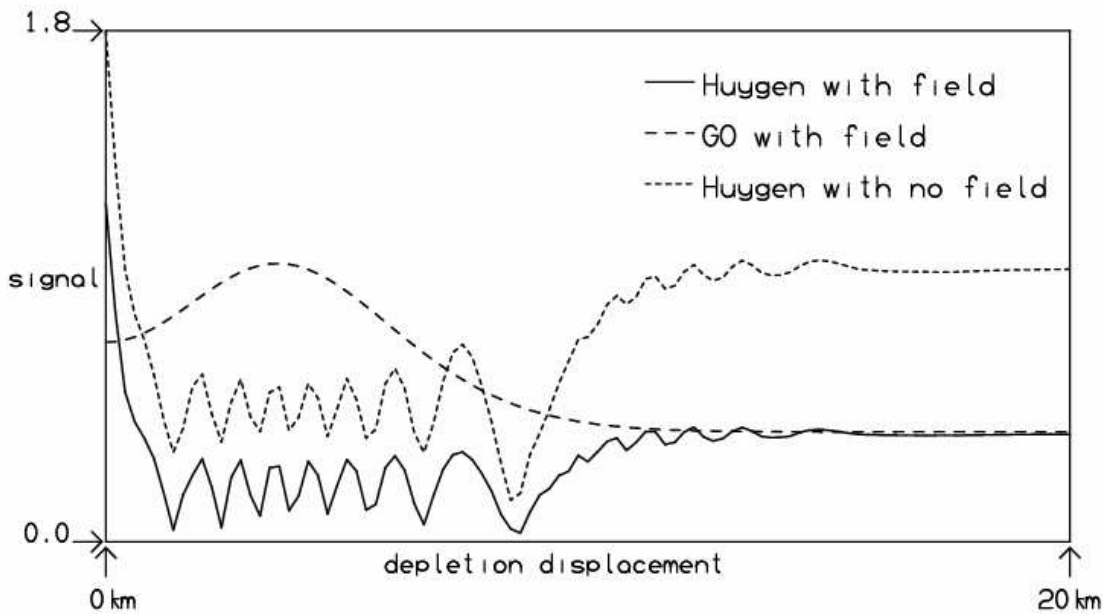

Figure 2 Signal strength, as a function of displacement from line of sight, for Huygen and GO approximations.