CONSTANT GAIN KALMAN FILTER APPROACH USING GENETIC ALGORITHM FOR ESTIMATING THE IONOSPHERIC TOTAL ELECTRON CONTENT OVER LOW LATITUDE INDIAN AND OTHER STATIONS DURING MAGNETIC STORMS

R. Anandraj[1], M. R. Ananthasayanam[2], P.R.Mahapatra[3], and P. Soma[4]
[1,4]ISRO Telemetry, Tracking and Command Network, Bangalore 560058, India,
emsraju@rediffmail.com, soma@istrac.org
[2]c/o Department of Aerospace Engineering, Indian Institute of Science, Bangalore 560012, India,
sayanam@aero.iisc.ernet.in
[3] Department of Aerospace Engineering, Indian Institute of Science, Bangalore 560012, India,
pravas@aero.iisc.ernet.in

ABSTRACT

In many science and engineering applications using the Kalman filter there could be unmodellable errors in the governing state and measurement equations of many systems. In such cases due to the introduction of process noise in the filter state equations the Kalman gain matrix elements tend to nearly a constant value after an initial transient. This observation provides a possible Kalman filter tuning approach in which instead of adaptively tuning P_0, Q and R respectively the initial state, process, and measurement covariances generally a smaller number of Kalman gain elements can be worked out. Another advantage of the Constant Gain Kalman Filter (CGKF) approach in dealing with the cost J function of the filter is that one need not propagate the covariance equations, which are responsible for the large computational load. The acceptable statistical consistency of the results from the adaptive technique and the constant Kalman gain approach provides the confidence in the latter procedure. There could be slight differences in the gain values between the above approaches due to the relative periods of the transient and the steady state conditions. In the present work dealing with the Ionospheric Total Electron Content (TEC) following Sardon et al formulation, the estimation of the above gains based on the cost function of the Kalman Filter utilizes the Genetic Algorithm. Using the above CGKF technique, the behaviour of the ionosphere over low latitude IGS station at Fortaleza, Brazil based on the data during the great magnetic storm on the 15th July 2000 has been analysed and the results were found to be consistent with the results of Basu et al. In addition, the effects on ionosphere over Indian region have also been analyzed during the ionospheric storm of October 29, 2003 using the dual frequency GPS data collected over Delhi and Ahmedabad. All these studies demonstrate the efficiency of Genetic Algorithm (GA) derived constant Kalman gains for real time applications in GPS.

INTRODUCTION

Satellite Based Augmentation System (SBAS) is one of the most recent innovative applications of Global Positioning system (GPS) for civil aviation. India is on its way in developing SBAS for civil aviation. SBAS has to meet the requirements like availability, accuracy, integrity and continuity, which are critical for safe civil aviation, in particular for Precision Approach. Ionospheric corrections play vital role in all the above four requirements for precision approach. However, in the low latitude, the ionosphere over Indian subcontinent is highly unstable and hence the diurnal variation of TEC has to be obtained precisely which determines the signal delay range. (1 TEC is 10^{16} electrons/m^2, 1 ns = 2.86 TEC and 1 TEC = 0.16 m). In order to obtain accurate TEC estimates the differential instrumental biases must be eliminated [1]. This paper describes a Genetic Algorithm tuned Constant Gain Kalman Filter approach for the estimation of TEC and the differential satellite plus receiver (SPR) biases.

THE STATE AND MEASUREMENT EQUATIONS

The state and measurement equation for every GPS satellite follows as in [1] using Kalman Filter in Estimation Theory (ET). However, they are modified to a four parameter problem and used for the present work since the receiver and satellite instrumental biases cannot be separately estimated in general unless one of the above components of the bias is known or assumed by introducing a reference receiver.

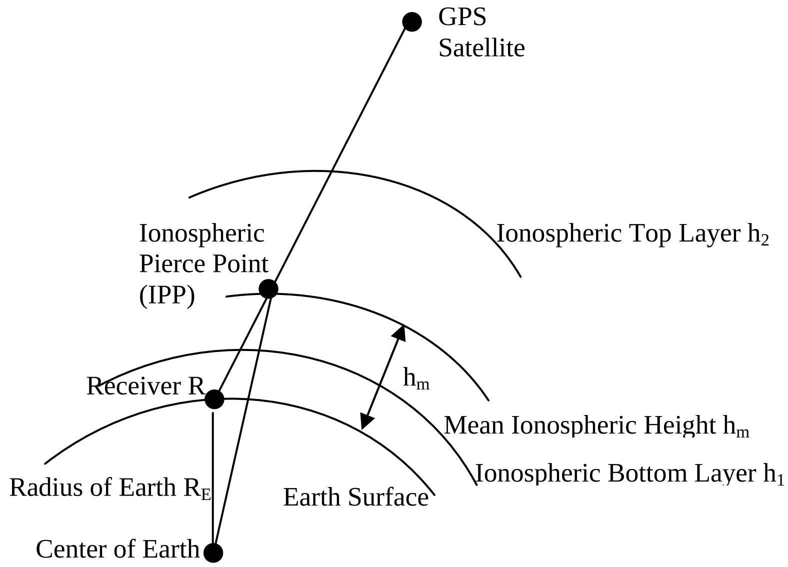

Figure 1. Schematic of the Ionospheric Pierce Point (not to scale).

The state equations are

$$A(t_k) = A(t_{k-1}) + B(t_{k-1}) [\ \varphi_{IR}(t_k) - \varphi_{IPP}(t_k)\ ] + C(t_{k-1}) [\ \lambda_{IR}(t_k) - \lambda_{IPP}(t_k)\ ] + w_1$$
$$B(t_k) = B(t_{k-1}) + w_2; \qquad C(t_k) = C(t_{k-1}) + w_3; \qquad SPR(t_k) = SPR(t_{k-1}) + w_4 \tag{1}$$

The A is the vertical TEC in the zenith direction of the station (receiver location). The B and C are coefficients of linear approximation of slant TEC. The 'SPR' is the differential instrumental biases of the receiver plus the satellite. The 'w's are the assumed white Gaussian process noise to account for unmodeled, and unmodellable features in the state equations. In fact it is this feature that is best handled by the Kalman filter equations unlike the earlier methods such as least square technique by Coco et al [5]. The parameters A, B, C, and 'spr' in the above equation can be estimated using Kalman Filter approach. The 'spr' can be taken to be a constant. The t_{k+1} and t_k refer to the measurement time instants. The quantities φ and λ refer to respectively the latitude and longitude [1]. The measurement equations are:

$$I\,(t_k) = S\,(e)\,.[\ A(t_k) + B(t_k).\ |\varphi_R(\ t_k) - \varphi_{IPP}(\ t_k)\ | + C(t_k).\ |\lambda_R(\ t_k) - \lambda_{IPP}(\ t_k)\ |\ ] + SPR(t_k) + v \tag{2}$$

where I (t_k) is the biased ionospheric term namely the Slant TEC, and 'e' is the satellite elevation. The symbol | . | denotes the modulus. The 'v' is the assumed white Gaussian measurement noise. The slant TEC namely I (t_k) at any time for a given GPS satellite is

$$I\,(t_k) = (L_1\,(t_k) - L_2\,(t_k)) / (\alpha_2 - \alpha_1) \tag{3}$$

where L_1 and L_2 are carrier phases and $\alpha_1 = 40.3/f_1^2$ and $\alpha_2 = 40.3/f_2^2$. (L_1 = 1575.42 MHz and L_2 = 1227.60 MHz). The S(e) the slant factor to account for the longer path of the GPS signal than overhead is given by

$$S(e) = \frac{\sqrt{R_E^2 \sin^2 e + 2R_E h_2 + h_2^2} - \sqrt{R_E^2 \sin^2 e + 2R_E h_1 + h_1^2}}{h_2 - h_1} \tag{4}$$

RESULTS AND CONCLUSIONS

A brief ET and GA is provided in the next section. However in this section the major results are provided. The bottom panels in Figure 2 compare the present vertical TEC, with that in [3] which is the important quantity obtained on July 15, 2000 from GPS measurements at Fortaleza, Brazil. Note the precipitous fall of 75 TECU during 2000-2200 UT. The top panels show that during the same period of time TEC decreases by only 26 TECU on the magnetically quiet day (Σkp = 17) of July 12, 2000. The Figure 3 shows the storm over Indian region from NOAA website on October 29, 2003 which occurred between 9 to 10 UT over Indian region and is quoted to move southward turning at 6.5 UT. The present approach picking up the above storm effect is shown very clearly in Figure 4. In particular the peak values of TEC can be seen to be comparable. All the above comparisons demonstrate the efficiency of the present constant Gain Kalman filtering approach to confidently estimate the variation of TEC over calm and storm conditions.

BRIEF ESTIMATION THEORY (ET) AND CONSTANT GAIN KALMAN FILTER APPROACH

In ET a nonlinear continuous system with time 't' with discrete measurements is described by

$$\mathbf{dX/dt = F(X, U, \Theta, t) + w(t);\ X(0) = X_0\ ;\ \ Z = G(X, U, \Theta, t_k) + v(t_k);\ \ \ k = 1,2,\ldots\ldots N} \tag{5}$$

where **X, Z, Θ, U, w,** and **v** denote respectively the states, measurements, unknown parameters, control inputs, state and measurement noise matrices. The process and measurement noise are assumed to be white with zero mean and covariance **Q** and **R** respectively. The time evolution of the estimate and covariance of the state **X** in terms of the transition matrices ϕ and ψ from local linearisation and the measurements are

$$\mathbf{X_k^-} = \phi_{k-1}\, \mathbf{X_{k-1}} + \psi_{k-1}\, \mathbf{U_{k-1}};\ \mathbf{X(0) = X_0}\ ;\ \ \mathbf{P_k^-} = \phi_{k-1}\, \mathbf{P_{k-1}}\, \phi_{k-1}^T + \mathbf{Q_k};\ \ \mathbf{P(0) = P_0}\ ;\ \ \mathbf{Z_k = H_k\, x_k + v_k} \tag{6}$$

The (-) denotes the estimate before using measurement information. Thus at time t_k an estimate $\mathbf{X_k^-}$ with covariance $\mathbf{P_k^-}$ and the measurement namely $\mathbf{Z_k}$ with covariance $\mathbf{R_k}$ can be combined statistically to obtain

$$\mathbf{X_k^+ = X_k^- + K_k[\, Z_k - H_k\, X_k^-\,] = X_k^- + K_k\, \nu_k\ ;\ \ P_k^+ = [\, I{-}K_k\, H_k\,]\, P_k^-\, [\, I{-}K_k\, H_k\,]^T + K_k\, R\, K_k^T} \tag{7}$$

where $\mathbf{\nu_k}$ called as the innovation. For minimum $\mathbf{P_k^+}$ the optimal Kalman Gain is given by

$$\mathbf{K_k = P_k^-\, H_k^T\, [\, H_k\, P_k^-\, H_k^T + R_k\,]^{-1}} \tag{8}$$

The innovation follows a Gaussian distribution whose probability when maximized leads to the Method of Maximum Likelihood Estimation (MMLE), which operationally is to minimize the cost function

$$\mathbf{J = (1/N)\, \textstyle\sum \nu_k\, [\, H_k\, P_k^-\, H_k^T + R_k\,]\, \nu_k^T = (1/N)\, \sum \nu_k\, [\, \Re\,]\, \nu_k^T = J\,(X_0, P_0, Q, R, \Theta)} \tag{9}$$

based on summation over all measurement times. When **Q** ≡ 0, and all the Kalman gains are zero!. In various applications the Kalman Filter needs compulsively the 'a priori' statistics $\mathbf{P_0}$, **Q** and **R**, which may not be known and difficult to estimate even adaptively [2]. Usually in many problems after the initial transients the Kalman Gain matrix tends to a constant value due to inaccurate system, input, and/or the measurement model. In such cases the constant Kalman gains can be worked out to minimise the above J assuming $\Re$ to be constant whence one need not propagate the covariance equations thus saving enormous computations.

Genetic Algorithm (GA) is an optimisation method mimicking nature and Darwin's theory of survival of the fittest [4]. It is efficient due to coding of the parameter set and not the parameters, uses cost but not its gradients, utilises probabilistic and not deterministic transition rules. In a simple GA an initial population are randomly chosen in the search space. The next generation is created after applying three operators in their order of execution such as (i) R*eproduction:* Individuals are copied to the next generation with probability relative to their fitness, (ii) *Crossover:* Pairs of strings drawn randomly from the population are subjected to crossover. They are randomly paired to create two new descendants. For each pair a crossover location in the bit string is selected at random, which divides the string into two parts. Swapping the remainder of the two strings with each other does crossover, and (iii) *Mutation:* After mutation a bit is randomly selected within the chromosome string and mutated. The termination criteria could be after a fixed number of generations, or the change in fitness value between populations. The parameters in the present GA implementation after some trial and error are the Population size = 200; Bit Length = 32; Probability of crossover = 0.90; Probability of Mutation = 0.05; Convergence: Number of generations = 10. Generate initial population from the search domains randomly. Evaluate each string of the population for fitness and iterate till convergence.

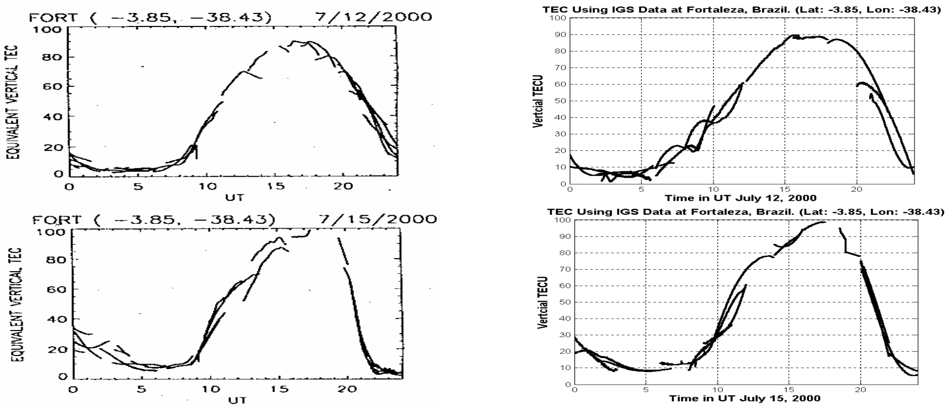

Figure 2. TEC variations over Fortaleza, Brazil for a normal day and during the day of ionospheric (Left column: Results of Basu et al; Right column: Present result)

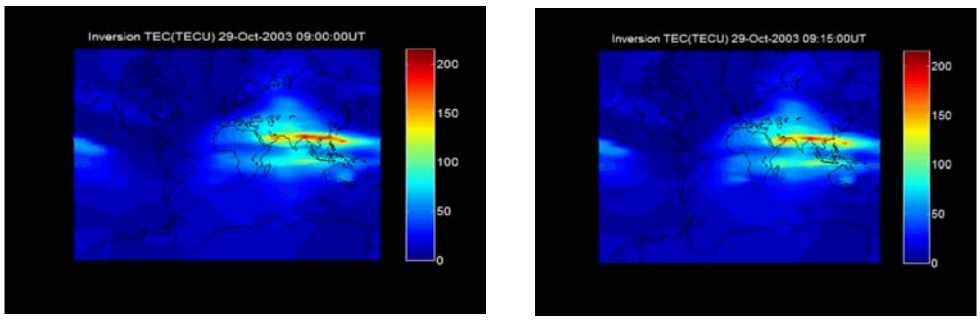

Figure 3. TEC enhancement over Indian region during the ionospheric storm on October 29, 2003. (Obtained from Hhttp://www.ngs.noaa.govH)

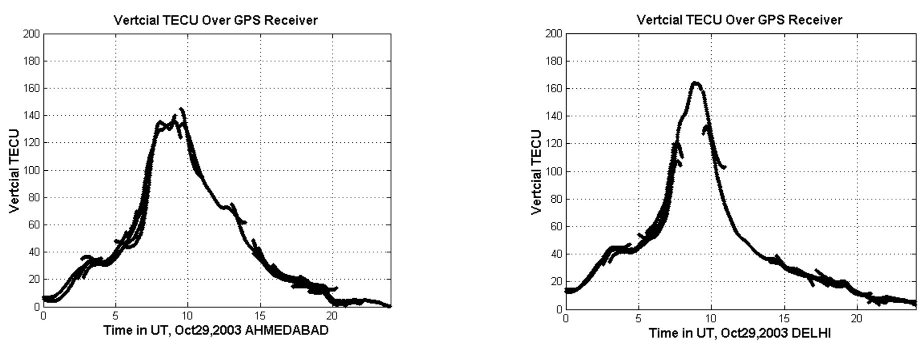

Figure 4. Vertical TEC over Ahmedabad and Delhi during the Ionospheric Storm on October 29, 2003.

REFERENCES

1. Sardon E, Rius A, and Zarraoa N, " Estimation of the Transmitter and Receiver Differential Biases and the Ionospheric Total Electron Content from Global Positioning System Observations", Radio Science, Vol. 29, Number 3, pp. 577-586, May-June 1994.
2. Gemson R.M.O and Ananthasayanam M.R., "Importance of Initial State Covariance Matrix For The Parameters Estimation Using An Adaptive Extended Kalman Filter", Proc. AIAA Conference on Atmospheric Flight Mechanics, AIAA-98-4153, 1998.
3. Basu S, Su Basu, Groves K.M, Yeh H.-C, Su S.-Y, Rich F.J, Sultan P.J and Keskinen M.J, "Response of the Equatorial Ionosphere in the South Atlantic Region to the Great Magnetic Storm of July 15, 2000", Geophysical Research Letters, Vol.28, pp 3577-3580, September 15, 2001.
4. Whitely, L.D (ed.), "Foundations of Genetic Algorithm. 2", Morgan Kauffmann Publishers, 1993.
5. Coco D.S, C.Coker, S.R.Dahlke and J.R.Clynch, "Variability of GPS Satellite Differential Group Delay biases", IEEE Trans. Aerospace. Electron. Syst., 27(6), 931-938,1991.