THE APPLICABILITY OF FRACTAL RAIN FIELD MODELS TO RADIO COMMUNICATIONS SYSTEM DESIGN.

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ABSTRACT

The radio spectrum is a finite resource, and one that is coming under increased pressure as the range of applications requiring large bandwidths, such as third generation mobile phones, become more prevalent. This has lead to a number of studies being performed in order to investigate methods of improving spectral efficiency, and opening up higher frequencies (10 GHz and above) to commercial exploitation.

Previous systems operating at these frequencies allocated a fixed fade margin to compensate for the attenuating effects of rain, clouds and atmospheric gases. However, as the operational frequency of radio systems increases, this method for compensating for fading increases in cost, until it is no longer economical, spectrally efficient or practical to implement in a working system.

For this reason, new techniques to compensate for attenuation, called fade mitigation techniques (FMTs) or fade countermeasures (FCMs), have been proposed in the literature. One sub-set of these techniques, sometimes known as diversity techniques, relies on the spatio-temporal inhomogeneity of rain fields for their effective operation. For example, satellite systems using site diversity have two (or more) ground stations receiving the same satellite signal, which are separated in space sufficiently so that the rain attenuation at the sites is de-correlated. In a properly configured arrangement the sites encounter intense rainfall at different times, and switching to the site experiencing the least fading improves system performance considerably. To correctly configure this and other FMTs requires a detailed knowledge of spatial and temporal rain field variation.

The fractal nature of rain has been extensively studied in past years by meteorologists and hydrologists, however the insights gained from these methods have been slow to integrate into the rain models used in radio communication systems planning. Fractal methods can be used to analyse and synthesise the spatial and temporal variation of rain fields, producing visually and statistically realistic synthetic rain fields, which may be customised for different climactic regions. These fields can then be converted to simulated attenuation time series and applied to communications engineering scenarios where measured data is not available.

This paper will demonstrate the fractal nature of rain fields and will present a fractal model for simulating rain fields in three dimensions, two spatial and one temporal. This monofracal, additive (in the logarithmic domain) discrete cascade can be used to create simulated attenuation time series which can then test the behaviour of radio systems operating with different FMTs, in order to determine their optimum use and effectiveness.

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THE FRACTAL NATURE OF RAIN FIELDS

The fractal nature of rain has been studied for many years, and its characterisation as a fractal and multifractal field is well documented [1,2]. Unfortunately, there is little consensus on the exact form of the fractal field, due to differing methods of calculating the fractal dimension and/or characteristic multifractal function. The majority of the published works use multifractal methods to deal with the intermittency and anisotropy of the rain field [2, 3, 4].

Data description

The rainfall fields analysed in this study were obtained by means of the Chilbolton Advanced Meteorological Radar (CAMRa), which is located in Chilbolton, Hampshire in the south of England, at latitude 51° 9' N and longitude 1° 26' W. The radar is a 25 m steerable antenna, equipped with a 3GHz Doppler-Polarization radar, and has an operational range of 100 km, and a beam width of 0.25°. The radar scans were interpolated onto a square Cartesian grid, with a grid spacing of 300m and a side length of 56.4km. The grids (also known as rasters) are separated in time by 2 minutes.

Multifractal analysis of the CAMRa meterological radar data

It is commonly taken that the *K(q)* function, sometimes known as the moment scaling function [2,6,7], can be viewed as a characteristic function of multifractal behaviour. The shape of the *K(q)* function specifies the type of scaling involved for a given dataset. A curved *K(q)* function indicates a multifractal structure, whereas a straight *K(q)* function indicates a monofractal structure. Schertzer and Lovejoy [6] present a method of calculating *K(q)* through investigation of the variation of statistical moments with scale. Fields produced by multiplicative cascade processes may have scaling behaviour that can be expressed by different scale-independent scaling relationships. One of these relationships is given by:

$$< \varepsilon_\lambda^q > \approx \lambda^{K(q)} \tag{1}$$

where $< \varepsilon_\lambda^q >$ is the ensemble average *q*th moment of the field studied at (i.e. averaged over) a scale specified by λ . λ, sometimes called the scale ratio, is defined as the ratio of the outer (maximum scale of the field) to the averaging scale. The following general forms have been proposed for the *K(q)* function [2,4]:

$$K(q) = \frac{C_1}{\alpha_L - 1}(q^\alpha - q) \qquad 0 \le \alpha_L < 1,\ 1 < \alpha_L \le 2 \tag{2}$$

$$K(q) = C_1 \log(q) \qquad \alpha_L = 1 \tag{3}$$

where C_1 is the co-dimension of the mean process and α_L, also known as the Lévy index, is related to the type of multifractal process involved. It should be mentioned that the generality of these forms has been questioned [8], as they are based on certain assumptions about the cascade structure of the fields.

The validity of (1) is tested by plotting the average moments $< \varepsilon_\lambda^q >$ as a function of λ on a log-log diagram. If the points fall on a straight line, then the value of *K(q)* can be estimated as the slope of line. The entire *K(q)* function can be estimated by performing the above procedure for different values of q. Fig.1 shows the *K(q)* functions calculated for three different rain events. The *K(q)* function is dramatically curved for the frontal event, though less so for the convective and stratiform events, indicating that the rain fields are multifractal. The same method was used to calculate the *K(q)* function for log rain fields, with the results shown in fig. 2. The *K(q)* functions presented in this case are straight lines, indicating that log rain fields can be analysed and simulated through the use of monofractal techniques with sufficient accuracy for our communications engineering purposes. This provides a justification for our use of a discrete cascade method of simulating rain fields which is based on a monofractal, additive process in the logarithmic domain.

RANDOM ADDITIONS ALGORITHM

To simulate the rain field we use the successive random addition algorithm introduced by Voss [5] to generate fractional Brownian motion. The algorithm is easily extended to higher dimensions and can produce surfaces with coastlines that are self similar fractals with a fractal dimension given by:

$$D = 2 - H \tag{4}$$

where *H* is the Hurst exponent, and is related to the power spectral density exponent of the measured rain fields.

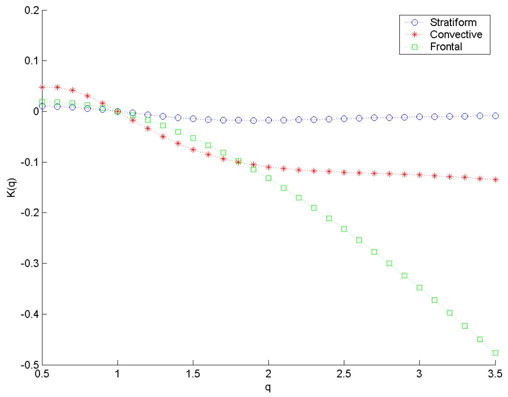

Fig. 1 *K(q)* functions plotted for rain rate events.

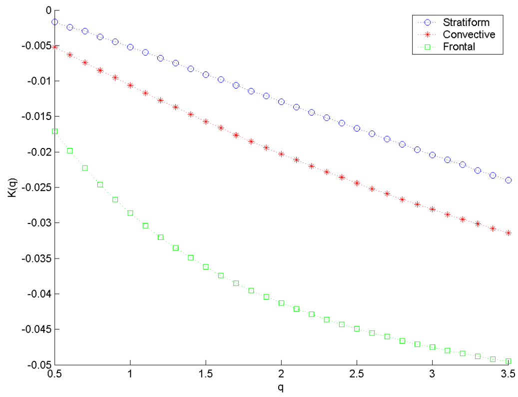

Fig.2 *K(q)* function plotted for log rain rate events.

The spectral density function for a two-dimensional isotropic random field is given by:

$$S(\omega) \propto \omega^{-2H-2} \tag{5}$$

H is also related to the fractal dimension of contour lines that enclose areas which have a (log) rain rate greater than a given threshold [5] and controls the fractal dimension of the resulting simulated rain field. *H*=1/3 for all the simulations.

In the successive random addition algorithm [5], all points are treated equivalently at each stage of the iteration, hence the resolution at the next stage can change by any factor $r_l < 1$. This parameter controls whether or not all the simulated rain occurs in one large area (as is the case for stratiform rain) or instead is broken up into convective type rain cells. It is related to the concept of "lacunarity" discussed in [9,10]. A fractal is called lacunar if it contains large intervals or gaps in the fractal set (or measure). In the case of our data and simulations, stratiform rain fields are more lacunar, i.e., the gaps between areas of stratiform rain tend to be larger than the gaps between the convective rain cells, which are more fragmented, but closer together. In some cases the gaps between areas of stratiform rain are so large that we only see one area of rain at a time in the radar scans. Varying the lacunarity does not alter the fractal dimension. For the simulations presented here, empirically chosen values of r_l=1/2 and r_l=1/3 are used for stratiform and convective rain respectively.

For simulating a rain field in two dimensions using the successive random algorithms technique, given a sample of N_n points at stage n with resolution λ, stage $n+1$ with resolution $r_l\lambda$ is determined first by interpolating the values at the new resolution from the old N_n values. The number of points plotted at the new resolution is:

$$N_{n+1} = \left(\frac{1}{r_l^{\,n+1}} + 1\right)^2 \tag{6}$$

A random element Δ_n is then added to all N_{n+1} points. Δ_n is a Gaussian random variable with zero mean, and at stage n, with scaling ratio $r_l < 1$, the variance is given by:

$$\Delta_n^2 \propto (r_l^{\,n})^{2H} \tag{7}$$

Simulated rain fields in three dimensions are easily generated using the same technique. This provides us with a temporal variation of the rain field (i.e. evolution of the field), but assumes that the spatial variation and the temporal variation of the rain field are equivalent. Overall advection of the rain field can be simulated through the application of Taylor's frozen storm hypothesis [11].

The resulting simulated arrays are equivalent to log rain rate. Looking at the cumulative distribution functions of the (log) measured and simulated arrays (fig 3), we can see that the cdfs of the simulated fields are offset, and hence need to be brought into line with the measured data. The equivalent simulated rain rates can be calculated as follows:

$$R_{sim} = 10^{(V+\log_{10}(R_{\max})-V_{\max})} \qquad for\, V_{\max} > R_{\max} \tag{8}$$

where V is the simulated array values resulting from the successive random additions algorithm, V_{max} is the maximum value in V, R_{max} is the desired maximum rain rate for the simulated event, taking into account the climate of interest, and R_{sim} is the resulting simulated array of rain rates. Fig. 4 shows the results of this conversion to equivalent rain rate for the simulated fields (The measured rain events have truncated cdfs for log rain rates less than -1.5 due to the limited sensitivity of the measuring radar.)

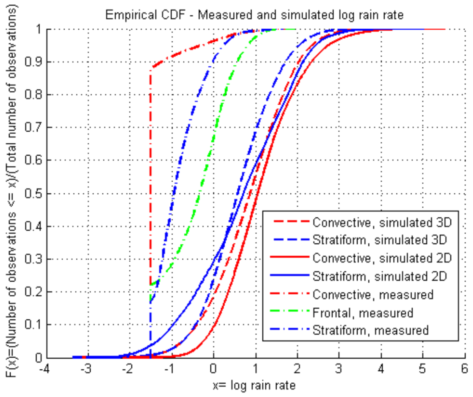

Fig.3 CDFs of measured rain events in comparison with simulated (2D and 3D) events.

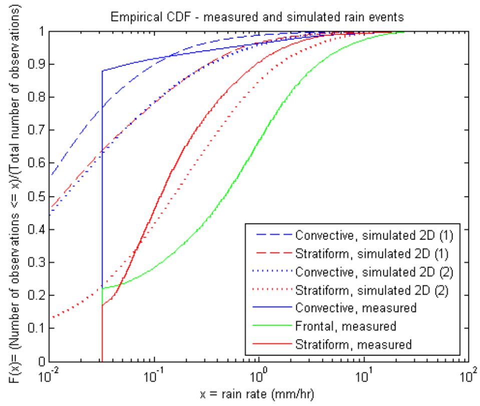

Fig.4 CDFs of example simulated rain rate events in comparison with measured events

As can be seen, the simulations are in reasonable agreement with the measured events, and the discrepancies between measured and simulated curves are of the same order as discrepancies between measured events resulting from inter-event variability. The resulting simulated rain field has appropriate spectral density exponent, fractal dimension, and behaviour that is visually consistent with experimentally observed convective or stratiform type of events (according to what is desired).

CONCLUSIONS

This paper has detailed a method of simulating fractal rain fields in two and three dimensions using the Voss random additions algorithm [6]. Multifractal analysis of rain rate and log rain rate fields shows that log rain rate fields can be approximated as monofractals, and hence provide a justification for using a discrete cascade method of simulating rain fields which is based on a monofractal, additive process in the logarithmic domain.

This work has formed part of a larger study [12], where the simulated rain fields have been used to create simulated attenuation time series, which have then been applied to a communications engineering case study; that of an Earth-space system operating with site diversity as a FMT. Further work with the simulated fields in on-going as part of a project to identify the spectrum efficiency gains resulting from the implementation of Adaptive Transmit Power Control (ATPC) on terrestrial links for bands above 18GHz.

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