Electric field modelling for the lower atmosphere due to cloud charge distribution

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INTRODUCTION

Electrified clouds and lightning play important role in the global electric circuit. The distributions of electric charges in the electrified clouds introduce important effects in the ionosphere and into the region between the ionosphere and the earth. Various models are there to examine the electrical coupling between the earth's upper and lower atmospheric regions. The main sources of electric current in these regions are thunderstorms which are distributed statistically in geographic areas [1-4].

A model for the penetration of DC thundercloud electric field at these regions has been presented in this paper. The model deals with the electromagnetic responses of the atmosphere which are simulated through Maxwell's equations together with a time-varying source charge distribution.

The modified ellipsoidal Gaussian profile has been taken for the charge distribution of the electrified cloud. The conductivity profile of the medium is taken as isotropic below 70 Km and anisotropic above 70 Km. The earth's surface has been considered to be perfectly conducting.

A general form of equation representing the thunder-cloud electric field components is deduced. The solution is expected to be more appropriate than the forms of monopole, dipole and tripole models. In spite of the assumptions for axial symmetry of thundercloud charge distribution considered in the model, the results may be obtained showing the electric field components in the ionosphere. The vertical component of the electric field would be related to global electric circuit while the radial component would show the electrical coupling between the lower atmosphere and the ionised earth environment.

MATHEMATICAL FORMULATION

The transmission of electric field in the steady state by thunderclouds with charge distribution has been investigated through the following equations :

$$\nabla^2\Phi = -\frac{r_T}{\in_0}(r,z) \quad \text{at } 0 \le z \le z_E \tag{1}$$

$$\nabla.\vec{j} = 0 \qquad \text{at } 0 \le z \le z_L \tag{2}$$

and

$$\vec{j} = (s_0 - s_1)\,(\vec{E}\hat{e}_0)\hat{e}_0 + s_1\vec{E} - s_2\vec{E}\times\hat{e}_0 + \vec{j}_s \qquad \text{at } z < z \le z_E \tag{3}$$

$$\vec{j}_s = \frac{Q(t)}{2pr}\partial(r)x(z - z_c)\hat{e}_s \tag{4}$$

Φ is the electric potential of the electric field of the electrified cloud; $\hat{e}_0$ is the unit vector in the direction of the external magnetic field; j_s is the source current density, assumed to generate a cantered charge at a height z_c in the cloud; s_0, s_1, s_2 are the longitudinal, Pedersen and Hall conductivities, respectively; ξ and δ are the unit Heaviside and Dirac functions [5]; $Q(t)$ is the source function which gives the total current injected at time t; $\hat{e}_s$ is the vertical unit vector. $z = 0$ is taken to be the surface level, z is the axis of symmetry of the charge density distribution; z_E is the height between 100 Km to 120 Km; z_L and z_U are the lower and upper boundaries of the electricfield cloud. The electrical conductivity is isotropic in the atmospheric region (0, z_I), z_I = 67 Km [6] and anisotropic above $z = z_I$; z_I is the lower boundary of the ionosphere. The radial and vertical components of the electric field are $E_r = -\frac{\partial\Phi}{\partial r}$ and $E_z = \frac{\partial\Phi}{\partial z}$

Above 70 Km height, the conductivity becomes tensorial in nature.

For 110 Km < z_E < 125 Km, the longitudinal conductivity σ_0 becomes very large compared to transverse conductivities. The charge is assumed to have a ellipsoidal Gaussian distribution given by [4].

$$r_T = r_{\text{model}} + r_S$$

$$r_{\text{model}} = g \exp\left[\frac{r^2}{a^2} - \frac{(z-z_0)^2}{b^2}\right] \qquad z_L \le z < z_U$$

$$\text{and } r_s = \frac{Q(t)}{2p\,r} d(r) d(z-z_0) \qquad 0 \le z < z_L$$

$$z_0 = (z_L + z_U)/2$$

g is the normalization constant.

α^2 and β^2 are the vertical dispersions of the charge density. The charge is higher for small α^2 and β^2. The maximum density is reached at $r = 0$, $z = z_0$.

From (2), in the region

$0 < z < z_L$ and $z_U < z < z_E$,

$$\nabla(s_1).\nabla\Phi + s_1\nabla^2\Phi + \nabla(s_2).(\nabla\Phi\times\hat{e}_0) + s_2\nabla.(\nabla\Phi\times\hat{e}_0) + \nabla.\vec{j}_s = 0$$

Now $\hat{e}_0 = a(r,z)\hat{e}_r + b(r,z)\hat{e}_z$

and let $\nabla.\vec{j}_s = f(z)$

$$\nabla\Phi = \frac{\partial\Phi}{\partial r}\hat{e}_r + \frac{\partial\Phi}{\partial z}\hat{e}_z$$

$$\nabla.\hat{e}_0 = \frac{1}{r}\left[\frac{\partial}{\partial r}(ra) + \frac{\partial}{\partial z}(rb)\right]$$

$$\nabla(s_1) = \frac{\partial s_1}{\partial z}\hat{e}_z$$

$$\nabla(s_2) = \frac{\partial s_2}{\partial z}\hat{e}_z$$

Using these, the equation (2) yields

$$\left[s_1+a^2(s_0-s_1)\right]\frac{\partial^2\Phi}{\partial r^2}+2ab\,(s_0-s_1)\frac{\partial^2\Phi}{\partial r\partial z}+\left[s_1+b^2(s_0-s_1)\right]\frac{\partial^2\Phi}{\partial z^2}+$$
$$+\left[\frac{s_1}{r}+ab\,\frac{\partial}{\partial z}(s_0-s_1\right]+(s_0-s_1)\left(\frac{a^2}{r}+2a\,\frac{\partial a}{\partial r}+b\,\frac{\partial a}{\partial z}+a\,\frac{\partial b}{\partial z}\right)\frac{\partial\Phi}{\partial r}+ \tag{6}$$
$$+\left[\frac{\partial s_1}{\partial z}+b^2\frac{\partial}{\partial z}(s_0-s_1)+(s_0-s_1)\frac{ab}{r}+2b\,\frac{\partial b}{\partial z}+a\,\frac{\partial b}{\partial r}+b\,\frac{\partial a}{\partial r}\right]\frac{dj}{dz}+$$
$$+f(z,r)=0$$

For isotropic region, $0<z<z_L$, $z_U<z<z_I$

$$s_0=s_1=c_1\exp\left(\frac{z}{h}\right).$$

The equation (6) yields,

$$\frac{\partial^2\Phi}{\partial r^2}+\frac{1}{r^2}\frac{\partial\Phi}{\partial r}+\frac{\partial^2\Phi}{\partial z^2}+\frac{1}{h}\frac{\partial\Phi}{\partial z}+f(r,z)=0 \tag{7}$$

For the anisotropic region,

$$s_0=c_A\exp(z/h_0)$$
$$s_1=c_A\exp(z/h_1)$$
$$\frac{\partial s_0}{\partial z}=\frac{s_0}{h_0}\quad,\quad \frac{s_0}{s_1}=\exp\left(\frac{z}{h_0}-\frac{z}{h_1}\right)=\exp(z/\ell)$$
$$\text{and}\quad \frac{\partial s}{\partial z}=\frac{s_1}{h_1}\qquad \frac{1}{l}=\frac{1}{h_0}-\frac{1}{h_1}.$$

When earth's magnetic field is considered,

$$a(r,z)=\frac{r(a+z)}{\sqrt{[r^2+(a+z)^2]\,[r^2+4(a+z)^2]}}$$
$$b(r,z)=\frac{r^2+2(a+z)^2}{\sqrt{[r^2+(a+z)^2]\,[r^2+4(a+z)^2]}}$$
$$a+z>>r,\quad \therefore\quad a(r,z)=-\frac{r}{2(a+z)}$$
$$\text{or,}\quad \frac{\partial a}{\partial r}=-\frac{1}{2(a+z)},\quad \frac{\partial a}{\partial z}=\frac{r}{2(a+z)^2}\equiv 0$$

and $\beta(r,z)=-1$,

$$\frac{\partial b}{\partial r}=\frac{\partial b}{\partial z}=0$$

also, $\alpha^2=1$, $\beta^2=1$

$\therefore$ The equation (6) yields

$$\frac{\partial^2\Phi}{\partial r^2}-\frac{r}{a}\left[1-\exp\left(\frac{z}{\ell}\right)\right]\frac{\partial^2\Phi}{\partial r\partial z}+\exp\left(\frac{z}{\ell}\right)\frac{\partial^2\Phi}{\partial z^2}+$$
$$+\left[\frac{1}{r}-\frac{r}{2a}\left\{\frac{1}{h_1}-\frac{1}{h_0}\exp\left(\frac{z}{\ell}\right)\right\}\right]\frac{\partial\Phi}{\partial r}+ \tag{8}$$
$$+\left[\left(\frac{1}{h_0}+\frac{r}{a}\right)\exp\left(\frac{z}{\ell}\right)-\frac{r}{a}\right]\frac{\partial\Phi}{\partial z}+g(r,z)=0$$

RESULT AND DISCUSSIONS

The electric field components can be obtained from (7) and (8) numerically. From the results, the height and region dependence for singly electrified cloud region on the variations of electric field may be analysed quantitatively for three different electrified areas. The effective magnitude of the electric field from these regions can be made useful to investigate the redistribution of the ionospheric electron density and the formation of the field-aligned irregularities in the upper atmosphere. For aeronautical and navigational purposes, the occurrence of such electric field by charged clouds may be very much useful for different predictions. The present model is sufficiently adequate to analyse the effect of thundercloud electrified charge distribution on the propagation of DC electric fields in the earth-ionosphere region. The results may be useful to study the time-dependent behaviour of the electron density in the atmosphere above a thundercloud.

ACKNOWLEDGEMENTS

This work has been funded by Indian Space Research Organization (ISRO) through S. K. Mitra Centre for Research in Space Environment, University of Calcutta, Kolkata, India.

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