ANALYSIS OF THE GALILEO REFERENCE TROPOSPHERE MODEL
(Session FG, Invited)

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ABSTRACT

The Galileo Reference Troposphere Model of ESA defines the algorithms for the calculation of zenith tropospheric delay and the mapping functions, to be implemented in navigation receivers. The model describes the dependence of delay on position, season, elevation angle and surface meteorological data. A critical analysis has been carried out of the model and the procedures to extract vertical and slant delay values, in particular with respect to consistency and scientific soundness. The main results of this study are discussed.

ZENITH DELAY

Refractivity model

The classical model for refractivity, formulated by Smith and Weintraub (7) is:

$$N = (n-1)\times 10^6 = N_{dry} + N_{vapour}$$

with

$$N_{dry} = K_1 \frac{P-e}{T}; \qquad N_{vapour} = K_2 \frac{e}{T} + K_3 \frac{e}{T^2}$$

where

$$K_1 = 77.6; \quad K_2 = 72.0 \quad K_3 = 375000$$

P, e – total pressure and water vapour pressure [mbar]
T – temperature [K]

Approximating the vapour part by

$$N_{vapour} \approx K_3' \frac{e}{T^2}$$

with

$$K_3' = 373200$$

results in the well-known 'ITU-R formula' (8):

$$N = \frac{77.6}{T}\left(P + 4810\frac{e}{T}\right)$$

Using the gas law for moist air, dry air and water vapour, respectively:

$$P = \rho_m R_m T Z_m; \quad P - e = \rho_d R_d T Z_d; \quad e = \rho_v R_v T Z_v$$

and writing

$$\rho_m = \frac{P}{R_d T} - \left(1 - \frac{R_d}{R_v}\right)\frac{e}{R_d T}$$

it is now usual to define

$$N = N_{hyd} + N_{wet},$$

with the "hydrostatic part"

$$N_{hyd} = k_1 R_d \rho_m$$

and the "wet part"

$$N_{wet} = \left\{\left(k_2 - k_1 \frac{R_d}{R_v}\right)\frac{e}{T} + k_3 \frac{e}{T^2}\right\} Z_v^{-1} = \left(k_2' \frac{e}{T} + k_3 \frac{e}{T^2}\right) Z_v^{-1}$$

Zenith delay model

Zenith total path delay follows from

$$ZTD = \int_{h_0}^{\infty} N(z)dz$$

Invoking the hydrostatic equation and assuming a constant effective mean gravity yields the hydrostatic and the wet zenith delay as:

$$ZHD = 10^{-6} k_1 \frac{R_d}{g_m} P_0$$

and $$ZWD = 10^{-6} \times Z_v^{-1} \int_{h_0}^{\infty} \left\{k_2' \frac{e}{T} + k_3 \frac{e}{T^2}\right\} dz,$$

Saastamoinen (9) models mean gravity as a function of latitude φ and station height h_0 (km)

$$g_m(h_0) = g_m^0 \cdot f(\varphi, h_0)$$

with $g_m^0 = 9.784$ [m/s^2] and $f(\varphi, h_0) = 1 - 0.00266\cos(2\varphi) - 0.00028 h_0$

Using a power law model for the water vapour mixing ratio, assuming a constant effective mean temperature of the atmosphere and invoking the hydrostatic equation with the approximation $\rho_d \approx \rho_m$ it follows that

$$ZWD = 10^{-6} \frac{R_d e_0}{(\lambda + 1) g_m}\left(k_2' + \frac{k_3}{T_m}\right)$$

Invoking, instead, a linear temperature profile

$$T(z) = T_0 - \alpha \cdot (z - z_0)$$

the zenith wet delay can then be written (3) as

$$ZWD = 10^{-6} \frac{R_d e_0}{g_m}\left(\frac{k_2'}{(\lambda + 1)} + \frac{k_3}{\left((\lambda + 1) - \frac{\alpha R_d}{g_m}\right) T_0}\right)$$

The ESA reference model (1) used this so-called two-parameter model as a basis, but simplified it into a one-parameter model by dropping the term with k_2' and redefining k_3 :

$$ZWD = 10^{-6}\frac{R_d}{g_m}\cdot\frac{k_3'}{\left((\lambda+1)-\frac{\alpha R_d}{g_m}\right)}\cdot\frac{e_0}{T_0}$$

The reference model uses as input parameters the temperature lapse rate α, the mixing ratio scale length λ and the "surface vapour temparature", defined as

$$T_{ms} = T_0\left(1-\frac{\alpha R_d}{(\lambda+1)g_m}\right)$$

and mapped for the world.

Results of the study

1) There is a fair amount of literature (and confusion) on the value for the constants k_i to be used. Following an analysis of the references, the values of Bevis (1) were recommended for adoption:

$$k_1 = 77.6 \pm 0.05$$
$$k_2 = 70.4 \pm 2.2$$
$$k_3 = 373900 \pm 1200$$

2) It was found that the reduction to a one-parameter model entails an undesirable loss of accuracy and the zenith delay model was redefined as

$$ZHD = 10^{-6}k_1\frac{R_d}{g_m}p_s \quad \text{and} \quad ZWD = 10^{-6}\frac{R_d}{g_m}\cdot\frac{(T_{ms}\times k_2'+k_3)}{(\lambda+1)}\cdot\frac{e_s}{T_{ms}}$$

MAPPING FUNCTIONS

The dependence of path delay for oblique paths is modelled by a mapping function $m(\varepsilon)$ defining the ratio of the oblique delay and zenith delay.

Using the Essen and Froome (10) model for refractivity the model by Saastamoinen (9) for the elevation dependence results in

$$\Delta s = 0.002277\sec\varsigma\left[p+\left(\frac{1255}{T}+0.05\right)e-1.16\tan^2\varsigma\right] \quad \text{[m]}$$

with P in [mb] and T in [K]. For other station heights, a table is given for the coefficient of the curvature term $\tan^2\zeta$.

ς is the true zenith angle which is higher than the apparent angle ς_1:

$$\Delta\varsigma = \varsigma-\varsigma_1 = \frac{16.0\tan\varsigma}{T}\left(P+\frac{4800e}{T}\right) \quad \text{[arc sec]}$$

The model is quoted to have an error of 0.1 m at $\varsigma = 80$ degrees, due to the omission of higher-order terms.

Using the ITU standard atmosphere (6) with:

- temperature at sea level $T_1 = 288.15$ K
- pressure at sea level $P_1 = 1013.25$ hPa
- temperature lapse rate: $\alpha = -6.5$ K/km
- tropopause height: $h^0 = 11$ km

- exponential pressure profile: $P(h) = P_1 \left(\frac{T_1}{T_1 + \alpha \cdot (h - h_1)} \right)^{\frac{g_m}{R_d \alpha}}$

we recalculated the Saastamoinen model as:

$$\Delta s = 2.278 \times 10^{-3} \sec \zeta_1 \cdot \left(P_1 + (1265 / T_1 + 0.071) e_1 - 1.164 \tan^2 \zeta_1 \right)$$

The ESA reference model for the mapping function is a continued-fraction model as proposed by Niell (5) :

$$m(\varepsilon) = \frac{1 + \frac{a}{1+b}}{\sin \varepsilon + \frac{a}{\sin \varepsilon + b}}$$

with empirical parameters *a* and *b,* given in the form of interpolation tables for wet and dry mapping separately.

This formulation was recalculated to be consistent with the assumptions and parameters of the zenith delay model as given by
- the Saastamoinen model (2)
- the reference atmosphere defined by the ITU (3)

This resulted in the following definition:

$$a = \frac{0.0164}{\left(P_1 + (1265 / T + 0.071) e_1 + 1.16 \right)}$$

$$b = 0.015$$

For the reference atmosphere at sea level, a = 0011 and b = 0.015:
The correction due to the addition of term *b* is 3 cm at 10 degrees elevation.

REFERENCES

1. ESA – Galileo (2004), Galileo Reference Troposphere Model for the User Receiver', *ESA Document* Reference ESA-APPNG-REF/00621-AM-Iss. 2 / Rev. 3
2. Bevis, M., S. Businger and S. Chiswell (1994), GPS Meteorlogy: Mapping Zenith Wet Delays onto Precipitable Water, *Jnl. Appl. Meteorology*, Vol 33, pp. 379-386
3. Askne, J. and H. Nordius (1987), Estimation of tropospheric delay for microwaves from surface weather data, *Radio Sci.*, Vol 22, pp. 379-386
4. Saastamoinen, J. (1970), The physical theory of astronomical refraction (abstract), *Eos Trans. AGU*, Vol 51, p. 742
5. Niell, A.E. (1996), Global mapping functions for the atmospheric delay at radio wavelengths, *Jnl. Geophys. Res.*, Vol 101, pp. 3227-3246
6. ITU (2004), Reference standard atmospheres, *ITU Recommendation,* ITU-R P.835-3
7. Smith, Ernest K. and S. Weintraub (1953), The Constants in the Equation for Atmospheric Refractive Index at Radio Frequencies, Proc IRE, Vol 41, pp. 1035-1037
8. Recommendation ITU-R P.453-9 (2004), The radio refractive index: its formula and refractivity data
9. Saastamoinen, J. (1972), Atmospheric Correct ion for the Troposphere and Stratosphere, AGU Geophys. Monogr. Series, 15, pp. 47-51
10. Essen, L. and K.D. Froome (1963), Dielectric constant and refractive index of air and its principal constituents at 24,000 Mc/s, Nature, Vol 167, pp. 512-513