A MULTICHANNEL AUTOREGRESSIVE MODEL OF RAIN ATTENUATION ON MULTIPLE RADIO LINKS AND ITS APPLICATION IN ASSESSMENT OF FADE MITIGATION SCHEMES IN FIXED WIRELESS SYSTEMS ABOVE 10 GHZ

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ABSTRACT

A multichannel autoregressive (AR) model for rain attenuation on a network of radio links is presented. Underlying assumptions are discussed, including the stationarity of rainfall rate in space and time within the region enclosing the links. Formulation of the model is described, together with some considerations for its application in assessing performance of radio links. An example of use of the model in evaluation of fade mitigation schemes involving adaptive M-QAM modulation and cell-site diversity on two short converging links operating at 30 GHz is given.

INTRODUCTION

In outdoor mm-wave radio communications, rain attenuation exhibits a devastating impact on the performance of a radio link. Mitigation of rain attenuation has long been studied with a goal of providing techniques to achieve reliable, high-quality radio links at millimeter-wave frequencies even under rainy conditions ([1], [2]). Such studies have become more urgent with the technological trend towards realizing broadband wireless access to multimedia services through a family of networks generally termed LMDS (Local Multipoint Distribution Services). Performance evaluation of adaptive rain fade mitigation techniques, especially those involving cell-site diversity and/or adaptive coding, modulation and power control, necessitates computer generation of time sequences of correlated rain attenuation on multiple links.

The lognormal approximation of rain rate and attenuation has been found from a lot of measurements in the past (e.g., [3]), with the multivariate form for rain on multiple points or links having recently been discussed in [4]. Accordingly, a set of time series of log attenuation occurring on multiple links can be generated through synthesis of a multi-channel autoregressive (AR) process. (AR processes have previously been used successfully to model rainfall rates [5]). This reasoning is valid only when the same rain event, stationary in time and space, encloses all of the links. This paper describes the assumptions and the model formulation as well as limitations of the model. Use of the model in performance assessment of some fade mitigation schemes is exemplified.

ASSUMPTIONS AND MODEL FORMULATION

It is assumed at the outset that rain attenuation in dB experienced by a radio link is lognormally distributed. In addition, the region that contains all of the links considered in the model is assumed to be shrouded by single rain events. Accordingly, links must be sufficiently short, which is usually the case for LMDS where links are 4 km or shorter to assure line-of-sight situation. Moreover, these rain events must be temporally and spatially stationary so that the same statistical characteristics apply homogeneously throughout the time and space domain of the events. Finally, the statistics of rainfall rate and specific attenuation in the region are assumed to be known. For instance, distribution of rainfall rate and attenuation can be obtained from measurements or application of prediction methods. Temporal autocorrelation of rainfall rate and specific attenuation can be acquired from measurements (e.g. [6], [7]), whereas (spatial) correlation between links can be obtained from radar measurements ([7]) or empirical models ([8]).

The proposed model involves synthesizing a set of J autoregressive processes that behave like time sequences $\mathbf{x}(k) \equiv \mathbf{x}(k\delta)$ of logarithmic value of rain attenuation $\boldsymbol{\alpha}(k)$ on J radio links, where $\mathbf{x}(k) = [x_1(k), x_2(k), \ldots, x_J(k)]^T$ and δ denotes the sampling period, according to the covariance functions $\phi^x_{lm}(\tau)$. A vector sequence of zero-mean normally distributed $\mathbf{x}_0(k) = \mathbf{x}(k) - \boldsymbol{\mu}_x$, where $\boldsymbol{\mu}_x = [\mu_{x1}, \ldots, \mu_{xJ}]^T$, is generated recursively as follows:

$$\mathbf{x}_0(k) = -\sum_{n=1}^{M} \mathbf{A}(n)\mathbf{x}_0(k-n) + \mathbf{C}^{\mathrm{T}}\mathbf{g}(k) \tag{1}$$

where $\mathbf{A}(n)$ with $n = 1, \ldots, M$ are $J \times J$ matrix coefficients of the AR processes,

$$\mathbf{A}(n) = \begin{bmatrix} a_{11}(n) & \cdots & a_{1J}(n) \\ \vdots & \ddots & \vdots \\ a_{J1}(n) & \cdots & a_{JJ}(n) \end{bmatrix} \tag{2}$$

whereas M denotes the order of the processes that depends on the maximum time lag of the autocorrelation function, $\mathbf{g}(k) = [g_1(k), \ldots, g_J(k)]^{\mathrm{T}}$ a vector sequence of independent white zero-mean unit-variance Gaussian random numbers, and $\mathbf{C}$ a $J \times J$ scaling matrix that determines covariances of the elements of Gaussian vector sequence $\mathbf{C}^{\mathrm{T}}\mathbf{g}(k)$. Once vector sequence $\mathbf{x}_0(k)$ is obtained, the rain attenuation sequence $\boldsymbol{\alpha}(k) = [\alpha_1(k), \ldots, \alpha_J(k)]^{\mathrm{T}}$ can be obtained:

$$\boldsymbol{\alpha}(k) = \exp\left(\mathbf{x}_0(k) + \boldsymbol{\mu}_x\right) \tag{3}$$

From $\phi^x_{lm}(n) \equiv \phi^x_{lm}(n\delta)$, *i.e.*, the discrete-time covariance functions of $x(k)$ (or equivalently, the correlation functions of $x_0(k)$), a set of $J \times J$ matrices can be formed:

$$\boldsymbol{\varphi}(n) = \begin{bmatrix} \phi^x_{11}(n) & \cdots & \phi^x_{1J}(n) \\ \vdots & \ddots & \vdots \\ \phi^x_{J1}(n) & \cdots & \phi^x_{JJ}(n) \end{bmatrix} \tag{4}$$

where $n = 0, 1, \ldots, M$. For real-valued AR processes, the AR matrix coefficients can subsequently be obtained from:

$$\tilde{\mathbf{A}} = -\mathbf{V}^{-1}\boldsymbol{\Phi} \tag{5}$$

where both $\tilde{\mathbf{A}} = [\mathbf{A}(1)^{\mathrm{T}}, \ldots, \mathbf{A}(M)^{\mathrm{T}}]^{\mathrm{T}}$ and $\boldsymbol{\Phi} = [\boldsymbol{\phi}(1)^{\mathrm{T}}, \ldots, \boldsymbol{\phi}(M)^{\mathrm{T}}]^{\mathrm{T}}$ are of $MJ \times J$ dimension, while the $MJ \times MJ$ matrix $\mathbf{V}$:

$$\mathbf{V} = \begin{bmatrix} \boldsymbol{\varphi}(0) & \boldsymbol{\varphi}(1) & \cdots & \boldsymbol{\varphi}(M-1) \\ \boldsymbol{\varphi}(1) & \boldsymbol{\varphi}(0) & \cdots & \boldsymbol{\varphi}(M-2) \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{\varphi}(M-1) & \boldsymbol{\varphi}(M-2) & \cdots & \boldsymbol{\varphi}(0) \end{bmatrix}. \tag{6}$$

Taking $\mathbf{A}_0 = [\mathbf{I}_J, \tilde{\mathbf{A}}^{\mathrm{T}}]^{\mathrm{T}}$ and $\boldsymbol{\Phi}_0 = [\boldsymbol{\phi}(0)^{\mathrm{T}}, \boldsymbol{\Phi}^{\mathrm{T}}]^{\mathrm{T}}$, the scaling matrix $\mathbf{C}$ is obtained through Cholesky decomposition:

$$\mathbf{C}\mathbf{C}^{\mathrm{T}} = \mathbf{A}_0^{\mathrm{T}}\boldsymbol{\Phi}_0. \tag{7}$$

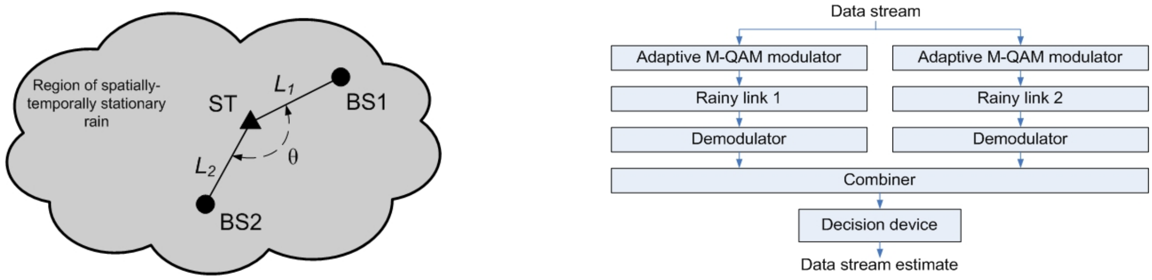
PRACTICAL CONSIDERATIONS AND AN EXAMPLE OF APPLICATION

A number of issues have to be dealt with prior to applying the model. Due to high correlation of rain attenuation for typical sampling intervals, both in space and time, correlation matrix **V** is generally not well-conditioned. We propose to use only selected sequences out of those generated by the computer which show correlation properties close to the desired. Also, since the probability of rain event is small, 1-5 % on average per year, we recommend that only portions of time in which a rain event occurs are examined to prevent time-consuming computation. For simulation purposes we define the occurrence of rain to be an event in which the rain rate exceeds that having the probability of occurrence specified in the ITU-R Rec. P.837 for a region of interest [9]. The starts and ends of rain events are then determined by linearly interpolating the attenuation series and locating upward and downward “threshold”-crossing points. In addition, due to the slow variation of rain attenuation relative to the transmission rate, we adopt quasi-analytical simulation technique for BER evaluation. Values of SNR are obtained at sampling times and subsequently translated to BER values using the analytical equation for the modulation scheme employed. We also emphasize that the spatial stationarity requirement requires that results for links longer than 1 km should be treated with great care.

As an example of application, sequences of rain attenuation generated from the AR model are used to study the performance of a cellular network with short 30 GHz links in tropical areas. Two converging links employ an adaptive transmission scheme, which involves adaptive M-QAM modulation and cell-site diversity, and which is activated only when rain fade on the default single link exceeds a specified level. The clear-sky SNR on a single 1-km link is assumed to be 44.8 dB. Segmentation of the SNR range for the adaptive M-QAM is made using BER of 10^{-6} for reference (see Table 1). Naturally, low-frequency low-rate feedback links are assumed to be available from involved base stations to enable this adaptive operation. Comparison of error rate performance with that of a single link system, with fixed or adaptive M-QAM, might provide an insight into whether or not realization of LMDS networks using the transmission technique being considered is worthwhile. All performance indicators are assessed only on the occurrence of rain. For this simulation, with the lack of space-time correlation data at hand, we assume $\phi^{x}_{lm}(n) = \phi^{x}_{lm}\,\phi^{x}(n)$, where $\phi^{x}(n)$ is the normalized temporal autocorrelation of rain attenuation and ϕ^{x}_{lm} is the normalized correlation of instantaneous attenuations on the l^{th} and m^{th} links. For the temporal autocorrelation we take the rainfall rate measurement results from Barcelona [6], whereas for the inter-link correlation coefficients we use Kanellopoulos method with Morita-Higuti spatial correlation model [8]. Figs. 1(a) and (b) depict the system to be evaluated with $L_1 = L_2 = 1$ km and $\theta = 180°$, whereas Figs. 2(a) and (b) show the closeness of distribution and temporal correlation of computer-generated rain attenuation on a single 1-km 30 GHz link to the desired curves from prediction and measurement. The inter-link average correlation coefficients of attenuation α are invariably found from the simulation to differ from the prescribed values by less than 0.01. For instance, the coefficients for two 1-km links with 45° and 180° separations are specified based on [8] to be 0.9563 and 0.8954, respectively, whereas the average values for the computer-generated attenuation sequences are found to be 0.9577 and 0.8998, correspondingly.

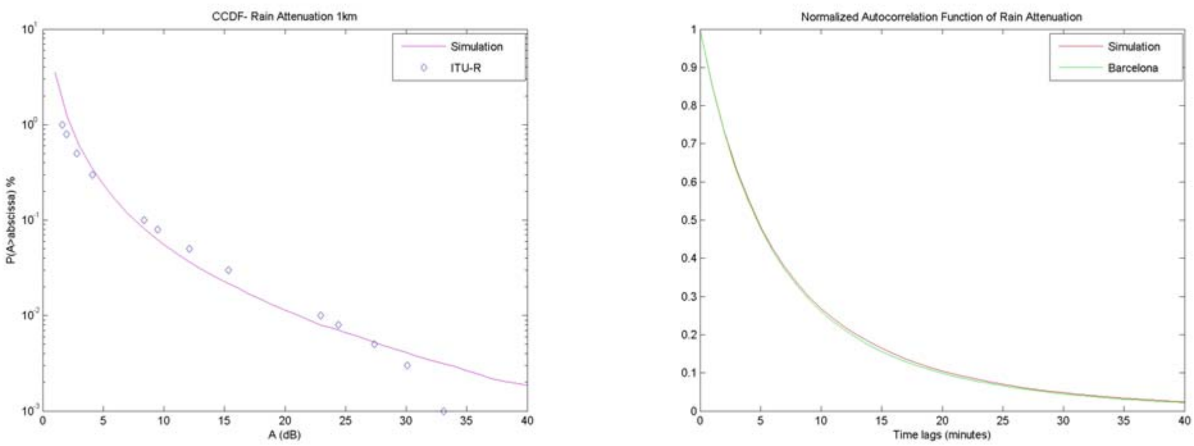
Table 1. Segmentation of SNR range

SNR range (dB)	< 13.6 dB	13.6 – 20.6	20.6 – 26.8	> 26.8
Modulation level	No transmission	4-QAM	16-QAM	64-QAM



(a) (b)

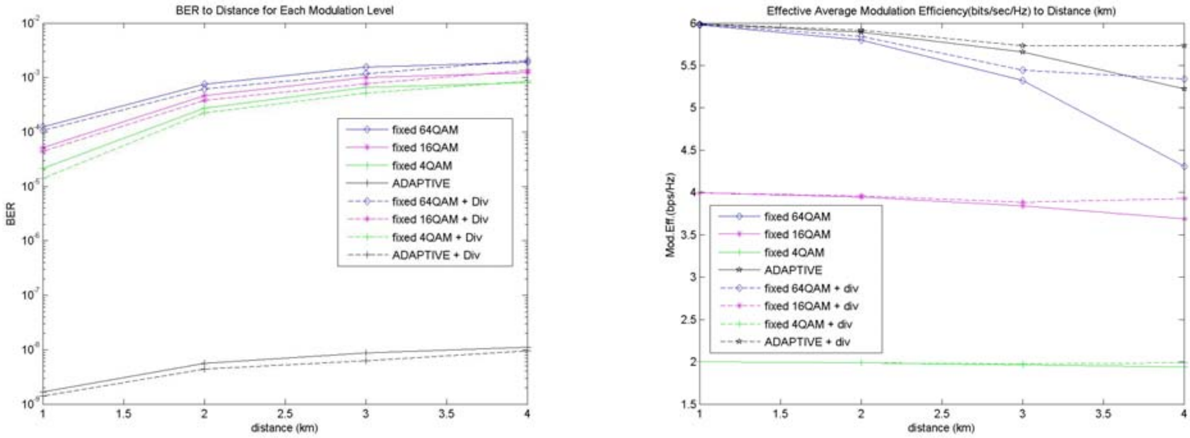
Fig. 1. The evaluated system: (a) Converging links of lengths L_1 and L_2 with separation angle θ between two base stations (BS1 and BS2) and a subscriber terminal (ST), (b) Adaptive M-QAM modulation with diversity combining.



(a) (b)

Fig. 2. Comparisons of simulated and desired (a) complementary cumulative distributions and (b) temporal correlations of rain attenuation on a 1-km 30-GHz link.

Fig. 3(a) and (b) illustrate the superiority of adaptive schemes in terms of average error rate and average spectral efficiency with respect to those with fixed modulation level, as found from the simulation. They further show potential improvement in both BER and capacity performances due to the use of cell-site diversity with selective combining. Use of other types of combining, such as switched, maximal-ratio, or equal-gain, might as well be considered for the diversity scenario.



(a) (b)

Fig. 3. (a) Average BER and (b) average spectral efficiency of adaptive and fixed M-QAM schemes under precipitation, with and without cell-site selective diversity.

CONCLUSIONS

The multi-channel AR model provides an alternative method to simulate the effects of rain on a network of radio links. With complete statistical knowledge of rainfall rates both in time and space, multiple sequences of rain attenuation occurring on a number of links, be they convergent, parallel, or connected in tandem, can be simulated and the performance of the entire multi-link network can be assessed. Use of the model, however, must consider that the assumptions stated above are fulfilled and that some limitations might cause inaccuracies in the results.

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