

# TO ESTIMATION OF THE STATISTICAL CHARACTERISTICS OF INTERFERENCES AT THE RECEIVER INPUT

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## INTRODUCTION

Continuous increasing in the number of electronic devices to be used in modern society, put forward the problem of EMC solution as one of the most interested ones in modern radio electronics. In particular, one of the actual problems in EMC ensuring is the problem of prediction of statistical characteristics of radio interferences at the receiver input. Actually, with such electromagnetic (EM) environment characteristics as probability density function (PDF)  $w(P)$  of radio interferences (RIs) power at the point of receiver location and PDF  $w(f)$  of interference sources in the frequent domain, we may estimate signal to noise+interference ratio at the receiver input as well as probability of non-linear distortion in the receiver or other characteristics.

The aim of the paper is the estimation of general character of the curves  $w(P)$  in the point of receiver location. To find solution, we must take into account at least two of components of model to be suitable for mathematical definition of problem. We must determine statistical model of EM environment in the points of location of interfering transmitters as well as we must know general elements of electromagnetic wave (EMW) propagation models. The connecting link between of these two arrays of datum is the Friis formula,

$$\Pi = \frac{P_{tr} G_{tr} y_{tr}(\alpha, \varepsilon)}{4D^n}. \quad (1)$$

It determines value  $\Pi$  of power density (watts/meters<sup>2</sup>) at the receiver location as a function on three random (in general case) values. There are distance  $D$  between of positions of receiver and transmitter, power  $P_{tr}$  to be radiated by the transmitter and transmitting antenna gain  $G$ . This expression is the initial stage of our investigation, it also explains us which parameters we need in knowing to find solution problem in general.

## EM ENVIRONMENT MODEL

Let assume that EM environment is formed by the some number  $N$  of radiation sources (transmitters) to be potentially dangerous to "our" receiver. The next supposition is that the parameters of RIs may conditionally be divided into two respectively independent groups. The first one consists of non-energetic signal parameters, such as the direction of EMW incoming and the polarization parameters as well as signal frequency and phase or time of the signal incoming. The second group is known as energetic parameters containing. There are amplitude of the signals, power of RI or the power flow in the point of the receiver, etc.

The operation of such interfering sources may in particular be determined by means of the multidimensional PDF  $w(x_1, x_2, \dots, x_n)$  of non-energetic parameters and PDF of energetic parameters. Dimension  $n$  of the PDF on non-energetic parameters is determined by the number of parameters of the desired signal, on which the selection in the receiver is implemented.

As a rule, the signal forming in the domain of non-energetic parameters is performing separately, therefore the multidimensional PDF may be presented as

$$w(x_1, x_2, \dots, x_n) = \prod_{i=1}^n w(x_i), \quad x_i \in Dx_i.$$

Here  $Dx_i$  are the ranges in which the interfering parameters are distributed. In view of that the ranges  $Dx_i$  are rather narrow and taking into account the tendency to effective utilization of EM resource, it is possible to postulate some of the EM environment parameters are uniformly distributed.

It may be concerned to such of EM environment parameters as azimuth  $\alpha$  and frequency  $f$ . So, the RI sources are supposed to be distributed uniformly on azimuth in the range  $[-\pi; \pi]$ , therefore  $w(\alpha)=1/2\pi$ ;  $w(f)=1/Df$ . PDF  $w(\varphi)$  of phase of the signal to be induced at the antenna aperture is also uniformly distributed,  $w(\varphi)=1/2\pi$ . The PDF of RI sources on angle of elevation is better to be studied separately.

Now discuss approach to estimation of PDF  $w(D)$  of distances  $D$  from receiver to arbitrary RI source. First of all we suppose the EM environment is formed by some number  $N$  of transmitters (Base Station and Mobile Stations as well as arbitrary RI sources) placed on the smooth surface. The model of uniform distribution of these RI sources within the ring shape area defined by the distances  $D_{\max}$  and  $D_{\min}$  is one of the most frequently used, [1], [2]. In this case, PDF  $w(D)$  of distance  $D$  from arbitrary interference source up to investigated receiver is determined by the expression

$$w(D) = \frac{2D}{D_{\max}^2 - D_{\min}^2}. \quad (2)$$

Here  $D_{\max}$  is the maximal distance of signal reception in view of receiving antenna height. When RI sources are uniformly placed in the space limited by the spheres with radiuses  $D_{\min}$  and  $D_{\max}$ , PDF of distances  $w(D)$  is defined as

$$w(D) = \frac{3D^2}{D_{\max}^3 - D_{\min}^3}.$$

If RI sources are uniformly placed along straight line within the range  $[D_{\min}; D_{\max}]$ , PDF  $w(D)$  has the view

$$w(D) = \frac{1}{D_{\max} - D_{\min}}.$$

Discussion of probable character of PDF  $w(D)$  is introduced in this subsection but fact that distance  $D$  is not non-energetic parameter directly. More over, distance  $D$  is one of the factors at estimation of RI power at the receiver input. But, therefore we try to use EM environment characteristics to be taken in the points of RI transmitters locations, here distance  $D$  play role of non-energetic EM environment parameter.

Now consider parameters of transmitting device of RI sources. As it is follows from (1), to our investigation be correct we need in knowing of such characteristics of RI sources as  $P_{tr}$ ,  $G_{tr}$  and two-dimensional antenna pattern  $y_{tr}(\alpha, \varepsilon)$ . So as all values of these parameters are random in view of different but true reasons, these values to have been estimated by PDFs. More over, our aim is in estimation of PDF of composition of random values  $(y_{tr}(\alpha, \varepsilon) \times G_{tr} \times P_{tr})$ .

First of all let estimate PDF of normalized antenna pattern  $w(y_{tr})$ ,  $0 < y_{tr} < 1$ .

In the UHF range two-dimensional pattern  $y_{tr}(\alpha, \varepsilon)$  may be performed as  $y(\alpha, \varepsilon) = y_1(\alpha)y_2(\varepsilon)$ , where  $y_1(\alpha)$  is the normalized azimuthal pattern,  $y_2(\varepsilon)$  is the normalized elevation radiation pattern. Now pay some attention to question of antenna pattern approximation. At definition of selectivity characteristics of spatial filters it is possible to use antenna patterns (APs) to be measured or AP approximations. In order to not restrict our research by type of concrete antenna device, we shall utilize of commonly used the pattern approximations.

Numerous approximations of AP are known. In the present section we use two-parameter approximation

$$y(\alpha) = \left| \frac{\sin(k\alpha / \Delta A)}{k\alpha / \Delta A} \right|^n. \quad (3)$$

Antenna beamwidth  $\Delta A$  and the level of the first side lobe  $d$  of the normalized AP are the input datum for definition of coefficients  $k$  and  $n$ , so

$$n = \ln(1/d) / \ln(3\pi/2), \quad k \approx 4.9\sqrt{1 - 2^{-1/n}}.$$

To our investigation not be very sophisticated, let study view of PDF  $w(y)$  for one-dimensional spatial filter to be described by the expression (3),  $\Delta A = 30^\circ$ ,  $d = 0.1$ . As it was mentioned above, consider  $w(\alpha) = 1/2\pi$ . Supposition about of uniform distribution of azimuth  $\alpha$  may be used for forming of two independent models of EM environment. First one describes random orientation of transmitting antenna; second one describes random orientation of receiving antenna.

A normalized histogram, i.e. approach to PDF  $w(y)$  is shown in the fig.1, abscisse axis is log-scaled. As it is followed from the figure,  $w(k)$  is rapidly decreasing function. It gives us very simple conclusion: if it is not electronic warfare (jamming), RI source irradiates "our" receiving antenna through side lobes of AP.

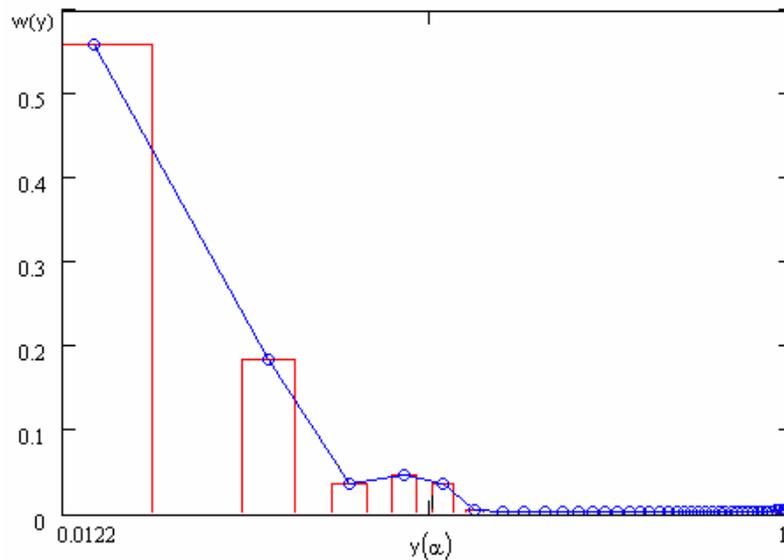


Fig.1. Normalized histogram (approach to PDF) of AP

Consider possibility of estimation of PDFs of random values  $w(P_{tr})$  and  $w(G_{tr})$ . It is known, value of  $G_{tr}$  varies in relatively large range of  $\pi/2$  to  $1E4-5$ . But at estimation of RI sources to be potentially dangerously for receiver, we must take into account possibility of RI suppression in the frequent, polarization and temporal domains. Type of signal to be used in communication system is of great importance. So, the parameters of RI transmitter are more closely to parameters of transmitter of desired signal, more such RI source damaged is (An example is the cellular RIs in the communication systems). In view of these reasons, we suppose the RI sources group is relatively homogenous in structure. Therefore we suggest the random value  $G_{tr}$  is of relatively short dynamic range. All these reasons may be referred to estimation of random value  $P_{tr}$ .

Now discuss PDF of composition of random values  $(y_{tr}(\alpha, \varepsilon) \times G_{tr} \times P_{tr})$ . There may be different point of view for character of PDF  $w(y_{tr}(\alpha, \varepsilon) \times G_{tr} \times P_{tr})$ . Sometimes have used log-normal or Rayleigh PDF, exponential or others. We suppose the function to be suitable for problem solution must take into account two main reasons. First one is the random

value  $(y_{tr}(\alpha, \varepsilon) \times G_{tr} \times P_{tr})$  has large dynamic range, second one is the more power signals are used by the communication system the less such a device is spread. Therefore in further we use uniform-logarithmical (Shannon) PDF,

$$w(P) = \frac{\beta}{P}, \quad \beta = 1/\ln(P_{\max}/P_{\min}), \quad P_{\min} < P < P_{\max}.$$

Here  $\beta$  is normalizing factor,  $P$  is newly introduced value

$$P = y_{tr}(\alpha, \varepsilon) G_{tr} P_{tr}. \quad (4)$$

## SIMPLIFIED EMW PROPAGATION MODEL

There are known some models for field prediction at the point of receiver location. So as we suppose to use expression (1) for estimation of power flow, our interest is in which measure these ones may be adopted for approach to be used. Consider COST-231-Hata's model, to be most frequently used for estimation of path loss in urban areas when cell radius of 1 to 20 km,

$$L = 43.6 + 33.9 \log(f_{[MHz]}) - 13.82 \log(h_{B[m]}) - H_M + [44.9 - 6.55 \log(h_{B[m]})] \log(D_{[km]}) + C_m \quad [dB]. \quad (5)$$

Here  $H_M$  is the correction factor for the mobile antenna height,  $C_m$  takes values 0 or 3 upon type of area. It is not difficult to reduce this expression into other one

$$L = \delta + \beta \log(D) = \delta + 10n \log(D), \quad [dB], \quad (7)$$

where

$$10n \equiv [44.9 - 6.55 \log(h_{B[m]})].$$

For example, if  $h_{B[m]}=30m$ , then  $10n=35.2$ ; when  $h_{B[m]}=300m$ , then  $10n=28.7$ .

After recalculation of dB-scale into decimal number system, we may find

$$Loss(D) = \delta' D^n. \quad (7)$$

Here  $\delta'=10^{\delta/10}$ . Substitution of (7) and (4) into (1) gives us Friis formula as

$$\Pi = \frac{P}{\delta' D^n}. \quad (8)$$

The main question in this section must be find is in what measure formula (7) may exchange (5) and how it may influence on our investigation in general. Hata's empirical formula [3] includes some of parameters besides of distance  $D$  and receiving antenna height. But all of these ones practically do not influence upon the slope of characteristic  $L(D)$ . In view of objective laws of the probability theory, some uncertainty we to have at determination of value  $\delta'$  will not of great importance on general character of curve  $w(I)$ . Therefore at estimation of PDF, EMW propagation distinguished features may be accounted by the value of exponent  $n$  only, see (8).

## ESTIMATION OF THE POWER FLOW AT THE POINT OF RECEIVER

Let find PDF of power flow  $w(I)$  in the point of receiver location. First of all let find general view of PDF  $w(I)$  for EM environment to be characterized by not wide dynamic range of distances  $D_{\max}/D_{\min}$  as well as interfering signal power  $P_{\max}/P_{\min}$ . For estimation have made by the direct statistical simulation at EM environment models components to be selected  $m=2.9$ ,  $P_{\max}/P_{\min}=1E6$ ,  $D_{\max}/D_{\min}=50$ ,  $\delta'=1$ .

A normalized histogram, i.e. approach to PDF  $w(I)$  is shown in the fig.2, ordinates axis is log-scaled. As it is follows from figure,  $w(I)$  is rapidly decreasing function. Histogram was built in equal intervals in the abscisse axis. In the figure we can see two regions in which behaviour of the curve  $w(I)$  is differed significantly. The first region is the area of very slow signals, but probability of appearance of these signals at the receiver input is in thousands times as than other all. Region II is characterized by the steady slope, that it may be correctly approximated by the hyperbolical function.

As it is revealed, the dependences  $w(I)$  found may be approximated by the series with the elements  $1/I^k$ ,  $k=4-5$ .

$$w(I) = a_0 + \frac{a_1}{I^1} + \frac{a_2}{I^2} + \frac{a_3}{I^3} + \dots \quad (9)$$

It is clear to say, the such approximation is very inapplicable for solving of such a problem as direct statistical simulation of random values  $I$  in accordance with PDF  $w(I)$ . Therefore we are needed in more exact but simple approximation of PDF  $w(I)$ .

To find more exact approximation to PDF  $w(I)$  we are interested in building of histogram in not-uniform intervals. The best solution for this problem may give us exponential step in the abscisse axis,  $I_i = I_{\min} e^i$ . Computational experiment has been made by the direct statistical simulation at EM environment models components to be selected  $m=2.5$ ,  $P_{\max}/P_{\min}=1E8$ ,  $D_{\max}/D_{\min}=50$ ,  $\delta'=1$ . Results of simulation are shown in figure 3, abscisse and ordinates axes are log-scaled.

In the figure we can see four regions in which behaviour of the curve  $w(I)$  is differed significantly. To explain character of the curve  $w(I)$  we must pay attention to specificity of random value  $I$ , see expression (9). It is distributed within the range  $[P_{\min}/\delta' D_{\max}^2; P_{\max}/\delta' D_{\min}^2]$ . As for points  $P_{\min}/\delta' D_{\min}^2$  and  $P_{\max}/\delta' D_{\max}^2$ , position of these ones in the abscisse axis can't be predicted without of knowledge of concrete values of  $P_{\min}$ ,  $P_{\max}$ , as well as  $D_{\max}^2$  and  $D_{\min}^2$ . It partially explains presence of four regions in the fig.3.

In the first region we have found increasing function. Even if result of investigation in this area is similar to be not very objective in view of limited number of simulation, we will have in real practice something around of it in view of threshold effect in receiver.

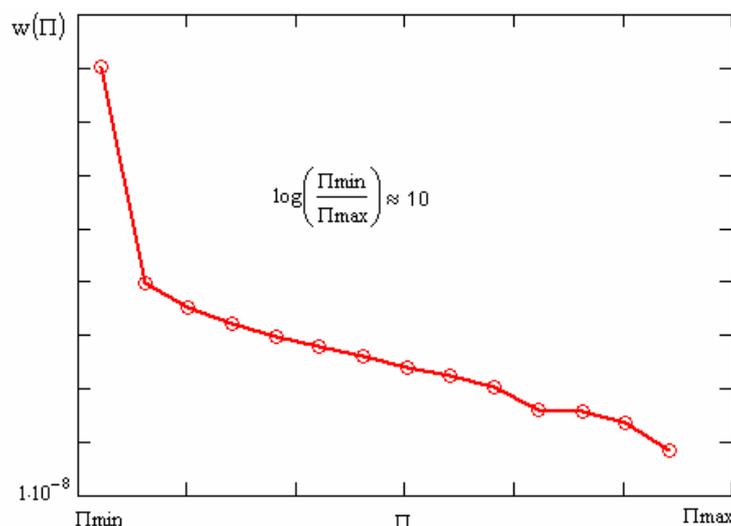


Fig.2. To estimation of PDF  $w(I)$

Region II is characterized by the uniform distribution of RI power. And in the region III we have straight line with the slope near -10dB/decade (function  $1/I$  gives us adequate approach for this situation). In the region IV we have very rapidly decreasing function.

Correspondingly, now process of generating of random numbers  $I$  to be in accordance with PDF  $w(I)$  may be reduced to forming of random value  $I$  to be distributed uniformly (regions I and II) and hyperbolically (III and IV).

It is much easy than forming of random values  $I$  taking into account expression (9).

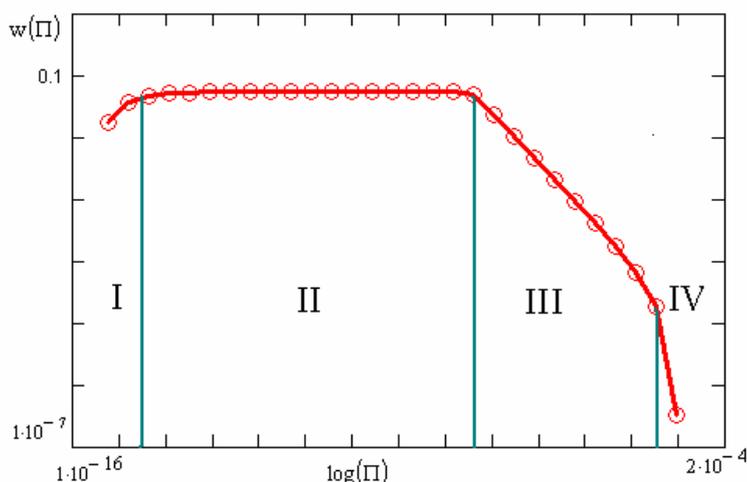


Fig.3. To discussion of PDF  $w(I)$

## CONCLUSION

In this paper an approach to estimation of general view of curves of PDF of power flow in the point of receiver has been discussed. This approach is based on statistical model of EM environment and simplified EMW propagation model. It is shown, the forming of random values  $I$  in accordance with PDF  $w(I)$  at direct Monte-Carlo simulation may be reduced to forming of random numbers with respectively simple PDFs.

The results of statistical simulation proved, in general, the hypothesis about of hyperbolical character of curves  $w(I)$ . PDF of interfering sources is a rapidly decreasing function having a large dynamic range. It is possible to suppose, that the character of dependence  $w(I)$  will not considerably change even with other PDFs  $w(P_{tr})$ ,  $w(G_{tr})$  and  $w(D)$  in a computational experiment.

The results of investigation may be used to predict EM environment at the receiver location, and for optimization of structure of communication systems on the stage of system structure design.

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