MUSIC LOCALIZATION OF
LOW-FREQUENCY MAGNETIC DIPOLE SOURCES

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ABSTRACT

It is important to identify the locations of electromagnetic (EM) noise sources within the electrical and electronic equipment, for the reduction of undesired noise emissions from it. In this study, by applying the MUSIC algorithm, we localize low-frequency (less than MHz) magnetic dipole sources (small current loops), which have incoherent or coherent time-series of waveforms. The numerical simulations and the experiments to localize incoherent or coherent magnetic dipole sources have demonstrated that the MUSIC algorithm is effective to estimate both the locations and orientations of the dipoles, as well as to recover their waveforms.

INTRODUCTION

In order to reduce the undesired electromagnetic (EM) noise emissions from the electrical and electronic equipment under the accrual operating conditions, it is necessary to measure the EM field distribution of the noise emissions externally, and identify the locations of the noise sources inside the equipment. Previously, for high frequencies (more than hundreds of MHz), the EM sources at finite distances have been localized for example with the MUSIC algorithms [1][2], and with holographic imaging [3]. On the other hand, at very low frequencies, the problem of localizing near-field EM sources has been solved for example in bioelectromagnetic inversion problems, such as the MUSIC localization of the current dipoles within human brains [4]. Recently the use of the MUSIC algorithm to characterize the near-field sources has been discussed in a more general way [5].

In this study we apply the MUSIC algorithm to estimate the 3-d locations and orientations of the low-frequency (less than MHz) *incoherent* or *coherent* magnetic dipoles (current loops), by measuring the magnetic field distributions around the dipoles with an array of magnetic vector sensors. The *incoherent* source dipoles can be localized by searching for the local maxima of a MUSIC cost function, which is scanned over unknown locations and orientations of the dipoles [6]. Such a MUSIC search can be easily extended to estimate additionally the "size" of large current loops [7]. For the *coherent* sources where two or more dipoles have correlated time-series of waveforms, however, the above MUSIC algorithm fails to distinguish them. The conventional method to find the coherent sources, such as the "spatial smoothing preprocessing" [8] cannot be applied here, because in our problem the measured magnetic field distribution depends nonlinearly on the relative arrangement of sensors and dipole sources. Instead, we adopt the spatio-temporal "independent topography" model used in biomagnetic inversion problems [9], where we search for the locations of multiple coherent dipoles simultaneously. The effectiveness of the method is evaluated by numerical simulations and experiments.

MUSIC ALGORITHM

As shown in Fig. 1, we have N_S incoherent magnetic dipole sources at locations $l_{S,i}$ $(i = 1, 2, \cdots, N_S)$, which have narrowband signal waveforms $s_i(t)$ and orientations $d_{S,i}$. The magnetic field distribution generated from the source dipoles is measured by N_A magnetic sensors, whose locations and orientations are given by $l_{A,j}$ and $d_{A,j}$ $(j = 1, 2, \cdots, N_A)$, respectively. Here the number of sensors N_A is assumed to be larger than the number of sources N_S. The magnetic field waveform measured by the j-th sensor is written as $x_j(t) = \sum_{i=1}^{N_S} a_{ji} s_i(t) + n_j(t)$, where $n_j(t)$ is the additive noise being zero mean and white with variance σ. The steering vector component a_{ji}

depends nonlinearly on the distance between the j-th sensor and the i-th source, as well as on their relative orientations.

MUSIC Localization of *Incoherent* Sources

With the eigenanalysis of the covariance matrix $\boldsymbol{R}_{xx}$ calculated from the magnetic field vector $\boldsymbol{X}(t)$ measured by the sensor array, we have $N_A - N_S$ noise eigenvalues. The noise subspace $\boldsymbol{E}_N$ spanned by the corresponding noise eigenvectors should be orthogonal to the steering vectors for the true sources, $a_i\left(\boldsymbol{l}_{S,i}, \boldsymbol{d}_{S,i}\right)$. Thus we can determine the source locations and orientations, by evaluating the local maxima of the MUSIC cost function P_{music} [10] given as

$$P_{music}\left(\boldsymbol{l}, \boldsymbol{d}\right) = \frac{\left\| a\left(\boldsymbol{l}, \boldsymbol{d}\right) \right\|^2}{\left\| E_N^H a\left(\boldsymbol{l}, \boldsymbol{d}\right) \right\|^2} , \tag{1}$$

where H means the Hermitian conjugate. Note that it is enough for us to scan (1) only over the locations $\boldsymbol{l}$, if we decompose the steering vector into the elementary steering vectors $\boldsymbol{a}_x$, $\boldsymbol{a}_y$, and $\boldsymbol{a}_z$, which correspond to x, y, and z-directional dipole sources, respectively [4][11]. Letting $\boldsymbol{a}\left(\boldsymbol{l}, \boldsymbol{d}\right) = [\boldsymbol{a}_x \mid \boldsymbol{a}_y \mid \boldsymbol{a}_z]\boldsymbol{d} \equiv \boldsymbol{a}_{xyz}\boldsymbol{d}$, the MUSIC cost function (1) is modified as the function of only the location $\boldsymbol{l}$, as

$$P_{music}\left(\boldsymbol{l}\right) = \frac{\boldsymbol{e}_{min}^H \boldsymbol{a}_{xyz}^H \boldsymbol{a}_{xyz} \boldsymbol{e}_{min}}{\lambda_{min}\left(\boldsymbol{a}_{xyz}^H \boldsymbol{E}_N \boldsymbol{E}_N^H \boldsymbol{a}_{xyz}\right)} , \tag{2}$$

where $\lambda_{min}(\ )$ takes the minimum eigenvalue of the matrix in the parenthesis. The modified cost function $P_{music}\left(\boldsymbol{l}\right)$ takes maximum at each of true source dipole locations, where the eigenvector $\boldsymbol{e}_{min}$ corresponding to the minimum eigenvalue represents the orientation of the dipole. The maximization of the MUSIC cost function (2) can be performed directly by a non gradient-based method, such as the Nelder-Mead simplex [12].

MUSIC Localization of *Coherent* Sources

For the *coherent* dipoles which have correlated time-series of waveforms, the conventional MUSIC algorithm fails to localize either of the dipoles because two or more coherent sources give a single eigenvalue. The conventional solution to this problem such as the spatial smoothing preprocessing [8] cannot be applied to the localization of the coherent dipoles, because their steering vectors change nonlinearly with the relative arrangement of sensors and dipoles. Instead, here we adopt the spatio-temporal "independent topography" model used in the biomagnetic inversion problems, such as the localization of the synchronous current dipoles within human brains [9]. We define a source with a single time-series of waveform, as a set of the coherent dipoles placed at different locations. To search for these dipole sources corresponding to the single signal eigenvalue, we modify the MUSIC cost function so as to be scanned over their locations simultaneously (i.e., a multiple dipole search).

Suppose there are two coherent dipoles at different locations ($\boldsymbol{l}_{S,1}$, $\boldsymbol{l}_{S,2}$) and orientations ($\boldsymbol{d}_{S,1}$, $\boldsymbol{d}_{S,2}$). Then we rewrite the MUSIC cost function (2) as,

$$P_{music}\left(\boldsymbol{l}_1, \boldsymbol{l}_2, \boldsymbol{d}_1, \boldsymbol{d}_2\right) = P_{music}\left(\boldsymbol{l}_1, \boldsymbol{l}_2\right) = \frac{\boldsymbol{e}_{min}^H \left[\boldsymbol{a}_{xyz}\left(\boldsymbol{l}_1\right) \ \boldsymbol{a}_{xyz}\left(\boldsymbol{l}_2\right)\right]^H \left[\boldsymbol{a}_{xyz}\left(\boldsymbol{l}_1\right) \ \boldsymbol{a}_{xyz}\left(\boldsymbol{l}_2\right)\right] \boldsymbol{e}_{min}}{\lambda_{min}\left(\left[\boldsymbol{a}_{xyz}\left(\boldsymbol{l}_1\right) \ \boldsymbol{a}_{xyz}\left(\boldsymbol{l}_2\right)\right]^H \boldsymbol{E}_N \boldsymbol{E}_N^H \left[\boldsymbol{a}_{xyz}\left(\boldsymbol{l}_1\right) \ \boldsymbol{a}_{xyz}\left(\boldsymbol{l}_2\right)\right]\right)} , \tag{3}$$

where an $N_A \times 6$ matrix $\left[\boldsymbol{a}_{xyz}\left(\boldsymbol{l}_1\right) \ \boldsymbol{a}_{xyz}\left(\boldsymbol{l}_2\right)\right]$ includes the elementary steering vectors at the source locations $\boldsymbol{l}_1$ and $\boldsymbol{l}_2$. We can find the locations of the two coherent dipoles simultaneously, by searching for the two sets of locations $\left(\boldsymbol{l}_{S,1}, \boldsymbol{l}_{S,2}\right)$ which maximize (3). The corresponding orientations of the dipoles $\left(\boldsymbol{d}_{S,1}, \boldsymbol{d}_{S,2}\right)$ can be found from the eigenvector $\boldsymbol{e}_{min}$ corresponding to the minimum eigenvalue in the denominator of (3). Even for such a multiple dipole search, the solutions can be found quite efficiently with the Nelder-Mead simplex.

For both the incoherent and coherent cases, once the dipole sources are localized, we can recover the signal waveforms of the individual sources [11].

NUMERICAL SIMULATION

The effectiveness of the MUSIC localization of *incoherent* dipoles has been demonstrated previously [6], so that here we show an example of the numerical simulations of the MUSIC localization of *coherent* dipoles. Suppose five magnetic vector sensors are placed at locations $(x, y, z) = (1, -0.5, -0.5), (2.5, 1, 0), (1, 2.5, 0.7), (-0.5, 1, 0), (1, -0.5, 0.7)$ [m], each of which measures the x-, y- and z-components of the magnetic field. We put two coherent magnetic dipole sources, #1 and #2, which have 10-kHz sinusoidal waveforms with different amplitudes and phases. The "true" locations and orientations of the dipoles are listed in Table 1, where the orientations are represented in direction cosines. The waveforms of the magnetic field measured by the sensors are sampled at 200 kHz, and the number of snapshots measured is 4096. When the SNR of the sensor output is 27 dB, which is defined as the ratio of the signal eigenvalue to the noise eigenvalue of the covariance matrix of the sensor output, the "estimated" locations and orientations are shown in Table 1. We can successfully estimate the locations and orientations of both of the coherent dipoles, with the estimation error less than a few mm for location and less than 1 degree for orientation. Such a location error should be consistent with the theoretical error variance of the MUSIC estimator [6][13]. Though not shown here, we can reproduce effectively the signal waveforms of the dipoles, successfully recovering their original amplitudes and phases.

Table 1: Source parameters (simulation)

Source #1	true	estimated	Source #2	true	estimated
location [m]	(1.740, 0.640, 0.120)	(1.741, 0.639, 0.119)	location [m]	(0.490, 0.900, -0.410)	(0.489, 0.903, -0.413)
orientation	(0.707, 0.000, 0.707)	(0.706, 0.001, 0.709)	orientation	(0.000, 0.000, 1.000)	(0.006, 0.001, 1.000)

EXPERIMENT

We have developed an experimental system to locate the low-frequency current loop (magnetic dipole) sources as shown in Fig. 2. Here, as the source dipoles we set up small current loops of diameters of about 10 cm, and we measure the magnetic vector filed with tri-axial search coil magnetometers. Table 2 shows an experimental result of the MUSIC localization of such current loop sources, where the configuration of the sources and sensors are identical with that in the simulation of the previous section. Compared with the "nominal" values, the "estimated" values exhibit the error as large as about 20 cm for location and 7 degrees for orientation, which is much larger than that in the simulation. The main reason of this is likely to be attributed to the inaccurate experimental setup of the "nominal" locations and orientations of the current loops and the tri-axial search coils.

Table 2: Source parameters (experiment)

Source #1	nominal	estimated	Source #2	nominal	estimated
location [m]	(1.740, 0.640, 0.120)	(1.789, 0.626, 0.089)	location [m]	(0.490, 0.900, -0.410)	(0.394, 0.735, -0.362)
orientation	(0.707, 0.000, 0.707)	(0.690, 0.025, 0.723)	orientation	(0.000, 0.000, 1.000)	(0.041, 0.114, 0.993)

CONCLUSION

We have applied the MUSIC algorithm to localize the multiple low-frequency magnetic dipole sources. The numerical simulations and the experiments to localize *incoherent* or *coherent* magnetic dipole sources (small current loops) have demonstrated that the MUSIC algorithm is effective to estimate both the locations and orientations of the dipoles, as well as to recover their waveforms. With the developed experimental system the estimated sources can be "visualized" on a computer screen, by superimposing them directly on the "real" camera image of the sources [14]. In the future we will extend the algorithm to deal with line and plane current sources, by possibly estimating their current distributions. We will also examine the influence of metal chassis or frames surrounding the EM noise sources inside electrical and electronic equipment, to localize the actual noise sources inside the equipment.

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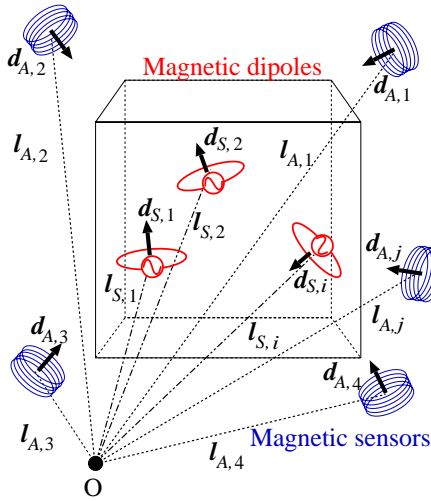
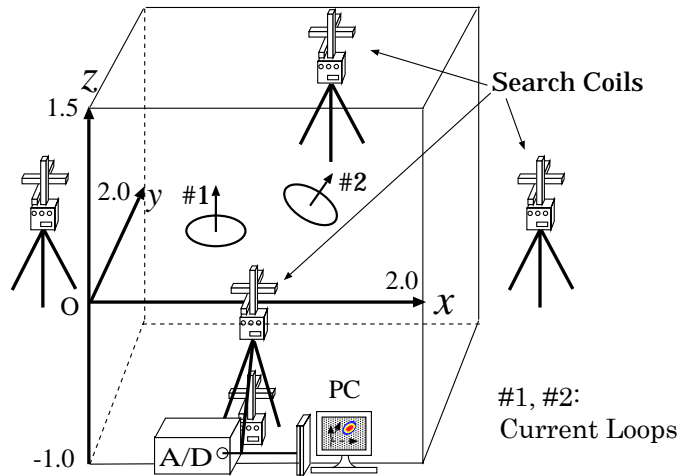Fig.1. Magnetic dipole sources
and magnetic sensors

Fig.2. Experiment configuration for low-frequency
current loop localization