# HIGH FREQUENCY WAVE PROPAGATION ALONG NON-UNIFORM TRANSMISSION LINES: A DIRECT ITERATION APPROACH

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## ABSTRACT

In this paper, we consider the propagation of current waves along the horizontal curved wire above the ground. On the base of the exact Mixed Potential Integral Equations (MPIE) for the current and scalar potential we have developed an iteration approach, which permits to obtain solutions including corrections accounting for line radiation. The zero iteration term correspond to the classical transmission line (TL) solution. The first iteration term gives corrections to the reflection and transmission coefficients associated with the line bend. The derived analytical expressions for the straight bend are compared with "exact" numerical results published by other authors and very good agreement is found. The developed analytical expressions can also be used to evaluate the radiated power associated with the line bend.

## INTRODUCTION

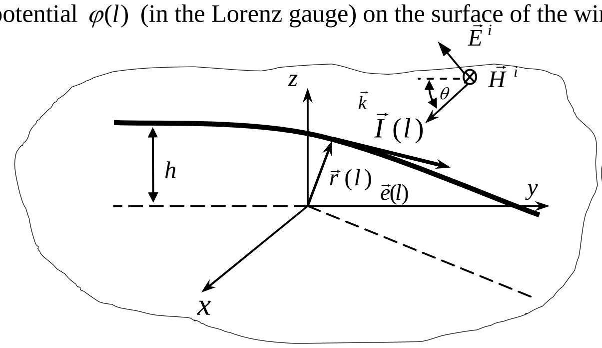
The evaluation of high frequency-operating signals propagation along transmission lines, as well as interferences induced by high frequency sources such as UWB systems require to take into account a radiation effects. It is well know that when current wave propagates along an infinitely-long straight wire parallel to a perfectly-conducting ground, the electromagnetic field around the wire has a TEM structure, and the current wave does radiate. The propagation of current waves can be described by the classical TL theory [1]. However, when non-uniformities (vertical risers, lumped impedances or source along the line, line bend, etc.) are present, the TEM structure of the electromagnetic field breaks down and other eigenmodes and radiation modes appear. As a result, the current wave near the non-uniformity scatters and radiates electromagnetic energy [2]. For high-frequencies, when the characteristic wavelength of corresponding electromagnetic field is comparable or less than the height of the wire, the classical TL approximation is not applicable. The processes of radiation and scattering by the non-uniformity can be described in terms of the complex reflection and transmission coefficients associated with the line nonuniformity [3], which can be defined by analogy with the one-dimension scattering problem in quantum mechanics [4].

In the present paper we consider the propagation of high frequency current waves along a horizontal curved wire above the ground. The analysis is based on the homogeneous Mixed Potential Integral Equations (MPIE), an integral-differential equation for the current and potential along the wire in the Lorenz gauge and assuming thin wire approximation. For the analytical solution of this system, we will generalize the method of direct iterations (perturbation theory) which was described in [5] for the case of a finite straight wire. The small parameters of this perturbation theory is the "thin-wire parameter for a horizontal line" $1/2\ln(2h/a)$, where $h$ is height of the wire, $a$ is the radius of the wire.

The developed method gives the possibility to obtain in a general form the complex reflection and transmission coefficients of the current waves through the transmission line non-uniformity, which characterize the radiation of the non-uniformity. For the case of a sharp bend, the method gives analytical results for the first iteration [3], which are in excellent agreement with "exact" numerical results published by another authors [6]. The proposed method will be used to investigate reflection and transmission coefficients associated with a smooth bend of arbitrary geometrical shape.

## MIXED POTENTIAL INTEGRAL EQUATION (MPIE) FOR A THIN CURVED WIRE ABOVE THE GROUND AND ITS ITERATIVE SOLUTION

Consider a lossless current filament of infinite length above a perfectly conducting ground (see Fig.1). We assume that the transmission line has a bend of arbitrary geometric form near the origin of coordinates (and straight otherwise). The curve of the wire axis can be described in the following parametric form [7]: $\vec{r}(l) = (x(l), y(l), h) = (\vec{\rho}(l), h)$, where $l$ is a the position along the curved wire and $h$ is the height of the wire above ground. We assume that the line is in the presence of an external electromagnetic field $E_l^e(l)$. Using the boundary condition for the total (scattered plus exciting) tangential electric field, the continuity equation for the induced current $I(l)$ and the charge $q(l)$ density, it is possible to obtain, under the thin-wire approximation, a system of integral-differential equations for the current $I(l)$ and scalar potential $\varphi(l)$ (in the Lorenz gauge) on the surface of the wire [8].



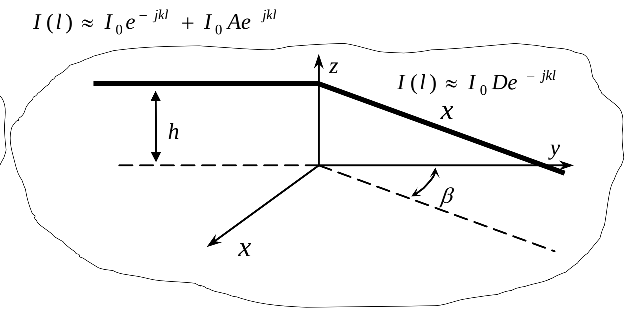




Fig. 1 Horizontal line excited by external field.

Fig. 2 Scattering of the TEM current wave on the horizontal straight bend.

$$\begin{cases} \dfrac{d\varphi(l)}{dl} + j\omega\dfrac{\mu_0}{4\pi}\displaystyle\int_{-\infty}^{\infty}\vec{e}_l(l)\cdot\vec{e}_l(l')\,g(l,l',k)\,I(l')\,dl' = E_l^e(l) \\ \displaystyle\int_{-\infty}^{\infty} g(l,l',k)\frac{dI(l')}{dl'}dl' + j\omega 4\pi\varepsilon_0\,\varphi(l) = 0 \end{cases};\qquad g(l,l',k) = \frac{e^{-jk\sqrt{(\vec{\rho}(l)-\vec{\rho}(l'))^2+a^2}}}{\sqrt{(\vec{\rho}(l)-\vec{\rho}(l'))^2+a^2}} - \frac{e^{-jk\sqrt{(\vec{\rho}(l)-\vec{\rho}(l'))^2+4h^2}}}{\sqrt{(\vec{\rho}(l)-\vec{\rho}(l'))^2+4h^2}} \qquad (1a,b)$$

Here $E_l^e(l)$ is an exciting tangential electric field (incident plus reflected), $\varphi(l)$ is the scalar potential along the wire (in the Lorenz gauge), $a$ the radius of the wire, the unit tangential vector $\vec{e}_l(l) = d\,\vec{r}(l)/d\,l$ of the curve is taken along the wire axis. The function $g(l,l')$ is the 3-dimension scalar Green's functions along the curved line, which takes into account the reflection of the ground plane. The current induced in the considered system is obtained by the solution of the MPIE (1) with proper boundary conditions in $\pm\infty$.

For low frequencies $2kh << 1$, and smooth curved wire $K(l)h << 1$ (where $K(l) = \left|d\vec{e}/dl\right|$ is the curvature of the bend of the wire axis [7]), it is possible to show that the integrals in (1) can be approximately written as:

$$\begin{aligned} &\int_{-\infty}^{\infty}\vec{e}_l(l)\cdot\vec{e}_l(l')\,g(l,l',k)I(l')\,dl' \approx I(l)\int_{-\infty}^{\infty}g_0(l-l',0)\,dl' = 2\ln(2h/a)I(l) \\ &\int_{-\infty}^{\infty}g_l^L(l,l',k)\frac{dI(l')}{dl'}dl' \approx \frac{dI(l)}{dl}\int_{-\infty}^{\infty}g_0(l-l',0)\,dl' = 2\ln(2h/a)\frac{dI(l)}{dl} \end{aligned} \quad (3);\quad g_0(l-l',k) = \frac{e^{-jk\sqrt{(l-l')^2+a^2}}}{\sqrt{(l-l')^2+a^2}} - \frac{e^{-jk\sqrt{(l-l')^2+4h^2}}}{\sqrt{(l-l')^2+4h^2}} \quad (4)$$

And as a result, Equations (1a) reduce to the classical TL coupling equations for an infinite wire exited by an external field:

$$\begin{cases} d\varphi^{(0)}(l)/dl + j\omega L_0' I^{(0)}(l) = E_l^e(l) \\ d\,I^{(0)}(l)/dl + j\omega C_0'\varphi^{(0)}(l) = 0 \end{cases};\quad \text{with } L_0' = \frac{\mu_0}{2\pi}\ln\left(\frac{2h}{a}\right), C_0' = \frac{2\pi\varepsilon_0}{\ln\left(\dfrac{2h}{a}\right)} \qquad (5)$$

Using the one-dimension Green's function, the solution of this system can be written in the following form,

$$I^{(0)}(l) = \frac{j}{2k}\int_{-\infty}^{\infty}\left(-j\omega C_0' E_l^e(l')\right)e^{-jk|l'-l|}dl' \qquad (6)$$

Now it is possible to describe a perturbation procedure to solve the exact MPIE system (1). We can re-write this system of equations in a TL-like form (equation (/) below) where $U_S'\{I(l)\}$ and $I_S'\{I(l)\}$ can be considered as additional distributed voltage and current source terms, respectively.

$$\begin{cases} \dfrac{d\varphi(l)}{dl} + j\omega L_0' I(l) = E_l^e(l) + U_S'\{I(l)\} \\ \dfrac{dI(l)}{dl} + j\omega C_0'\varphi(l) = I_S'\{I(l)\} \end{cases};\qquad \begin{aligned} U_S'\{I(l)\} &= -j\omega\frac{\mu_0}{4\pi}\int_{-\infty}^{\infty}\left(\vec{e}_l(l)\vec{e}_l(l')g(l,l',k)I(l') - g_0(l,l',k)I(l)\right)dl' \\ I_S'\{I(l)\} &= -\frac{1}{2\ln(2h/a)}\int_{-\infty}^{\infty}\left(g(l,l',k)\frac{dI(l')}{dl'} - g_0(l,l,k')\frac{dI(l)}{dl}\right)dl' \end{aligned} \qquad (7a,b)$$

Now it is possible to solve the system of equations (7) using an iterative technique. The system of equation for the zero-order iteration is the classical homogeneous TL system with constant parameters (5). The corresponding zero-order iteration solution for the current is given by (6). To obtain the solution for the iteration $n \geq 1$, we have to solve the classical system of telegrapher's equations (9) which include additional distributed voltage and current sources dependent on the previous iteration.

$$\begin{cases} I(l) = I^{(0)}(l) + I^{(1)}(l) + ... + I^{(n)}(l) + ... \\ \varphi(l) = \varphi^{(0)}(l) + \varphi^{(1)}(l) + ... + \varphi^{(n)}(l) + ... \end{cases} \quad (8); \qquad \begin{cases} d\varphi^{(n+1)}(l)/dl + j\omega L_0' I^{(n+1)}(l) = U_S'\left\{I^{(n)}(l)\right\} \\ d\, I^{(n+1)}(l)/dl + j\omega C_0' \varphi^{(n+1)}(l) = I_S'\left\{I^{(n)}(l)\right\} \end{cases} \quad n \geq 1 \quad (9)$$

Using the one-dimensional Green's function, the general solution for the n$^{th}$ iteration can be written as

$$I^{(n+1)}(l) = \frac{j}{2k} \int_{-\infty}^{\infty} \left( -j\omega C_0' U_S' \left\{I^{(n)}(l)\right\} + \frac{d}{dl'} I_S' \left\{I^{(n)}(l)\right\} \right) e^{-jk|l'-l|} dl' \qquad (10)$$

It is possible to show that the n$^{th}$ iterations for the current and potential have an order of magnitude $I^{(n)}, \varphi^{(n)} \sim \alpha^n$, where $\alpha = \left(2\ln(2h/a)\right)^{-1}$ is a small parameter for a thin-wire horizontal line. For example, for $h = 1$ m, $a = 1$ mm $\alpha \sim 0.065$

**REFLECTION AND TRANSMISSION COEFFICIENTS ASSOCIATED WITH A LINE BEND**

The developed method will be now applied to evaluate the current reflection and transmission coefficients associated with the line bend. In this case the external electromagnetic field is assumed to be absent $E_l^e(l) = 0$. The TEM current wave $\exp(-jkl)$ traveling from $l = -\infty$ (what formally corresponds to a source located in -∞) is partially reflected from the bend and partially transmitted to $l = +\infty$. In the zone of the bend, the TEM structure of electromagnetic field is disturbed. The magnitudes of the reflected and transmitted waves are described by complex reflection and transmission coefficients $A$ and $D$ (11). It is possible to show that the radiation from the bend can be expressed in terms of these two coefficients. The associated radiated power $P_{rad}$ can be evaluated using (12).

$$I(l) = \begin{cases} I_0 \exp(-jkl) + I_0 A \exp(jkl) & -l >> 2h, 1/|K(0)| \\ I_0 D \exp(-jkl), & l >> 2h, 1/|K(0)| \end{cases} \quad (11); \qquad P_{rad} = \frac{1}{2} Z_c |I_0|^2 \left(1 - |A|^2 - |D|^2\right) \quad (12)$$

Using the perturbation theory and similarly to the equations (8) for the induced current and potential, the reflection and transmission coefficients can be decomposed to

$$A = A^{(0)} + A^{(1)} + A^{(2)} + ... ; \qquad D = D^{(0)} + D^{(1)} + D^{(2)} + ... \qquad (13\text{ a,b})$$

The zero-iteration term for current is $I^{(0)}(l) = e^{-jkl}$ and the zero-iteration reflection and transmission coefficients have the trivial values $A^{(0)} = 0$, $D^{(0)} = 1$. The general solution for the nth-iteration term for the reflection and transmission coefficients $A^{(n)}, D^{(n)}$ can be obtained by the using a general analytical solution for the current (10) for the current. For the first nontrivial correction term $n = 1$, the results are:

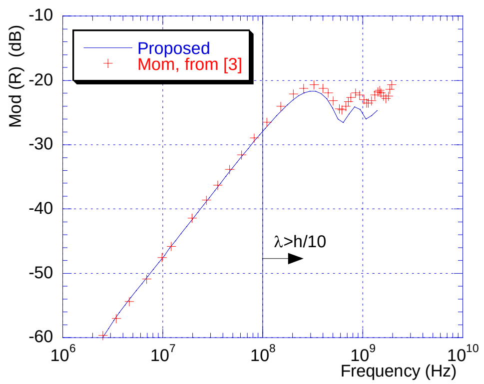
$$A^{(1)} = \frac{k}{4j\ln\left(\frac{2h}{a}\right)} \int_{-\infty}^{\infty} dl \int_{-\infty}^{\infty} dl' e^{-jk(l+l')} \left\{ (\vec{e}_l(l)\vec{e}_l(l') + 1) g(l,l',k) - g_0(l-l',k) \right\}; \qquad D^{(1)} = \frac{k}{4j\ln\left(\frac{2h}{a}\right)} \int_{-\infty}^{\infty} dl \int_{-\infty}^{\infty} dl' e^{-jk(l'-l)} (\vec{e}_l(l)\vec{e}_l(l) - 1) g(l,l',k)$$

(14 a,b)

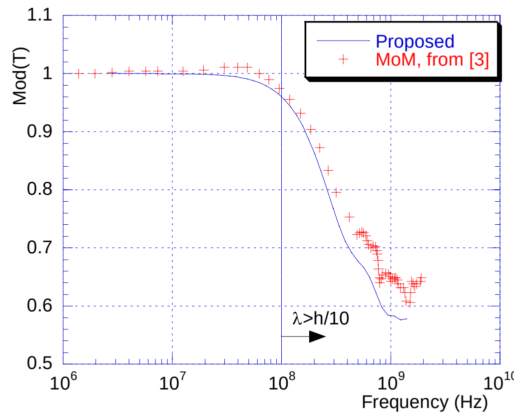
The formulae (14 a,b) are the main result of the present paper. They yield reflection and transmission coefficients associated with a wire bend of arbitrary shape. For the simplest case of a straight bend (see Fig. 2), it can be shown that Equations (14a,b) reduce to equations (15a,b), already presented in [3] (here $\beta$ is the angle of the bend as shown in Fig. 2). The comparison of our results with numerical simulations obtained using the method of moments [6] gives a good agreement (see Fig. 3 a,b)

$$A^{(1)} = \frac{j}{2\ln(2h/a)} \left[ 2\int_0^{\infty} \sin kl \cdot g_0(l,k) dl - k(1+\cos\beta) \cdot \int_0^{\infty} dl \int_{-\infty}^{0} dl' \cos(k(l+l')) \cdot g_0\left(k\sqrt{l^2 + l'^2 - 2ll'\cdot\cos\beta}, k\right) \right] \qquad (15\text{ a})$$

$$D^{(1)} = \frac{j}{2\ln(2h/a)} k(1-\cos(\beta)) \int_0^{\infty} dl \int_{-\infty}^{0} dl' \cos(k(l-l')) \cdot g_0\left(\sqrt{l^2 + l'^2 - 2ll' \cdot \cos\beta}, k\right)\Bigg] \qquad (15\text{ b})$$




a)




b)

Fig. 3 Magnitudes of reflection (a) and transmission (b) coefficients as a function of frequency (Horizontal wire with a radius $r = 5 \cdot 10^{-4}$ is located at a height $h = 0.3m$ above a perfectly conducting ground. The bend angle is $\beta = 90^0$) .

## CONCLUSION

A Mixed Potential Integral Equation describing the coupling of an electromagnetic field with a non-uniform infinite horizontal line is derived. Using perturbation theory, iterative solutions have been obtained where the zero iteration tem corresponds to the classical TL coupling equations with constant parameters and with tangential component of the exciting electric field as the voltage source. The general $n^{th}$ iteration is described by similar TL equations with constant parameters but with additional source terms which depend on the solution of the $(n\text{-}1)^{th}$ iteration. General analytical expressions for the $n^{th}$ iteration have been obtained.

Using the proposed method, analytical expressions for the reflection and transmission line coefficients associated with a line bend of arbitrary shape are also derived. It is shown that, with only one iteration, an excellent approximation to the 'exact' numerical results for the reflection and transmission coefficients can be obtained. The developed analytical expressions can be used to evaluate the radiated power associated with the line bend.

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