CALCULATION OF LIGHTNING ELECTROMAGNETIC FIELDS: A REVIEW

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ABSTRACT

Lightning return stroke is a self-propagating discharge with path length measured in kilometres, which extends at high speeds, sometimes approaching a significant fraction of the speed of light. Usually one is interested in the electromagnetic fields produced by lightning several tens of meters to some kilometres away. In order to calculate fields from lightning, it is modelled as a linear travelling-wave antenna with some current distribution or line charge density distribution that changes rapidly with time. Lightning discharge is a special source usually not found in traditional electromagnetic texts. During the last several years, new analytical techniques were developed for calculating electromagnetic fields from lightning, which are reviewed in this paper.

Calculation of lightning electromagnetic fields requires careful consideration of the retardation effects due to finite travel time of the signals. The analytical expressions for far radiation fields often contain the so-called *F* factor, $(1-(v/c)\cos\theta)^{-1}$, where v is the speed of the return stroke wave front or speed of travelling waves in the return stroke channel (these two speeds are not necessarily the same), c is the speed of light, and θ is the angle formed by the return stroke channel with the line connecting the field point and the retarded position of the wavefront. However, the general expressions for electromagnetic fields from arbitrarily-specified current distributions that vary in time and space do not require any explicit correction involving *F* factor. Different situations in which an explicit *F* factor can arise and their implications for the radiation field pattern from lightning with different return stroke speeds are reviewed.

The electric fields from lightning can be calculated using three different approaches, resulting in three different but equivalent expressions that yield the same total fields and the same Poynting vectors. These general expressions are applicable to any line source distribution that varies with time. The first two approaches are the traditional dipole (Lorentz condition) technique and the monopole (continuity equation) technique. The third approach is based on a mathematical transform that relates the retarded currents and charge densities as would be 'seen' by an observer at the field point. The three approaches yield three different expressions for the scalar potential, but are analytically equivalent and do satisfy the Lorentz condition. In the three approaches, expressions for the individual electric field components in the time domain, traditionally identified by their distance dependence as electrostatic, induction, and radiation terms, are different, suggesting that explicit distance dependence is not an adequate identifier of these components. Calculations of electric fields at different distances from the lightning channel show that the differences between the corresponding field components identified by their distance dependence in different techniques are considerable at close ranges but become negligible at far ranges.

In the transmission line model with the return stroke speed assumed to be equal to the speed of light, c, the shapes of electric field, magnetic field, and current waveforms are identical at all points in space. In that case, the magnetic fields and electric fields are related through the free-space impedance (or through the speed of light) and the Poynting vector is directed radially outward from the origin. When the return stroke speed is less than c (typical values range from c/3 to 2c/3), the electric and magnetic field waveforms within a few tens of kilometres are quite different from each other and from the causative current waveform.

INTRODUCTION

Electromagnetic fields from lightning can couple to electrical circuits and systems and produce transient overvoltages, which can cause power and telecommunication outages and destruction of electronics. Therefore calculation of the electric and magnetic fields from different lightning processes has important practical applications. Some aspects of the analytical methods for calculating the electromagnetic fields from the lightning return stroke are reviewed here.

1. TREATMENT OF RETARDATION EFFECTS AND THE NEED FOR F FACTOR

Lightning return stroke channel extends typically at a speed of one-third to two-thirds of the speed of light. Due to the finite travel time of the electromagnetic waves between the source and observer, at any given time the remote observer ‘sees’ the current on the return stroke channel from an earlier time. Similarly, the observer at a remote point does not see the true length of the channel, that is, at a given time only some portion of the lightning channel traversed by the return stroke contributes to the field at the remote point. Therefore retarded sources and retarded channel lengths are to be used in the calculation of return stroke fields. An extensive treatment of retardation effects in calculating the electromagnetic fields from the lightning discharge is found in Thottappillil et al. [1].

Consider a return-stroke channel with one end fixed at A as shown in Fig. 1. It takes a time r/c for the information from A to reach the observer at P and hence the observer “sees” the channel emerging from A at time r/c. The actual length $L(t)$ of the channel at a time t is given by $L(t) = v \cdot t$, where v is the return-stroke front speed. The apparent length of the channel at time t ‘seen’ by the observer at P is different from $L(t)$.

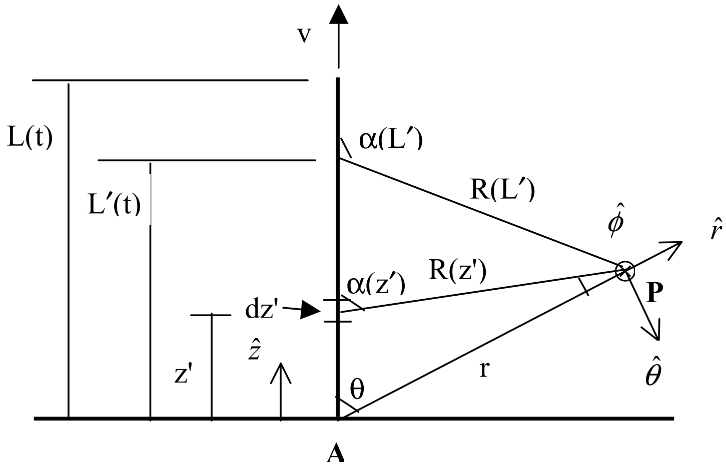

Fig. 1 Geometry of the problem and definition of symbols

The time t can be expressed as the sum of the time required for the return-stroke wavefront to reach a height $L'(t)$ and the time required for the electromagnetic signal to travel from the wavefront at $L'(t)$ to the observer at P,

$$t = \frac{L'(t)}{v} + \frac{R(L')}{c} \qquad \text{where } R(L') = \sqrt{r^2 + L'^2(t) - 2L'(t) r \cos\theta} \tag{1}$$

The apparent length L′(t) can be obtained from (1), and is given by

$$L'(t) = \frac{r}{1-(v^2/c^2)}\left(-\frac{v^2}{c^2}\cos\theta + \frac{vt}{r} - \frac{v}{c}\sqrt{\left(1-\frac{v^2}{c^2}\right) + \frac{v^2 t^2}{r^2} + \frac{v^2}{c^2}\cos^2\theta - \frac{2vt}{r}\cos\theta} \right) \tag{2}$$

If the ground is treated as perfectly conducting, (2) can also be used, with θ replaced by (180°-θ) to find the apparent length $L''(t)$ of the channel image ‘seen’ by the observer. If the channel length is very small compared to the distance to the observer, that is, if $L'(t) \ll r$, then $R(L')$ can be approximated as $R(L') = r - L'(t)\cos\theta$, substitution of which in (2) gives

$$L'(t) = \frac{v}{1-(v/c)\cos\theta} \cdot (t - r/c) \tag{3}$$

and the image channel length is obtained from (3) by replacing θ by 180°-θ. The factor $(1-(v/c)\cos\theta)^{-1} = F$ in (3) above is due to retardation effects and would be equal to unity if retardation effects could be neglected, that is, if $v \ll c$. This factor, sometimes called F factor, can also arise in the expression for the apparent speed of a moving point charge or of a wavefront. The exact expression for apparent speed of the return stroke wavefront is given by [1],

$$\frac{dL'(t)}{dt} = v.\left[1 + \frac{v}{c}.\frac{L' - r\cos\theta}{\sqrt{L'^2 + r^2 - 2L' r\cos\theta}}\right]^{-1} = v.\frac{1}{1-(v/c)\cos\alpha(L')} = v \cdot F(L') \tag{4}$$

So far we have seen that F factor can appear explicitly in equations for a) the length of a moving line as seen by a remote observer (apparent length) and b) the speed of a travelling wave as seen by a remote observer (apparent speed). F factor can also arise when one carries out certain analytical simplifications of field expressions. The case of far electromagnetic fields from a travelling current discontinuity, first considered by Rubinstein and Uman [2] and later discussed by Rubinstein and Uman [3] and [1], is one example of that. Current distributions associated with certain return stroke models, for example, the TL and TCS models, also may give rise to the F factor in the analytical simplification of far radiation field expressions (e.g., LeVine and Willett [4], [1]). Other situation when the F factor can come out explicitly is the gradient of retarded time, as discussed by Shao et al. [5].

The implications of F factor in determining the radiation beam pattern from lightning return stroke have been discussed in Krider [6], [1], Thottappillil et al. [7], and [5]. Using the radiation or far field approximation, [6] showed that for the transmission line model with speed *v*, with the presence of ground taken into account, the radiation field has its maximum along the ground surface ($\theta = 90^o$) for $v < c/\sqrt{2}$. For speeds greater than $c/\sqrt{2}$, the maximum field occurs for an angle between 0 and 90^o, closer to 0^o for higher speeds. For return stroke speeds near c, maximum fields radiated in the upward direction can far exceed in amplitude those at ground level at similar distances.

2. **NON-UNIQUENESS OF ELECTROSTATIC, INDUCTION, AND RADIATION FIELD COMPONENTS**

The problem of calculating the electric and magnetic fields from a known source distribution is discussed extensively in the literature (e.g., Uman et al. [8]). Usually the fields are calculated by using scalar and vector potentials, which are directly related to the source charge and current density distributions, respectively. There are three equivalent approaches to calculating the electric fields produced by a specified source. Two of these equivalent approaches are discussed in Rubinstein and Uman [9], and Thottappillil and Rakov [10, 11], and the third equivalent approach in Thottappillil et al. [12] and [10].

Approach 1: The first approach, the so-called dipole technique or Lorentz condition (LC) technique, involves the specification of current density $\overline{J}$, the use of $\overline{J}$ to find the vector potential $\overline{A}$, the use of $\overline{A}$ and the Lorentz condition to find the scalar potential ϕ, the computation of electric field $\overline{E}$ using $\overline{A}$ and ϕ, the computation of magnetic field $\overline{B}$ using $\overline{A}$. In this technique, the source is described only in terms of current density, and the field equations are expressed only in terms of current. The use of the LC eliminates the need for the specification of the line charge density along with the current density and assures that the current continuity equation, which is not explicitly used in this technique, is satisfied.

Approach 2: The second approach, the so-called monopole technique or the continuity equation (CE) technique, involves the specification of current density $\overline{J}$, the use of $\overline{J}$ and the continuity equation to find ρ, the use of $\overline{J}$ to find $\overline{A}$ and ρ to find ϕ; the computation of electric field $\overline{E}$ using $\overline{A}$ and ϕ, and the computation of magnetic field $\overline{B}$ using $\overline{A}$. In this technique, the source is described in terms of both current density and line charge density, and the field equations are expressed in terms of both line charge density and current. The current continuity equation is needed to relate the current density and charge density, and it is given by

$$\frac{\partial \rho^*\left(z',t-R(z')/c\right)}{\partial t} = -\left.\frac{\partial i\left(z',t-R(z')/c\right)}{\partial z'}\right|_{t-R(z')/c=const} \tag{5}$$

where ρ^* is the local line charge density evaluated at retarded time, *i* is the local current at retarded time, and the spatial derivative of the current is evaluated keeping the retarded time constant. There is no need for the explicit use of the LC in this technique, although properly specified scalar and vector potentials do satisfy the LC.

Approach 3: In the third approach, the electric fields are expressed in terms of the apparent charge density, that is, the charge density that would be 'seen' on the lightning channel at the field point [11], which is given by

$$\frac{\partial \rho\left(z',t-R(z')/c\right)}{\partial t} = -\frac{\partial i\left(z',t-R(z')/c\right)}{\partial z'} \tag{6}$$

where ρ is the 'apparent' charge density and the spatial differentiation of retarded current is carried out without keeping the retarded time constant. The differences between this apparent charge density ρ and the charge density ρ^* in Approach 2 above are explained in [10]. Field equations obtained in either of the first two approaches can be converted into this third form, completely in terms of apparent charge density.

Magnetic fields are expressed completely in terms of current in the first two approaches and completely in terms of apparent charge density in the third approach [10, 12].

Fields calculated using the LC approach (Approach 1 above) and the CE approach (Approach 2 above), should be equivalent since both are based on rigorous application of electromagnetic principles and use the same basic assumptions. However, their appearances are different. It is shown in [10] that while the total fields are identical, the individual field components (electrostatic, induction, and radiation terms identified by their dependence on R) in the field equations from these two methods are different. In equation from the CE approach, the electrostatic and induction terms are given completely by the gradient of the scalar potential, while the radiation term is completely given by the time derivative of the vector potential. In contrast, in equation from the LC approach, both the gradient of the scalar potential and the time derivative of the vector potential contribute to the radiation field term. Thus the electrostatic (R^{-3}

dependence), induction ($c^{-1}R^{-2}$ dependence), and radiation ($c^{-2}R^{-1}$ dependence) terms in the equation from the LC approach are different from the corresponding terms in the equation from the CE approach. The difference is considerable very close to the channel (e.g., at 50 m) and almost negligible far away from the channel (e.g., at 100 km). Very close to the channel, the electrostatic term (R^{-3} dependence) in the equation from the LC approach is larger than its counterpart in equation from the CE approach. The difference between the two formulations of the continuity equation in Approaches 2 and 3 is related to different treatments of retardation effects.

In summary, even though the total electric field from a current or charge distribution is unique, the division of this electric field in the time domain into so-called electrostatic (R^{-3} dependence), induction ($c^{-1}R^{-2}$ dependence), and radiation ($c^{-2}R^{-1}$ dependence) components is not unique. Further details on different methods for field calculations can be found in [14].

3. TRANSMISSION LINE MODEL FOR RETURN STROKE SPEED EQUAL TO THAT OF LIGHT

Lightning return-stroke transmission-line model calculations indicate that for a return stroke speed less than the speed of light, c, the electric and magnetic field waveforms within a few tens of kilometers are quite different and are different from the causative current waveform. For a vertical return stroke channel, whose radius is zero in the limit, above perfectly conducting ground, the exact expressions for total electric and magnetic fields for return stroke speed equal to *c* are derived by [7] and [13], and are given by,

$$\overline{E}(r,\theta,t) = \frac{1}{2\pi\varepsilon_0 cr\sin\theta} i(0,t-r/c)\hat{\theta}, \quad \theta \neq 0 \tag{7}$$

$$\overline{B}(r,\theta,t) = \frac{1}{2\pi\varepsilon_0 c^2 r\sin\theta} i(0,t-r/c)\hat{\phi}, \quad \theta \neq 0 \tag{8}$$

It is seen from (7) and (8) that for such an idealized return stroke, the electric field, magnetic field, and current waveforms have identical shapes at all points in space. Equations (7) and (8) suggest the formation of a spherical TEM (transverse electromagnetic) field structure in the surrounding space [13].

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