Hierarchical Models of Complex Systems in Time-Domain

C. Christopoulos, P. Sewell, K. Biwojno, Y. Liu, C.J. Smartt

George Green Institute for Electromagnetics Research – GGIEMR
The University of Nottingham, UK
University Park, Nottingham NG7 2RD

ABSTRACT

It is generally accepted that modelling complex electromagnetic interactions in electromagnetic compatibility problems is an extremely demanding task and that no amount of increased computational power will ever be sufficient for tackling the complexity of modern engineering designs. Therefore major innovations in the way we model systems are also urgently required. Scientists have recognised the need to develop models of appropriate complexity to describe the response of particular physical systems to a range of excitations by devising a hierarchy of models. For example, much first pass design in industry is based on semi-empirical models exploiting the intuition and experience of the designer. More formal expert systems may also be designed which embody the experience of the design engineers in software and thus offer a check and guidelines for less experienced designers. As new design specifications are developed, such expert systems become insufficiently accurate and new more complete models based on sound physics are needed. For electromagnetic systems whose dimensions are smaller than the wavelengths of interest lumped circuit equivalents provide a sound alternative and if only one spatial dimension is larger than the wavelength, a distributed model is sufficient. However, as designs migrate to higher frequencies (with clock rates in the GHZ range) full field models are required which due to the geometric complexity of many systems result in not only prohibitively large computations but which are also not ideal for illuminating the physical mechanisms at play. It is therefore useful to consider a global model of complete systems which includes within it a hierarchy of sub-models each tuned to the particular requirements of particular features of the system. In this way both computational efficiency and deeper physical understanding are achieved since the details do not obscure the overall nature of model. It is in this area that this contribution is focused.

The possible hybrid environments for modelling complex systems will be described by examining techniques such as multi-scaling, structured/unstructured meshes and multi-gridding. Embedding local field solutions in multi-scale models will be described based on both Modal Expansion Techniques (MET) and Digital Filter Interfaces (DFI). The relative merits of using structured and unstructured meshes will also be described and the way they may be combined to produce a multi-grid modelling environment consisting of regions of structured and unstructured mesh of different density and embedded local field solutions will be discussed and illustrated by results.

INTRODUCTION

Significant effort has been made over the last decade to develop electromagnetic field modelling techniques that can deal efficiently with problems of rapidly increasing scale and geometric complexity. Particularly demanding for design and analysis are systems containing a diverse range of physical scales separated or surrounded by a significant amount of space, all of which meaningfully contribute to the overall response. This is commonly epitomised in electromagnetic compatibility (EMC) predictions where sub-wavelength features, such as thin wires or interconnects, are embedded in an otherwise large body (e.g. an equipment cabinet) and their mutual interaction and interference with other elements in the environment has to be accounted for in the design. The key issue for modelling this type of multi-scale system lies is the method of discretisation. It is readily apparent that the classical approach of employing a fine mesh for the whole problem results in excessive computational run time and memory consumption and hence from a practical point of view, the use of locally distorted and refined grids that characterise small elements in sufficient detail is often inevitable. To accommodate this requirement, the general-purpose simulation methods that are already in widespread use due to their flexibility and relative ease of use [1,2], have been evolved for use with hybrid meshes. It is also noted that EMC simulations are also challenging because of their inherent wide-band nature, operating well into the microwave frequency range, and consequently often encounter the need to model highly inhomogeneous, nonlinear and frequency dispersive materials [3] which further increases the model complexity for already sophisticated systems. In this contribution, the capabilities of various hybridisations will be demonstrated based on the TLM method, a well established time domain technique in electromagnetic field simulations.

MODAL EXPANSION TECHNIQUES FOR MULTI-SCALE SYSTEMS

As described in the previous section, the presence of thin wires in large systems requires an engineer to utilise alternatives to high resolution meshes in order to enable practical configurations to be simulated at reasonable computational cost, or in many cases even guarantee their feasibility. One solution is to set the global spatial step at typically one-tenth of the wavelength and embed into this relatively large mesh a special node enclosing the sub-wavelength features. Different techniques based on this concept have already been proposed, implementing various algorithms to interface the novel node into the numerical network [4]4,5]. However in many of these proposals the equivalent capacitances and inductances of the transmission lines have been obtained from the experience of the designer and an empirical factor established from experimental data or fine mesh solution, which undermines the generality of the method. Also the position of the wire has been restricted to the centre of computational cell and thus these approaches can not be applied to wire looms or bundles where several components need to be placed within a single node demanding that the near field coupling of the objects is adequately represented. Our MET method offers much more flexibility when dealing with thin wires and allows an arbitrary number of wires of arbitrary position and material type to be inserted within single coarse cell. In this technique a set of local solutions to Maxwell's equations is identified and mapped onto a discrete number of ports, 4 in 2D TLM and 12 for 3D TLM. The field around each wire can be expressed in many ways, but due to the round geometry of the object of interest, a cylindrical expansion is the natural choice. Using cylindrical harmonics, the electric and magnetic field for z-directed centrally placed wire of radius 'a' can be represented as:

$$E_z = \sum_{n=-\infty}^{\infty} B_n e^{jn\theta}\left[J_n(k_o r) - \frac{J_n(k_o a)}{N_n(k_o a)} J_n(k_o r) \right] \quad \text{and} \quad H_\theta = \frac{1}{j\omega\mu_o}\frac{\partial E_z}{\partial r} \tag{1}$$

where $k_o = \omega / c$ is the free space wavenumber.

Interfacing with quantities in the numerical mesh requires linking the analytical solution with the four ports of the TLM node. Therefore the infinite number of cylindrical modes is truncated and only the lowest order contributions shown in Fig. 1. will be included in a final mapping process. It is noted that the selected modes exhibit either an inductive $\underline{E} = L \cdot d\underline{H} / dt$ or a capacitive $\underline{H} = C \cdot d\underline{E} / dt$ behaviour.

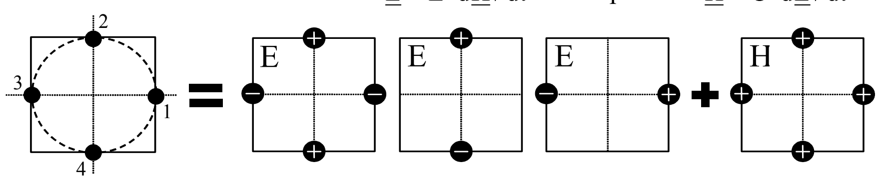

Fig. 1. Modal representation of cylindrical harmonics for 2D TLM node

In the analytical model the tangential electric and magnetic field components on the surface of the node are related through an admittance operator $\underline{H} = \hat{Y}\underline{E}$ and mapped onto the circuit quantities using $\underline{E} = (\underline{V}_i + \underline{V}_r)$ and $\underline{H} = y_L \underline{\underline{I}}(\underline{V}_i + \underline{V}_r)$, where V_i and V_r are the incident and reflected voltages travelling in the TLM mesh. Combining those two expressions leads to: $(y_L \underline{\underline{I}} + \hat{Y})\underline{V}_r = (y_L \underline{\underline{I}} - \hat{Y})\underline{V}_i$ where y_L is a scalar and $\underline{\underline{I}}$ is the identity matrix. Clearly, this equation describes the scattering relationship between the quantities $\underline{V}_i$ and $\underline{V}_r$ which define the electric and magnetic fields in a manner analogous to the forward and backwards travelling voltages on a transmission line with characteristic admittance y_L. The most straightforward method of solving this scattering problem for $\underline{V}_r$, given the incident voltage $\underline{V}_i$, is to consider the eigenvalue problem: $\hat{Y}\underline{U}_n = \gamma_n \underline{U}_n$, where $\underline{\underline{U}}$ is a square matrix whose columns are eigenvectors and γ_n are the eigenvalues of $\hat{Y}$. As a result, the incident and reflected voltages can be defined in terms of modal components $\underline{X}_i$ and $\underline{X}_r$ via the relationship $\underline{X}_{i,r} = \underline{\underline{U}}^H \underline{V}_{i,r}$, and finally the scattering process in modal space is recovered $X_{rn} = \frac{y_L - \gamma_n}{y_L + \gamma_n} X_{in}$ which can be identified as corresponding to a phase delay of $e^{-j\sigma_n}$ for mode 'n'. The overall mapping process is summarised as: (i) transform the real space port voltages into modal space (ii) perform the scattering of each modal amplitudes, (iii) convert the reflected modal amplitudes back to port voltages. In the presence of offset or multiple conductors within a single node the formula for the total field contains additional elements. The field scattered from each wire is represented as a sequence of local cylindrical harmonics and the global incident field on the node in terms of cylindrical harmonics centred on the node centre. The use of the Bessel summation theorem [6] is then applied to determine the net incident field on each wire, allowing identification of the local scattering coefficients and the wire currents, from which the global nodal scattering parameters can be determined.

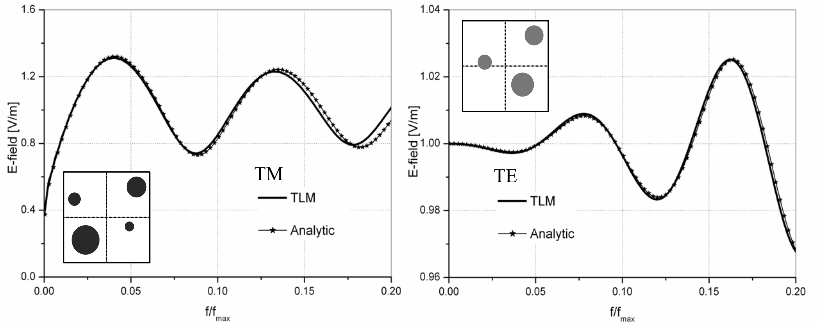

Fig. 2. Comparison of field representation for TM polarisation of 4 metal wires and TE for 3 dielectric rods between analytical and TLM solution

To demonstrate the capability and accuracy of the MET two different simulations have been carried out for two different nodes embedded into TLM grids: one containing metal conductors and the second dielectric rods. Fig. 2. shows the two types of special nodes and the total electric field excited by an incident plane wave observed 8 numerical nodes (node radius Δ = 0.05m) in front of the special nodes for both TM and TE polarisation. Four metal wires of radii $a_1 = 0.5\Delta$, $a_2 = 0.45\Delta$, $a_3 = 0.2\Delta$, $a_4 = 0.15\Delta$ were incorporated in a single cell for TM case where the electric field is polarised parallel to axes of wires. The positions of the wires within the cell are given by (0.6Δ;225°), (0.5Δ;45°), (0.5Δ;140°), (0.4Δ;320°) where (R,θ) denotes: R the distance between each wire and the cell centre and θ the angle between R and the positive vertical axis of the node. Secondly 3 dielectric rods of radii $a_1 = 0.35\Delta$, $a_2 = 0.45\Delta$, $a_3 = 0.3\Delta$, positions (0.7Δ;45°), (0.45Δ;315°), (0.5Δ;175°) and permittivity ε=10 are considered for the case of TE polarisation where the electric field is polarised perpendicularly to the axes of the rods. The comparison between analytical and numerical solution developed using the MET approach is exact even for the relatively weak field scattering that occurs for TE polarisation. Moreover, very good agreement is observed well above a frequency corresponding to 10 % of the maximum frequency, the limit below which the basic TLM method is generally regarded to be accurate [1].

DIGITAL FILTER INTERFACE TECHNIQUES

A further commonly encountered problem in EMC studies is the treatment of electrically small features with frequency dependent properties. Typical examples are thin sheets, e.g. ferrite absorbers, thin panels and perforation screens. The complexity of the models for such features is exacerbated when working in time domain, as not only must small scales be dealt with, but also an equivalent algorithm to describe the frequency-dependent parameters must be devised. Compared to other time-domain methods, such as FDTD, TLM is highly amenable to accommodating this frequency-dependent description as the sampling of all the components of the electric and magnetic fields are coincident in time and space. For empty space, the TLM "connection" process is a simple exchange between two neighbouring nodes of the reflected voltages at time step "k" with the incident voltages at the time step "k+1" [1]. In the presence of perforated thin screen placed in between adjacent cells the connection scheme must be generalised to account for transmission and reflection coefficients which are both frequency dependent. One can view these coefficients as transfer functions specified in the (analogue) frequency domain and then seek to develop equivalent (digital) functions in the discrete time domain which can then be embedded in time-domain field solver. This digital filter interface (DFI) has been presented in [7] and can be summarised as follows: (i) the scattering coefficients for the particular features are obtained from analytical solutions, experimental measurements or from refined simulations of the basic shape of the feature; (ii) the poles and zeros are extracted from a Padé approximation of the frequency domain response; (iii) the bilinear Z-transform is then applied to yield a procedure in the discrete time domain; (iv) this procedure is then embedded into the time-domain algorithm. The algorithm allows efficient description of complex geometrical features with frequency-dependent characteristics. It is also suitable for multi-scale environments as the size or periodicity of perforations is not restricted to the spatial resolution of a mesh. Detailed description of digital filter interface method can be found in [7] and the extension to modelling of bulk frequency-dependent materials in [8].

UNSTRUCTERED AND HYBRID MESHES

In the previous two sections, examples of embedding local solutions in a coarse mesh have been shown using the modal expansion technique or digital filter interfaces. In this section focus will be placed on distorted and unstructured meshes and their ability to deal with fine features in EMC problems.

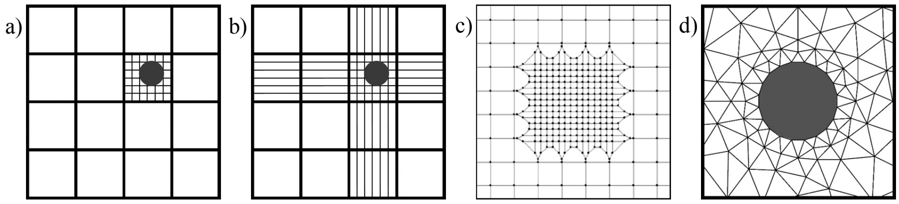

Fig. 3. Different configuration of distorted meshes

In Fig.3. different types of meshes have been illustrated. Configuration a) shows the case of a uniform coarse mesh with an embedded refined grid to describe a small object (wire) with sufficient detail. Unfortunately this standard multi-gridding method is susceptible to instabilities due to the loss of one-to-one connectivity between the fine and coarse meshes [9]. The graded mesh shown in Fig.3 b) circumvents this problem by extending the cells refinement to the boundary of the discretised space. Naturally the fine mesh is now not confined to the desired region and for many practical structures this may still exceed the available computational resources. The advantage of employing unstructured meshes is observed in Fig.3 c) where the irregular mesh is used as a "glue" to seamlessly interface two types of structured grids [10]. Another interesting example which is presented in Fig.3 d) is an application of irregular triangles to mesh to the whole problem with a grid density depending on the amount of details the particular areas needs to be described with [11].

CONCLUSIONS

As has been described above is difficult to devise a universal model that deals efficiently with the complexity of the systems, in particular those typical of EMC environments. Nevertheless a number of techniques have been shown that are able to speed up the simulations in a practical and elegant manner and that are both numerically stable and guarantee accuracy comparable to that obtained from fine meshes

[1] C. Christopoulos, "The transmission-Line Modeling Method: TLM", Piscataway, NJ: IEEE Press, 1995
[2] A. Taflove, S.C. Hugness, "Computational Electrodynamics: The Finite Difference Time-Domain Method, 2nd edition. Norwell, MA: Artech House, 2000
[3] C. Chrisopoulos, "Principles and techniques of Electromagnetic Compatibility", CRC Press, 1995
[4] A.J. Wlodarczyk, and D.P. Johns, "New wire interface for graded 3D TLM", Electron. Letters, vol. 28(2), pp. 728-729 , 1992
[5] P. Naylor, C. Christopoulos, "A new wire node for modelling thin wires in electromagnetic field problems solved by transmission line modelling", IEEE Trans, on MTT, vol. 38(3), pp. 328-33, 1990
[6] I.S. Gradshteyn, I.M. Ryzhik, "Table of integrals, series, and products", Boston Academic Press, 5th ed., 1994
[7] J. Paul, V. Podlozny and C. Christopoulos, "The Use of Digital Filtering Techniques for the Simulation of Fine Features in EMC Problems Solved in the Time Domain", IEEE Trans. on EMC, vol 45(2), pp 238 -244, 2003
[8] J. Paul, C. Christopoulos, D.W.P. Thomas, "Generalized material models in TLM – pat I: materials with frequency-dependent properties", IEEE Trans. on Ant. and Propag. vol. 47(10), pp 1528-1534, 1999
[9] C. Christopoulos, "Multi-Scale Modelling In Time-Domain Electromagnetic", Int. J. Electron. Commun. (AEÜ), vol. 57, pp 100-110, 2003.
[10] P. Sewell et all, "Multi-grid interface in computational electromagnetics,", IEE Electron. Letters, Vol. 40 (3), pp 162-163,2004
[11] P. Sewell et al, "Transmission Line Modeling Using unstructured Meshes", IEE Proc.-Sci. Meas. Technol., vol. 151, pp 445-448, 2004