Reciprocity, Energy, and Norms for Propagation on Nonuniform Multiconductor Transmission Lines

Carl E. Baum
University of New Mexico
Dept of Electrical and Computer Engineering
Albuquerque, NM 87131, USA
carl.e.baum@ieee.org

A previous paper [1], briefly summarized in [2], considers and generalizes various conservation theorems of electromagnetic theory. These are based on reciprocity, conservation of energy, and relativistic invariance. In recent years much research has been devoted to the understanding of the properties of nonuniform multiconductor Transmission lines (NMTLs) [3]. An interesting question concerns the general properties of NMTL solutions based on conservation theorems of electromagnetic theory. The present paper explores this question and obtains some interesting results.

First, some general properties of the product-integral solution of the supermatrizant (2N×2N) for an N-conductor (plus reference conductor) NMTL are reviewed. This gives the well-known results of unit determinant from the zero-trace property of the product integrand. This provides a general constraint on NMTL product integrals.

Reciprocity leads to many results. We first find what can be called *differential reciprocity* between any two voltage/current waves on and NMTL. Next, by terminating the NMTL in a passive reciprocal impedance matrix, we find the input impedance matrix which must also be passive and reciprocal. This is found in terms of the four submatrices (blocks) of the matrizant. Since the input impedance matrix must be symmetric this gives a constraint on the submatrices. By selecting various acceptable choices for the termination (such as short circuit and open circuit) this simplifies the results giving various symmetries to combinations of the matrix blocks.

Turning to energy conservation, for lossless NMTLs we first find a result with a striking similarity to the time-domain Poynting-vector theorem. Going into complex frequency ($s = \Omega + j\omega$) domain we find power conservation relations which simplify as one would expect for the lossless case. Next we embark on some general properties for the lossless case based on Foster's theorem. Applying this to lossless impedance matrices, all elements are odd functions of s. Returning to our reciprocity results we find in the lossless case that various combinations of the product-integral submatrices are odd functions of s in some cases, and even functions in others. This severely constrains the form the product integral can take. By an inspection of the product integral as the limit of a product we find a very general property. Specifically the diagonal blocks (N×N) are even functions of s, while the off-diagonal blocks are odd functions of s.

Finally we obtain some bounds on the product integral by use of associated matrix norms. Due to its analytic properties the 2-norm is used. This leads to a result involving an integral of the maximum eigenvalue of the product integrand plus its adjoint.

REFERENCES:

1. C.E. Baum, "Electromagnetic Reciprocity and Energy Theorems for Free Space Including Sources Generalized to Numerous Theorems, to Combined Fields, and to Complex Frequency Domain, " Mathematics Note 33, 1973
2. C.E. Baum and H.N. Kritikos, "Symmetry in Electromagnetics," ch. 1, pp. 1-90, in C.E. Baum and H.N Kritikos (eds.), *Electromagnetic Symmetry,* Taylor & Francis, 1995
3. C.E. Baum, J.B. Nitsch, and R.J. Sturm, "Analytical Solution for Uniform and Nonuniform Multiconductor Transmission Lines With Sources," pp. 433-464, in W. R. Stone (ed.), *Review of Radio Science 1993-1996*, Oxford U. Press, 1996