ANALYSIS OF OPTICAL FIELD PROPAGATION AND OPEN RESONATOR MODES IN THE CONTEXT OF VERTICAL CAVITY SURFACE EMITTING LASERS (VCSELS)

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ABSTRACT

Evolution of the VCSEL structure over the years has resulted in a device geometry that is difficult to analyse/model using traditional modal analysis. Present day (e.g., oxide aperture) VCSELs resemble cylindrical (open) optical resonators that have essentially diffracting optical field propagation determined by the oxide aperture and the equivalent locations of the finite sized end mirrors. This paper presents a quasi-analytic function expansion method based on the use of a set of Laguerre-Gauss (LG) functions, combined with the numerical collocation method, to achieve a versatile, effective and efficient scheme for computing the optical fields in inhomogeneous open (VCSEL) resonators.

INTRODUCTION

The Vertical Cavity Surface Emitting Lasers (VCSELs) have gone through several generations of evolution in the device structure. The main objectives of the developments have been to achieve single mode operation to reduce modal dispersion and to increase coupling efficiency to optical fibres. Further, high power applications have also spurred interest in achieving stable operation in higher order transverse modes.

Early generation VCSELs take the form of Circular Buried-Heterostructure (CBH) devices, [1], Fig.1(a). This structure is akin to a cylindrical (e.g., optical fibre) waveguide where modal analysis[2] is conveniently applicable to find the optical field modes in the VCSEL cavity. Recent developments have resulted in the design of oxide aperture VCSEL (Fig.1(b)) which give excellent single mode operation. However, the oxide aperture devices have no lateral optical guiding and hence the optical field distribution is best described by a diffraction-type analysis, [3]. Such present-day VCSELs are essentially open resonators with finite diameter mirrors.

This paper presents the results of investigations on the optical field characteristics of VCSEL resonators. For the present analysis the VCSEL is considered as a passive device, i.e., zero gain in active layer, Fig.1(b). For stable open resonators [4], e.g., planar mirrors with index-guided cavity, or, appropriately curved mirrors in a homogenous medium, the ‘steady state’ optical field profiles and resonant conditions can be obtained analytically. However, the oxide aperture VCSEL resembles an open resonator with finite radius planar mirror where analytic field solutions are not readily available. In the model presented here the weakly diffracting optical field can be analysed by expanding the total field expression in terms of a complete basis set of Laguerre-Gauss Functions [7],[8]. This leads to a simplified formulation which is applicable even to inhomogeneous media. When combined with the numerical collocation method [7],[8],[9], this expansion scheme proves to be effective and efficient in computing optical field propagation in inhomogeneous open resonators with finite size mirrors,and hence is applicable to the analysis of present-day VCSELs.

OPTICAL FIELD MODEL

Since oxide aperture VCSELs are essentially homogeneous open resonators, i.e., with no radial (r) index guiding, the ‘traditional’ modal analysis [2] for the optical field is not applicable. The optical field propagation in the homogeneous medium is generally described as a diffracting optical beam. Since the present model is being developed for the design of present generation VCSELs, it is necessary to account for lateral (radial, azimuthal) refractive index variations due to injected carriers and/or temperature gradients.

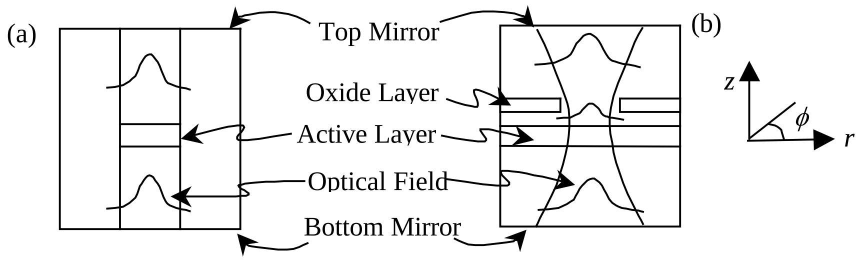

Fig. 1. Schematic of (a) CBH VCSEL and (b) Oxide aperture VCSEL

Harmonic time dependent optical (electromagnetic) field propagation in a medium with a refractive index profile $\eta(r,\phi,z)$, is described by the scalar wave equation[5]:

$$\nabla^2 E(r,\phi,z) + k_o^2 \eta^2 (r,\phi,z) E(r,\phi,z) = 0 \tag{1}$$

where k_0 is the free-space wave number. The total (electric) field E consists of components for forward(E^+) and backward(E^-) travelling optical field, with the former expressed as a basis set expansion:

$$E^+(r,\phi,z) = F^+(r,\phi,z)\, e^{-jpz} \tag{2}$$

where $$F^+(r,\phi,z) = \sum_{m=0}^{\infty}\sum_{k=0}^{\infty} A_k^+(z)\psi_k^m(r) e^{jm\phi} \tag{3}$$

in which p is an arbitrarily but judiciously chosen constant to account for the fast-varying longitudinal phase term while $F^+(r,\phi,z)$ represents a slowly varying envelope which satisfies the paraxial approximation[4]. $\psi_k^m(r)$ are an appropriate set of orthonormal basis functions. A similar expression to (2) can also be obtained for the reverse travelling wave.

From (1) and (2) and applying the paraxial approximation [4] yields

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial F}{\partial r} \right) + \frac{\partial^2 F}{\partial \phi^2} - 2jp\frac{\partial F}{\partial z} + \left(k_o^2 \eta^2 - p^2 \right) F = 0 \tag{4}$$

It is proposed that in the context of VCSEL modelling the following Laguerre-Gauss (LG) functions[10] are appropriate

$$\psi_k^m(r) = C_{mk} e^{-r/2}\, r^{m/2}\, L_k^m(r) \tag{5}$$

where L_k^m is the Laguerre Polynomial and C_{mk} is a normalisation constant. The LG functions are a discrete and complete set of orthonormal functions and are suitable for describing both the weakly diffracting fields in a homogeneous region and waveguide-like fields in laterally inhomogeneous media.

For this paper it is assumed that the optical field (E) is azimuthally invariant, i.e. $\partial/\partial\phi = 0 \ \ (= m)$. Considering the axial (z) dependence of the refractive index profile $\eta(r,\phi,z)$ as piecewise constant in each discrete segment q, $\eta(r,\phi,z) = \eta_q(r,\phi)$, where $z_q < z < z_{q+1}$. When applied to (4) and utilising the orthogonality properties [10] of LG functions, results in the following ODE for initial value problem (IVP):

$$\frac{dA_k^{(q)}(z)}{dz} = \frac{jp}{2} A_k^{(q)}(z) - \frac{j}{2p}\left[\sum_i^{\infty} A_i^{(q)}(z) \int_o^{\infty} \left(\frac{1}{4} - \frac{2i+1}{2r} + k_o^2 \eta^2(r) \right) \psi_k^0(r)\psi_i^0(r)\, dr \right] \tag{6}$$

The solution of (6) describes the field propagation in each segment q in the axial direction (z). This formulation can be further optimised by applying the Collocation Method [7],[8],[9] which utilises the Gaussian quadrature formula. This method, LG Collocation Method (LGCM), involves discretisation of the radial axis (r) and transformation of the numerical integrations into matrix multiplications, resulting in faster computation time.

OPEN RESONATOR

Only homogeneous media resonators are addressed in this paper. For ‘steady-state’ resonator modes to exist the stability criterion[4] must be satisfied - with reference to Fig.2 the stability criterion is given by

$$0 \le g_1 g_2 \le 1 \tag{7}$$

where $$g_1 = 1 - L/R_1 ,\quad g_2 = 1 - L/R_2 ,\quad \text{and}\quad L = z_2 - z_1 \tag{8}$$

Although the Fresnel-Kirchhoff Diffraction Integral[5] is applicable for describing optical field propagation in homogeneous resonators, the LGCM is used here to establish its validity as the first step to extend the procedure to analyse inhomogeneous resonators.

For geometrically stable resonators, a stable transverse field distribution at a reference location z_R in the cavity can be found by the Fox-Li iteration scheme [4],[6]. The stable field profile is defined as the eigenmode of the resonator structure when, apart from changes in amplitude and phase-shift, the transverse field profile remains unchanged at z_R after each round-trip.

Comparison of 3 Types of Stable Open Resonator

Confocal Mirror Resonator

The confocal mirror resonator (Fig.2) is geometrically stable and the eigenmode shape is given as the Gaussian beam function. The mode size is dependent on the mirror curvature where $R_c = R_1 = R_2$ and the mirrors located at the focal points of each other. In this case the resonant wavelength can be determined analytically[4].

Planar (Circular) Mirror Resonator – Infinite Diameter

The optical field in this structure (Fig.3) takes the form of standing (plane) wave solution with no transverse variation, i.e. infinite mode size. Hence the simple plane wave resonant condition can be applied where $2\pi\eta L/\lambda = m\pi$.

Planar (Circular) Mirror Resonator – Finite Diameter

The eigenmode shape for this structure (Fig.4) cannot be determined a priori [4]. Although the mode size is dependent on mirror diameter, there is no analytic expression for the field distribution. The resonant condition, which deviates from the case of plane wave, is given by the following expression:

$$4\pi\eta L/\lambda = 2m\pi + \Delta\varphi \tag{9}$$

where $\Delta\varphi$ is the round-trip propagation phase difference. The resonant wavelength and eigenmode profile can be obtained by the Fox-Li iteration scheme with optical field propagation described by LGCM. Referring to the parameters given in Fig.4, the value of $\Delta\varphi$ is found by examining the difference between round-trip phase front of the optical field in Fig.5, hence giving resonant wavelength by (9). Alternatively, the resonant wavelength can also be determined by the Fabry-Perot (F-P) Etalon [5] calculation. In this case, the peak of the ratio between the output intensity (I_o) and input intensity (I_{in}) gives the corresponding resonant wavelength (Fig.6), where

$$I_{in} = \int_0^{r_m} F_{in}(r)F_{in}^*(r)rdr \;,\; I_o = \int_0^{r_m} A_o(r)A_o^*(r)rdr \quad \text{and} \quad A_o(r) = \sum_n F_{on}(r) \tag{10}$$

The example of fundamental eigenmode shape for m = 3 is given in Fig.7. Meanwhile Table 1 gives a comparison of resonant wavelengths obtained by the above methods for m = 1 to 3.

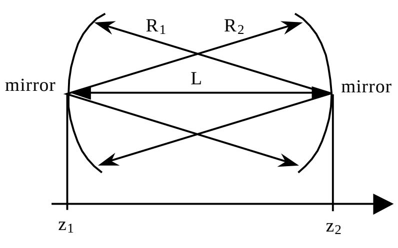

Fig.2. Confocal Mirror Resonator

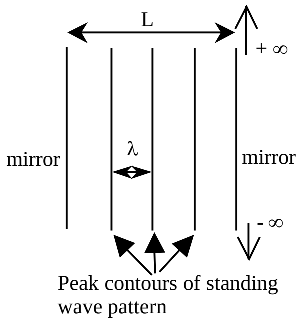

Fig.3. Planar (Circular) Mirror Resonator of infinite diameter

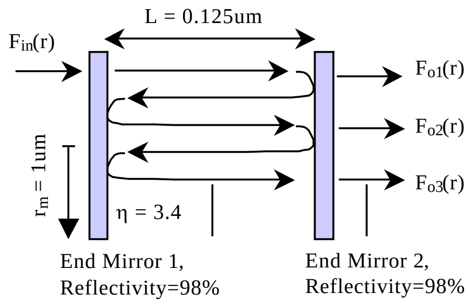

Fig.4. Planar (Circular) Mirror Resonator of finite diameter

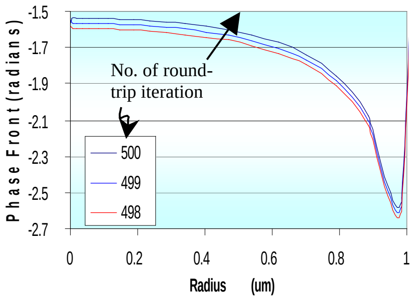

Fig.5. Round-trip phase fronts for Planar (Circular) Mirror Resonator

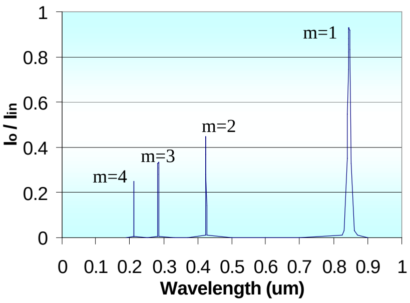

Fig.6. Resonant wavelengths obtained by Fabry-Perot Etalon

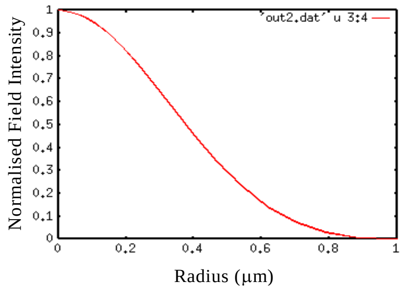

Fig.7. Fundamental eigenmode solution for structure in Fig.4

Table 1. Comparison of resonant wavelengths for (a) planar mirror resonator of infinite diameter, and those for finite mirror size obtained using (b) estimation by (9) and (c) Fabry-Perot Etalon.

	(a)Plane-wave	(b)Finite mirror size planar resonator		(c)F-P etalon
	λ(μm)	$\Delta\varphi$ (rad)	λ (μm)	λ (μm)
m=1	0.8500	0.0281	0.8462	0.8458
m=2	0.4250	0.0152	0.4245	0.4245
m=3	0.2833	0.00843	0.2832	0.28315

CONCLUSIONS

This paper demonstrates the advantages of analysing optical field characteristics in an open resonator with the Laguerre-Gauss Collocation Method (LGCM). This method is proposed the for the analysis of present-day VCSELs which have, in general, inhomogeneous resonators.

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