

# SURFACE PLASMON WAVES IN MULTILAYERS WITH LOSS / GAIN

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## INTRODUCTION

Surface Plasmon Waves (SPWs) have gained immense prominence in present day optical technology particularly because of their unique feature of being able to tightly bind the electromagnetic (optical) field at the interface between two dielectrics having relative permittivities,  $\epsilon$ , of opposite signs [1]. SPWs can also be supported by multi-layer media [1] and this paper presents the quantitative analysis of SPWs by applying the convenient yet rigorous technique of transverse resonance ( T-R ) condition to the equivalent transmission-line ( T-L ) representation of the multi-layer media. Experimental and theoretical results that quantify the operational characteristics to be expected of prism coupled SPW sensors that use semiconductor optical sources form the first part of the paper. The second part of the paper presents the specially interesting results from investigations of SPW propagation in multilayer media with one layer having optical gain, as a means of counteracting the typical, undesirably large loss associated with SPWs in conventional structures.

## BACKGROUND

At optical frequencies (wavelengths,  $\lambda_0$ ) a single interface formed by a dielectric and a metal with relative permittivities  $\epsilon_d$  and  $\epsilon_m = -|\epsilon_{mr}| - j|\epsilon_{mi}|$ , respectively, is capable of supporting SPWs which are transversely ( x ) confined, longitudinally propagating (  $\exp[-j\beta z]$  ), TM-polarised electromagnetic (optical) waves. The dispersion relation satisfied by the SPW (complex) propagation constant [2],

$$\beta = k_o \sqrt{\frac{\epsilon_d \epsilon_m}{\epsilon_d + \epsilon_m}} \quad (1)$$

For ideal metal (  $\epsilon_{mi} = 0$  ) propagating solutions exist only if  $\epsilon_d < |\epsilon_{mr}|$ , where  $k_o = 2\pi / \lambda_0$ . At optical frequencies the most commonly used metals for SPWs typically have  $|\epsilon_{mi}| \ll |\epsilon_{mr}|$ . In general, the lossy nature of the metal yields a complex  $\beta = \beta_r - j\beta_i$ ; the longitudinal decay rate,  $\beta_i$ , limits the SPW propagation distance to  $L_z = 0.5\beta_i^{-1}$  [2]. Increase in propagation length  $L_z$  can be achieved by reducing  $\epsilon_d$  or by operating at longer wavelengths where metal losses are smaller.

Multiple ( $> 2$ ) layer media are used for most practical SPW operations and the T-L / T-R provides a very convenient method to quantitatively analyse the modal characteristics. The 3-layer structure, for example, permits two distinct configurations, namely, metal-dielectric-metal (M-D-M) and dielectric-metal-dielectric (D-M-D) [3]. Both configurations support, in general, symmetric and antisymmetric SPW modes.

For the symmetric D-M-D configuration the symmetric and anti-symmetric SPW modes have no cut-off, with the antisymmetric being the fundamental and experiencing the highest losses. Although a single interface (2-layer) structure sustains SPWs only when  $|\epsilon_{mr}| > \epsilon_d$ , the finite thickness of the central (M) layer permits SPWs to be excited even for  $|\epsilon_{mr}| \leq \epsilon_d$  and in this special situation only two, both symmetric, SPWs, each with an upper cut-off frequency, are supported; in this case, interestingly, the fundamental symmetric mode gives a Poynting vector which is oppositely directed to the phase velocity for that mode. For the general and the special structures a comparison between different permittivity combinations shows that higher field intensities at the two interfaces are generated when  $\epsilon_d \rightarrow |\epsilon_{mr}|$ , [3].

By comparison, the M-D-M structure in general supports two categories of electromagnetic waves - the 'conventional' (parallel metal plate waveguide) modes [4], and the SPWs. Referring to the SPW modes, if  $|\epsilon_{mr}| > \epsilon_d$ , the symmetric SPW is the fundamental mode whereas the antisymmetric has a low cut-off and high modal losses. However, only a

single (the antisymmetric) SPW mode is supported when  $|\mathcal{E}_{mr}| \leq \mathcal{E}_d$ ; this mode has an upper cut-off frequency and a Poynting vector which is oppositely directed to the phase velocity [3].

The following particularly interesting features occur also with multilayer media but are far more readily visualized with reference to the two-layer (single interface), dielectric / ideal-metal structure. Note that when  $\mathcal{E}_d \approx |\mathcal{E}_{mr}|$ ,  $\beta_r \rightarrow \infty$  which corresponds to very short effective modal wavelength,  $\lambda_{eff}$ , and a very short transverse field decay distance,  $k_x^{-1}$ , for the SPW. This indicates the potential to design for sub-wavelength resolution probes and nanoscale optical integrated circuits [5], [1]. The realization of these features will be greatly enhanced if the longitudinal attenuation rate, due to (non-ideal) metal losses, can be reduced. Inspection of the expression for  $\beta$ , (1), suggests that reducing  $\mathcal{E}_d$  or operating at longer wavelengths (smaller metal losses) reduces SPW attenuation. An alternative method would be to introduce optical gain ( $g > 0$ ) in one of the layers of the structure. The latter part of this paper presents computed results of SPWs in structures with a layer having  $g > 0$  and discusses the corresponding properties.

### TRANSMISSION LINE REPRESENTATION

The use of a prism to phase velocity match an incident plane wave to couple to an SPW is very often employed for a type of well known optical sensor [1], Fig.1a where the core layer permittivity is equal to that of the substrate  $\epsilon_a = \epsilon_s$  and both are real. Then, at a particular incident angle,  $\theta = \theta_p$ , dependent on the operating wavelength and the parameters of the structure, Fig.1a, the SPW is optimally excited and the reflected optical intensity,  $I_r$ , attains a minimum which shows as a ‘dip’, Fig.1c. Considering unattenuated z-directed propagation constant, that is,  $\beta = \beta_r = k_o n_p \sin(\theta)$  where  $n_p$  is the (purely real) refractive index of the prism, the transversely directed impedances, [4], for the various layers, prism, metal, core, substrate, can be evaluated, Fig.1b. Thus the four layer structure shown in Fig.1a can be represented as a 4-section T-L system which can be reduced to a single T-L terminated by an impedance  $Z_L$ , [4]. The reflectivity is given by [4]

$$I_r / I_i = \left| \frac{(Z_L - Z_p)}{(Z_L + Z_p)} \right|^2 \quad (2)$$

which attains a minimum at  $Z_L = Z_p$ , representing transverse resonance [6], and occurs at  $\theta = \theta_p$ . Equation (2) demonstrates that the excitation of the SPW implies transverse impedance matching (resonance) and corresponds to maximum power transfer from the plane wave to the structure.

### EFFECT OF SPECTRALLY BROAD & DIVERGENT BEAM SOURCE

The analysis so far of the optical sensor system, Fig.1a, has assumed a monochromatic source with a parallel beam which is very nearly the case with gas lasers. However, for a compact system it would be desirable to use semiconductor LASERS and LEDs where both have divergent beams and the LED is not monochromatic. The following presents an analysis and experimental results for the system when excited by sources (LEDs and LASERS) having a spectrally broad and divergent output beam.

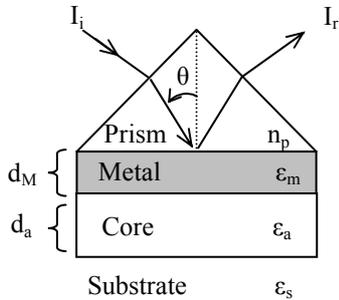


Fig.1a.

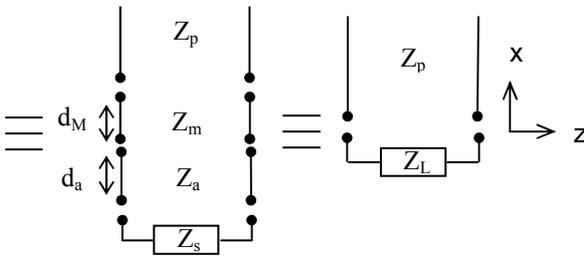


Fig.1b

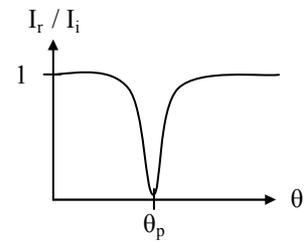


Fig.1c

For an optical source with beam divergence angle  $\delta\theta$ , spectral width  $\delta\lambda$  and total intensity  $I_s$  incident on a SPW prism coupled sensor system, Fig.1a, at central angle  $\theta_i$ , the intensity of the modified reflected signal  $\hat{I}_r$  can be represented as a convolution between  $I_s$  and  $I_r$  of an ideal optical source ( $\delta\theta = 0$  and  $\delta\lambda = 0$ ), as follows

$$\hat{I}_r(\theta_i, \lambda_o) = \int_{\lambda_-}^{\lambda_+} \int_{\theta_-}^{\theta_+} I_s(\theta, \lambda) I_r(\theta_i - \theta, \lambda) d\theta d\lambda \quad (3)$$

where  $\theta_+ = \theta_i + \delta\theta$ ,  $\theta_- = \theta_i - \delta\theta$ ,  $\lambda_+ = \lambda_o + \delta\lambda$  and  $\lambda_- = \lambda_o - \delta\lambda$ .

Comparisons between theoretical and experimental results of the effect for a commercial LASER pointer ( $\lambda_o = 630nm$ ,  $\delta\lambda \approx 1nm$ ), observed with the coupler of Fig.1a assuming  $\epsilon_a = \epsilon_s = 1$ ,  $n_p = 1.5$  and gold metal film with  $d_M = 47nm$ , are presented in Fig.3a. Note that the theoretical results include the effects of both  $\delta\lambda$  and  $\delta\theta$  as specified for the sources.

The results show that, as intuitively expected, with increasing beam divergence the depth reduces and the width widens of the 'dip' in the reflected signal. This worsening of the resolution sensitivity of the system is better seen in Fig.3b which corresponds to a typical experiment where the sensor is used to detect a change of 0.1% in the permittivity,  $\epsilon_s$ , of the bottom (substrate) layer. Clearly, with a larger incident beam divergence the change in the position of the 'dip' is more difficult to resolve. Hence these analyses supported by experimental results provide the means for specifying the optical source properties needed to design for a required system sensitivity.

## OPTICAL GAIN

As indicated earlier in the paper the reduction in the longitudinal decay rate,  $\beta_i$ , (i.e., increase  $L_z$ ) is very desirable to significantly enhance the practicality of using SPWs for an increasing number of applications. The analyses and results presented here investigates the possibility of achieving that goal by counteracting the losses inherent in the metal by introducing a layer with optical gain ( $g > 0$ ) [7]. It is suggested that an optically active layer such as is commonly used for semiconductor laser diodes would be suitable and computed results are presented for the defined structure. It is found that when a suitable layer with  $g > 0$  is introduced in the prism coupled sensor, Fig.1a, the reflected signal is, in fact, amplified compared to the input at the optimum excitation incidence angle,  $\theta_p$ . The computations for SPW modes for structures with a layer having  $g > 0$  shows that an increase in several orders in the value for  $L_z$  can be achieved and, in addition, the field is more tightly confined to the interface, i.e.,  $k_x^{-1} \rightarrow 0$ .

With reference to the prism coupled sensor, Fig.1a it is assumed that the substrate layer is a semiconductor with real permittivity,  $\epsilon_s$ , while the core is the same semiconductor but with  $g > 0$  so that the corresponding relative permittivity is complex,  $\epsilon_{ac} = \epsilon_{ar} + j\epsilon_{ai} = \epsilon_{ar} + j(\sqrt{\epsilon_{ar}}/k_o)g$ , with  $\epsilon_{ar} \gg \epsilon_{ai} > 0$ . With the following numerical

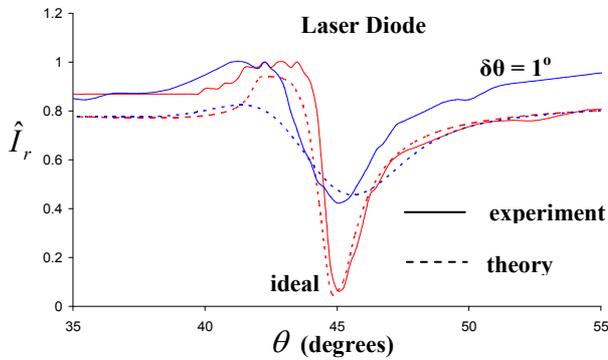


Fig.3a.

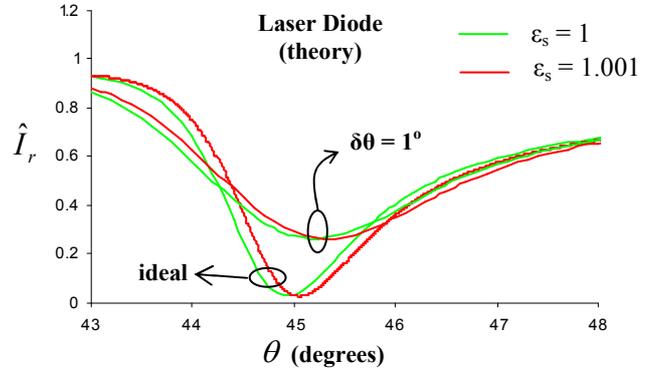


Fig.3b.

values for the parameters:  $d_M = 37\text{nm}$ ,  $d_a = 25\text{nm}$ ,  $n_p = 4.5$ ,  $\epsilon_m = -49.9 - j3.84$ ,  $g = 10^4 \text{ cm}^{-1}$ ,  $\epsilon_{ar} = 12.25$ ,  $\epsilon_s = 12.25$ , the computed results reveal that the reflected signal increases by two orders of magnitude, and because of amplification is, in fact, larger than the input intensity, Fig.4a. Further increase in the value of  $g$  to  $1.65 \times 10^4 \text{ cm}^{-1}$  leads to extremely large amplification of the reflected signal (five orders of magnitude increase).

The effect on SPW mode propagation characteristics due to the introduction of optical gain in one of the layers is investigated using essentially a 2-layer (single interface) structure although, strictly speaking, there are 3-layers, Fig.4b. The lowest substrate layer is taken as a passive semiconductor ( $g = 0$ ) with real permittivity  $\epsilon_s = 12.25$ . The thin ( $d_a = 25\text{nm}$ ) semiconductor active layer has a complex permittivity,  $\epsilon_{ac} = \epsilon_s + j\epsilon_{ai}$ , with  $\epsilon_{ai} = \left(\sqrt{\epsilon_s} / k_o\right)g$  at  $\lambda_o = 1\mu\text{m}$  and the metal is gold.

The computed results yield  $L_Z = 1.2\mu\text{m}$  when  $g = 0$ , increasing to  $10 \mu\text{m}$  when  $g = 7.5 \times 10^3 \text{ cm}^{-1}$  but very rapidly increasing thereafter, e.g.,  $L_Z = 200 \mu\text{m}$  at  $g = 8.5 \times 10^3 \text{ cm}^{-1}$ . This illustrates that optically active layers with thickness similar to that for typical Quantum Well (QW) semiconductor material and optical gain,  $g$ , comparable to that used for high-power edge-emitting, superluminescent LEDs, can lead to enormous improvements in SPW characteristics.

## CONCLUSION

A brief review of some of the basic but important properties of SPWs in multilayer structures has been outlined. The Transmission Line representation and Transverse Resonance analysis in modelling such structures has been described and demonstrated. It is recognised that T-R corresponds to impedance matching and maximum power transfer. The reduction in prism coupled sensor sensitivity due to spectrally broad and diverging beam sources (such as semiconductor LASERS and LEDs) has been theoretically and experimentally investigated. Finally, with an eye to the future, it is proposed and theoretically investigated that the introduction of a thin layer of semiconductor with optical gain (as is typically used in semiconductor LASERS) can counteract the detrimental effects of loss associated with the metal layer and greatly enhance the characteristics of SPWs. If such conditions can be realised then very much improved prism coupled sensor response can be achieved while optical probes based on Raman scattering will have greatly increased sensitivity.

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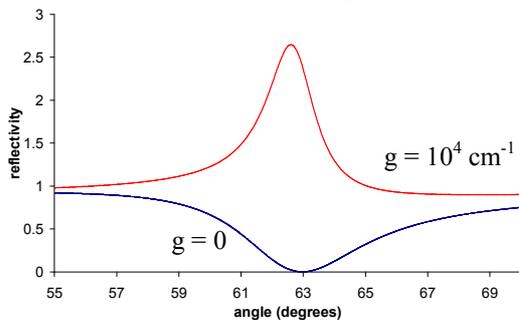


Fig.4a

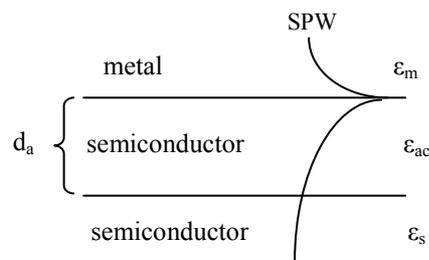


Fig.4b