A Novel Analysis and Design Recipe of Optimized Star Couplers for Arrayed-Waveguide Grating

G. Hugh Song[1]* and Mahn Yong Park[2]

[1] *Dept. of Information & Communications, Gwangju Institute of Science & Technology, Gwangju 500-712, KOREA*
[2] *Electronics and Telecommunications Research Institute, Daejeon 305-700, KOREA*

***Abstract*—A new functional analysis of the optimized star coupler utilizing directional coupling in the input array is given providing a physical insight into the unique diffraction properties of the device. The result of this analysis has unique features which can be clearly distinguished from the analysis based on an existing theory. From the new analysis, a novel design recipe which can be applied to star couplers and arrayed waveguide gratings, which are of prime importance as a WDM optical network component, is given.**

I. INTRODUCTION

A planar-optic star coupler is one of the basic elements of the more integrated structure such as an arrayed waveguide grating (AWG) which is one of the key components for wavelength division multiplexing (WDM) optical networks. In fact, in its own sake as either an $N \times N$ coupler or a $1 \times N$ power splitter, it may well surpass any other basic optical element in number of deployments in future optical networks. In spite of its importance, we believe that its optimization has not been fully understood for so long. This article addresses this issue and will suggest a different viewpoint based on a solid mathematical ground with a straight-forward design recipe.

There exist two schemes for ultimate optimization of such an $N \times N$ star coupler with uniform power splitting. The first scheme was suggested by C. Dragone [1,2] with an argument of adiabatic mode conversion of Bloch modes into plane waves being collected within the first Brillouin-zone in the Fraunhofer-diffraction pattern [3]. The second scheme utilizes a special diffraction property of a field pattern at the end of directionally-coupled waveguides in the input array. The unique Fresnel-diffraction pattern in this case was first reported with a beam-propagation (BPM) simulation by K. Okamoto *et al*. [4]. We would like to mention that the two optimization methods are *different in their design principles*, although the design drafts on paper may appear similar. We have found from literature [6,7] that this difference has not been very well appreciated.

II. BESSEL-FUNCTION FORMULA AND THE DESIGN RECIPE

Fig. 1 illustrates the basic design of a star coupler utilizing directional coupling in the input array. It was recognized by S. Somekh *et al*. [8] that, with a sufficient number of parallel waveguides in close proximity, the field profile at the end of the array in directional coupling, if a single waveguide at the center ($m = 0$) is excited initially at $z = -L_\mathrm{c} < 0$, is given by

$$V_m \simeq \mathrm{i}^m \, J_m(2\,\varkappa), \tag{1}$$

where $J_m(\cdot)$ is the mth-order Bessel function of the first kind, and $\varkappa \equiv K\,L_\mathrm{c}$ is the integration of the conventional coupling coefficient K over the effective coupling length L_c. Under a reasonable approximation, even when the waveguides converge as in Fig. 1, the field at line A in Fig. 1, defined by $z = 0$, has been verified by a wide-angle BPM analysis to still maintain the above field amplitude expressed by (1) if one exercises a reasonable care in the design of the array. Thus, to a good approximation, the field at $z = 0$ can be represented by a series of Gaussian beams weighted by V_m with the common waist of w_0 as

$$\phi(y,\,0) \simeq \sum_{m=-\infty}^{\infty} \mathrm{i}^m \, J_m(2\,\varkappa)\, A \exp\!\left(-\,\frac{[y - m\,h]^2}{w_0^2}\right), \tag{2}$$

where $A \equiv \sqrt[4]{2/\pi\,w_0^2}$ is a normalization constant letting $\int_{-\infty}^{\infty} |\phi(y,\,0)|^2 \,\mathrm{d}y = 1$. Here, h is the center-to-center distance between the neighboring waveguides at the waveguide opening region. In (1)–(2), by choosing $-\infty < m < \infty$, we have assumed that the field coupled to neighboring waveguides does not reach either fringe waveguide of the array, which is valid by designing a sufficient number of dummy waveguides in the two sides. Then, as we move along the propagating field with increasing z, the diffraction pattern slowly evolves from a two-pronged pattern to a rectangular pattern and then to a Gaussian pattern as illustrated in Fig. 2.

*hughsong@gist.ac.kr, myparkgo@etri.re.kr. This work was supported in part by IITA through the CHOAN-ITRC Program.

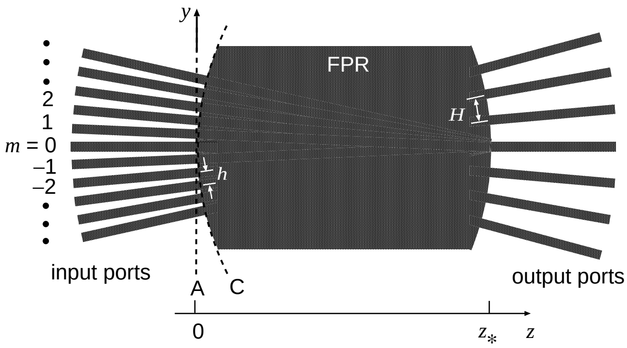

Fig. 1. Structure of an optimized star coupler utilizing directional coupling in the input array. FPR: free-propagation region.

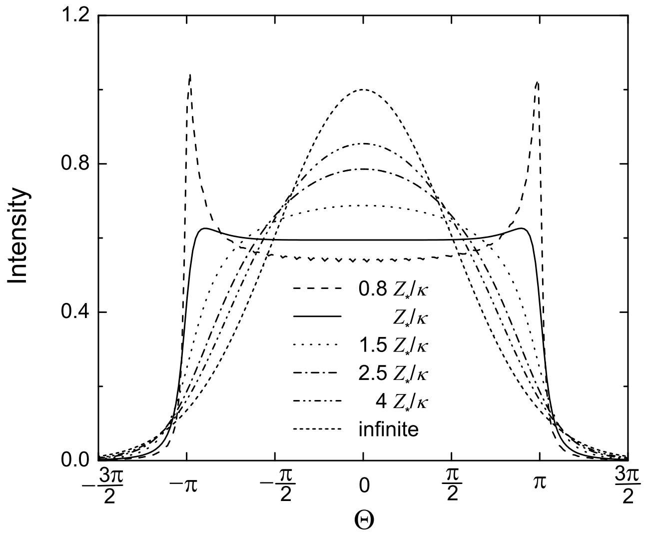

Fig. 2. Diffraction patterns with respect to the renormalized diffraction angle $\varTheta \simeq \bar{k}\,h\,y/z$ at several distances of $Z/\varkappa$ with $\varTheta_{\rm w} = \pi$ fixed. The renormalized distance is denoted by the optimum distance $Z_\star$ (given in (6)) times some number in order to emphasize the sole dependence of the diffraction pattern on $Z/\varkappa$, explained by (4).

In the Fresnel-diffraction range, the pattern evolves according to

$$|\phi(y,\,z)|^2 \cong |q(0)/q(z)|\, A^2 \exp\!\big(-\,2\,\varTheta^2/\varTheta_{\rm w}^2(Z)\big)\, |\breve{\phi}(\varTheta,\,Z)|^2, \tag{3}$$

where $q(z) \equiv z - {\rm i}\,z_0$ with $z_0 \equiv \bar{k}\,w_0^2/2$ of the Rayleigh range of the Gaussian beam. After a somewhat lengthy mathematical derivation, we have reduced the last factor to

$$|\breve{\phi}(\varTheta,\,Z)|^2 \simeq \sum_{p\,=\,-\infty}^{\infty} {\rm e}^{{\rm i}\,p\,\varTheta} \left\{ \left[1 - p^2\,\frac{\varTheta_{\rm w}^2}{8\,Z^2}\right] J_p\Big(-[p\,\varTheta_{\rm w}^2 + {\rm i}\,4\,\varTheta]\,\frac{\varkappa}{Z}\Big) + 2\,\varTheta_{\rm w}^2 \left[\frac{\varkappa}{Z}\right]^2 J_p''\Big(-[p\,\varTheta_{\rm w}^2 + {\rm i}\,4\,\varTheta]\,\frac{\varkappa}{Z}\Big) \right\} \tag{4}$$

by retaining terms of orders up to Z^{-2}. Here, we have used a set of renormalized variables for convenience such as

$$Z \equiv \frac{z}{z_0}, \qquad \varTheta \equiv \frac{\bar{k}\,h\,y}{z\,[1+Z^{-2}]}, \qquad \varTheta_{\rm w} \equiv \frac{2\,h/w_0}{\sqrt{1+Z^{-2}}}. \tag{5}$$

The mathematical derivation of (4) relies on the addition theorem for Bessel functions and two-term Taylor-series expansions based on ${\rm e}^x \simeq 1 + x$ for $|x| \ll 1$ [9]. Although (4) is not a closed-form expression, its form and a few plots in Fig. 2 let us understand the unique diffraction property of the field as it propagates through the FPR. That is, the diffraction pattern basically depends only on two parameters $\varTheta_{\rm w}$ and $Z/\varkappa$ in the range of interest $Z \gg 1$,

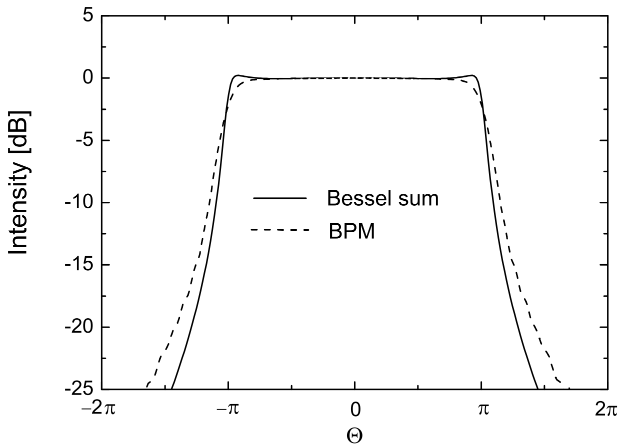

Fig. 3. Plot for the result of the paraxial equation (solid curve) and its approximation by (3)–(4) (solid curve, overlapped completely within the scale of the plot) for an idealized array of parallel input waveguides in comparison with a beam propagation simulation (dashed curve) with a realistic array of the converging input waveguides. The distance and the coupling strength have been chosen per the recipe, i.e., $z = z_\star$ and $\Theta_{\rm w} \simeq \pi$.

as the term $n^2\ \Theta_{\rm w}^2/Z^2$ is found negligible. Moreover, it adequately provides a foundation on which we can find a closed-form expression for $\left[\partial^2|\phi|^2/\partial\Theta^2\right]_{\Theta=0}$, which is essential for establishing the following design recipe of the optimized star coupler:

1) We find the parameter $Z/\varkappa = Z_\star/\varkappa$ at which the curvature of the diffraction pattern becomes flat at $y = 0$, i.e., $\Theta = 0$, which results in

$$Z_\star/\varkappa \equiv \left[\Theta_{\rm w}^2/4 - 1\right]\,\Theta_{\rm w}^2. \tag{6}$$

2) We then adjust $\varkappa$ by matching the Gaussian waist with the zone-boundary value of the first-Brillouin zone which comes from the periodic array of input waveguides:

$$\Theta_{\rm w} = \pi. \tag{7}$$

3) We let $z_\star$ and w_0 be matched with N, i.e., the number of fan-outs and the basic waveguide structure allowed from the material system of the platform.

Based on the above mathematical findings, a practical design procedure should be as follows: From the basic structure of the platform, w_0 is almost given. According to (5) and (7), h, i.e., the center-to-center distance of waveguides in the input ports is determined by $h \simeq \pi\, w_0/2$. Because $w(z_\star) \equiv w_0\sqrt{1 + z_\star^2/z_0^2} \simeq w_0\, z_\star/z_0$ should encompass all the output-port waveguides, the number of fan-outs, N, times the distance between waveguides in the output array, H, will determine overall distance $z_\star$. Finally, the amount of the needed coupling strength $\varkappa$ is determined accordingly:

$$z_\star \gtrsim \frac{N\,H\,\bar{k}\,w_0}{4}, \qquad \varkappa = \frac{4\,Z_\star}{\Theta_{\rm w}^2\left[\Theta_{\rm w}^2 - 4\right]} = \frac{8\,z_\star}{\bar{k}\,w_0^2\,\pi^2\left[\pi^2 - 4\right]}. \tag{8}$$

The latter quantity should carefully be matched with the basic waveguide design of the platform. One may employ the design of tapered waveguides to achieve the necessary coupling strength. Either positive or negative tapering should work as long as the diffraction field from a single waveguide is not too different from the Gaussian profile. A BPM analysis may be needed to check the validity of tapering and the result as illustrated in Fig. 3. A wide-angle two-dimensional simulation on one of the most typical star coupler structure whose design is based on the above-proposed design recipe demonstrates the validity of the analytic formula of (4).

III. DISCUSSION AND CONCLUSION

It should be mentioned that, in the optimization scheme of C. Dragone, the field at the end of mode conversion into the plane-wave modes, as well as at the end of directional coupling in the other optimization scheme of K. Okamoto *et al.*, the resulting field is aligned not along the curve (curve C in Fig. 1), which defines the FPR-array interface, but very much along a straight line (line A in Fig. 1) defined by $z = 0$, as long as the structure of directional coupling is not overly spoiled by tapering or by the arrangement of converging input waveguides with an excessive angle spread. This forces us to consider the pattern in the Fresnel diffraction range rather than the far-field pattern. Since the theory of the first-Brillouin zone relies on the Fourier transformation which explicitly gives the far-field pattern, the accuracy of the Brillouin-zone analysis applied to the Fresnel range becomes questionable. For this reason, we believe that, in the optimized star coupler of [2], designed under the first scheme that aimed uniform power splitting, what actually happened could have been better explained by directional coupling enhanced by the positive taper in the input array, rather than by a near-ideal adiabatic transformation of mode fields from excitation of a single waveguide, i.e., uniformly-excited entire Bloch modes, to plane-wave modes in the first-Brillouin zone only. Such a design of spoiled waveguides, i.e., zero-gap in the end, with a sophisticated taper scheme in [1], which was considered as the ultimate optimization in the first scheme under the principle of mode conversion from Bloch modes to plane-wave modes, can be redesigned by a different principle with the recipe-provided parameters without taper. The new design will thus make the diffraction pattern as much rectangle-shaped as possible *in the Fresnel range*, in addition to reducing the scattering-induced insertion loss from the abrupt refractive-index interfaces from the known effect [10, 11] of the spoiled-waveguide array structure for optimized star couplers and AWG's.

Superiority of the directional-coupling scheme with design parameters optimally chosen according to the recipe presented here is best demonstrated by Fig. 3 in comparison with the simulation plots of some recent works by J. C. Chen *et al*. [10, 12]. In their simulated results, it appears that the effect of directional coupling showed up with less efficacy in comparison to ours that is represented by the dashed curve in Fig. 3. Comparing the BPM plot appeared in Fig. 2 in [12], achieved without the help from such an itemized recipe procedure presented here, we believe that our new procedure actually provides a way of producing a nearly-optimum simulation result, as the one shown in Fig. 3, only after a few number of trials. As is evident by now, the recipe can directly be applied to the optimized design of the second star in an AWG configuration for uniform WDM channel responses. Indeed, J. C. Chen *et al.*'s introduction of auxiliary guides [12] can be understood as a variation to the directional-coupling scheme.

In conclusion, we have found a new mathematical expression for the diffraction pattern which should be obtained more or less in general in optimized planar-optic star couplers with directional coupling in the input array. Those new expressions have made us understand the unique diffraction property in the Fresnel range in concrete terms with a few design parameters. Out of those expressions, we have found a novel optimization recipe, which gives the desired properties of nearly-ideal uniform power splitting in the star coupler and of WDM channel uniformity in the AWG.

G. H. S. would like to appreciate all the inspirational discussions with the late Prof. Hermann A. Haus of MIT on subjects including the one in this paper.

References

[1] C. Dragone, "Efficiency of a periodic array with a nearly ideal element pattern," *IEEE Photon. Tech. Lett.*, vol. 1, no. 8, pp. 238–240, 1989.

[2] C. Dragone, C. H. Henry, I. P. Kaminow, and R. C. Kistler, "Efficient multichannel integrated optics star coupler on silicon," *IEEE Photon. Technol. Lett.*, vol. 1, pp. 241–243, 1989.

[3] C. Dragone, "Optimum design of a planar array of tapered waveguides," *J. Opt. Soc. Am. A*, vol. 7, no. 11, pp. 2081–2093, 1990.

[4] K. Okamoto, H. Takahashi, S. Suzuki, A. Sugita, and Y. Ohmori, "Design and fabrication of integrated-optic 8×8 star coupler," *Electron. Lett.*, vol. 27, no. 9, pp. 774–775, 1991.

[5] M. Y. Park, G. H. Song, K. Hwang, H. J. Lee, and K.-B. Chung, "Design of waveguide-grating routers with minimal insertion-loss over all channels based on coupling between adjacent waveguides," *OSA* Trends in Optics and Photonics (TOPS) Vol. 56, Conference on Lasers and Electrooptics (CLEO 2001), *Technical Digest*, vol. 56, pp. 127–128, 2001.

[6] Y. P. Li and C. H. Henry, "Silicon optical bench waveguide technology," *Optical Fiber Telecommunications*, vol. IIIB, I. P. Kaminow and T. L. Koch, Eds., Academic Press, San Diego, USA, pp. 351–353, 1997.

[7] C. R. Doerr, "Chap. 9. Planar lightwave devices for WDM," *Optical Fiber Telecommunications*, vol. IVA, I. P. Kaminow and T. Li, Eds., Academic Press, San Diego, USA, 2002.

[8] S. Somekh, E. Garmire, A. Yariv, H. L. Garvin, and R. G. Hunsperger, "Channel optical waveguide directional couplers," *Appl. Phys. Lett.*, vol. 22, no. 5, pp. 46–47, 1973.

[9] G. H. Song and M. Y. Park, "Bessel-function analysis of the optimized star-coupler for uniform power splitting," *J. Opt. Soc. Amer.*, vol. 21, no. 8, pp. 1529–1544, 2004.

[10] J. C. Chen and C. Dragone, "A proposed design for ultralow-loss waveguide grating routers," *IEEE Photon. Technol. Lett.*, vol. 10, no. 3, pp. 379–381, 1998.

[11] A. Sugita, A. Kaneko, K. Okamoto, M. Ito, M. Himeno, and Y. Ohmori, "Very low insertion-loss arrayed-waveguide grating with vertically tapered waveguides," *IEEE Photon. Technol. Lett.*, vol. 12, no. 9, pp. 1180–1182, 2000.

[12] J. C. Chen and C. Dragone, "Waveguide grating routers with greater channel uniformity," *Electron. Lett.*, vol. 33, pp. 1951–1952, 1997.