NON-LINEAR EFFECTS IN NEGATIVE MAGNETIC META-MATERIALS

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ABSTRACT: It is well-known that sub-wavelength sized metallic structured materials such as the Split Ring Resonator (SRR) can have an effective non-zero magnetic susceptibility. The magnetic permeability of such material can also become negative in the vicinity of the internal LC resonance of the SRR structure due to the over-screened under-damped response of the system. When imbedded in a Kerr non-linear medium, such structures show interesting non-linear behaviour such as bistability etc. The non-linear material changes the capacitance of the structure, and hence the effective medium permeability can be switched from negative to positive depending on the intensity of the electromagnetic wave.

In recent years, the effective medium properties of sub-wavelength sized metallic composite materials have been the subject of intense investigation. When the size of the structures are much smaller than the wavelength of electromagnetic radiation, typically at least smaller by one or two orders of magnitude, then the structured composite medium appears effectively homogeneous to the radiation. A set of effective response functions, the effective di-electric permittivity ε and the effective magnetic permeability μ can then describe the electromagnetic response of such materials. Note that these effective media parameters are different from the permittivity and permeability of the underlying constituent materials of the composite.

Structures comprising of arrays of the Split Ring Resonator (SRR) have been of particular interest [1,2]. These SRR are resonant structures which behave as an artificial magnetic atom due to a over-screened response to an applied magnetic field. When the level of damping is low, driven by the back emf, the structure exhibits an anti-phased response in a frequency band above the resonant frequency and makes it possible for the structure to have a negative effective magnetic permeability. It is now known that the SRR is one of a class of resonant structures that exhibit such a resonant response which includes the Omega-particles etc. The common feature of all these structures is that all of them have a closed geometry around which the magnetic fields induce currents to flow and thus have a finite inductance, and there is a certain amount of capacitance incorporated into these particles as well. The combined inductance and capacitance results in a LC resonance, which is the key element responsible for the negative magnetic permeability.

The possibility of negative magnetic permeability combined with the plasma-like behaviour of thin-wire structures [3] has given rise to a new class of materials termed negative refractive index materials [4]. These negative refractive index materials support the propagation of left-handed electromagnetic waves with negative phase velocity in a band of frequencies and give rise to novel effects such as a negative refraction at interfaces, negative Doppler shifts, and the possibility of imaging without a limit on the resolution, i.e., perfect lenses [5] etc. (See [6,7] for recent reviews on negative refractive index materials).

Here we present a nano-structured SRR meta-material that possesses negative magnetic permeability at near-infra-red (NIR) frequencies. We analyse the non-linear behaviour of the SRR when imbedded in a non-linear medium. The non-linearity of the matrix affects the capacitance of the SRR structure depending on the field intensity. There is immense concentration of electromagnetic fields in the capacitive gaps of the SRR and this amplifies the non-linearities of the imbedding medium. We present an expression for the change in the resonance frequency of the structure as a function of the incident magnetic field strength and give an estimate for the characteristic magnetic field strength which leads to a bistable response and switching of the effective magnetic permeability from negative to positive. Such a non-linear meta-material have immense potential in photonic applications as even a slab with only four layers of SRR can have very small transmission in the negative magnetic permeability regime while being highly transparent when it can be switched to a positive magnetic permeability material.

First let us analyse a modified form of the SRR as shown in the inset of Fig. 2. This SRR consists of a single ring with four splits placed symmetrically about it. The splits provide the capacitance in the system and the magnetic fields should be visualised to be perpendicular to the plane of the ring which drive the LC resonance and this results in a dispersive magnetic permeability. The symmetrically placed splits avoid problems of bi-anisotropy encountered in other varieties of split rings [7]. The capacitance per unit length of the cylindrical structure $C = \varepsilon_0\varepsilon_s\, \tau/nd_c$ can now be tuned by changing the thickness (τ), the spacing of the splits (d_c), the number of splits (n)or the dielectric constant (ε_s) of the medium in the capacitive gaps. The parameters shown in the figure enable operation of the SRR medium at NIR frequencies. The permeability of this effective medium of SRR was shown [8] by a quasi-static calculation to be

$$\mu_{eff} = \frac{B_{eff}}{\mu_0 H_{eff}} = 1 + \frac{f\,\omega^2}{\omega_0^2 - \omega^2 - i\,\gamma\,\omega} \tag{1}$$

where f= $L_g f/(L_g+L_i)$, $\gamma = L_i\gamma/(L_g + L_i)$ *and* $\omega_0^2 = 1/C\,(L_g + L_i)$ and $L_g = \mu_0 A_g$ is the geometrical inductance, A_g being the geometrical area of the SRR, and $L_i = l_c/\epsilon_0\omega_p^2\,\tau$ is an additional inductance that stems solely from the reactive behaviour of the metal at high frequencies and that we call the *inertial inductance* as it is directly proportional to the electronic mass. Here l_c is the perimeter length of the SRR.

A photonic band structure calculation based on the transfer matrix method yielded the reflection and transmission characteristics (solid lines) as shown in Fig. 1 for four layers of such infinitely long split cylinders. A plasma form for the dielectric permittivity of the constituent metal *viz.* Silver was taken to be of the form

$$\varepsilon(\omega) = \varepsilon_\infty - \frac{\omega_p^2}{\omega(\omega + i\,\gamma_s)} \tag{2}$$

where the empirical values of $\hbar\omega_p = 9.013$eV, $\gamma_s = 0.018$eV and $\varepsilon_\infty = 5.7$ were used. The figure focusses on the stop-band between 190 to 210 Terahertz frequencies caused by the negative magnetic permeability of the structure. One should note that just four layers of the structure is sufficient to lower the transmission coefficient to about 10^{-7}. The reflectivity is still about 85% to 90% indicating that the losses are not extremely high. The dashed lines show the response of an ideal homogenised material from the quasi-static calculation [8] which agree reasonably well with the numerical calculations.

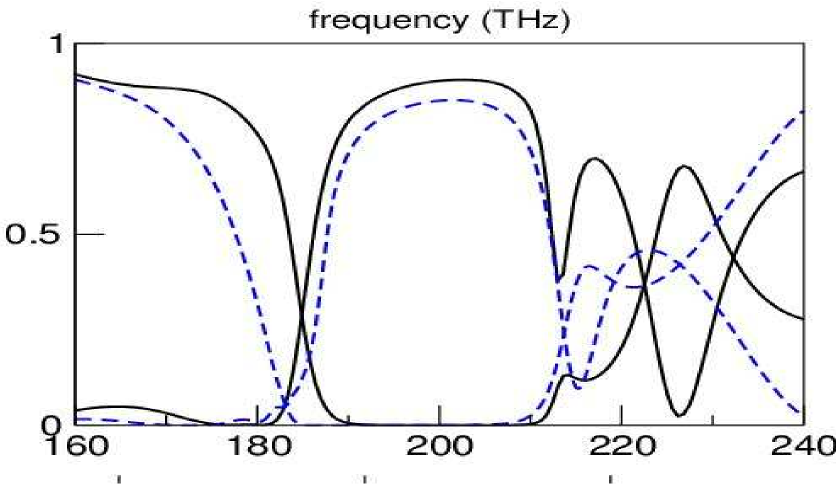

Fig. 1: Reflectance and Transmittance of a slab of four layers of SRR as a function of frequency obtained by the transfer matrix method (solid lines) and of an homogeneous slab of equivalent thickness and effective response function determined by a quasistatic calculation (dashed lines).

Now consider the case when the material in the capacitive gaps were to exhibit a Kerr non-linearity. The inset in Fig. 2 shows the SRR just being enclosed by the non-linear matrix as considered by us. The refractive index of the non-linear material has a dependence

$$n_s = \sqrt{\varepsilon_s} = n_0 + n_2 I \tag{3}$$

on the field intensity (I) where n_0 is the refractive index in the linear limit of low intensity. Materials with a defocussing non-linearity ($n_2 < 0$) are of particular interest here. In such a case as the intensity of the electromagnetic field increases, the dielectric constant decreases, in the process reducing the capacitance of the SRR and thereby switch to a regime of positive permeability below the (now intensity dependent) resonance frequency. Thus we can switch from a high reflectivity behaviour to a highly transmitting one with low losses. A relation between the incident field strength and the non-linear resonance frequency can be obtained as [9]

$$|H_{ext}|^2 = \frac{Z_s n_c^2 d_c^2}{4 n_2 L_g^2 \omega_0^2} \frac{(1 - X^2)}{Y^2 X^6} \left\{ \left[X^2 - Y^2 \right]^2 + Y^2 \gamma'^2 \right\} \tag{4}$$

where $Y = \omega/\omega_0$, $X = \omega_{NL}/\omega_0$, $\gamma' = \gamma/\omega_0$, ω_0 is the resonant frequency for the linear SRR and ω_{NL} is the resonant frequency of the SRR imbedded in the non-linear medium. The plot of the non-linear resonance frequency with the incident field strength is shown in Fig. 2. The bistable behaviour is found to be fairly stable against dissipation. The curve with open circles is for a situation when the intrinsic dissipation rates in the material were taken to be three times that of silver. Realistic values of $n_2 = -2.5 \times 10^{-12}$ cm^2/W which have been recently obtained were used for these calculations. The incident switching field strength of only about 20 kA/m implies that a source with about 100 mW peak power should be enough. This underlines the potential of non-linear SRR for switching applications

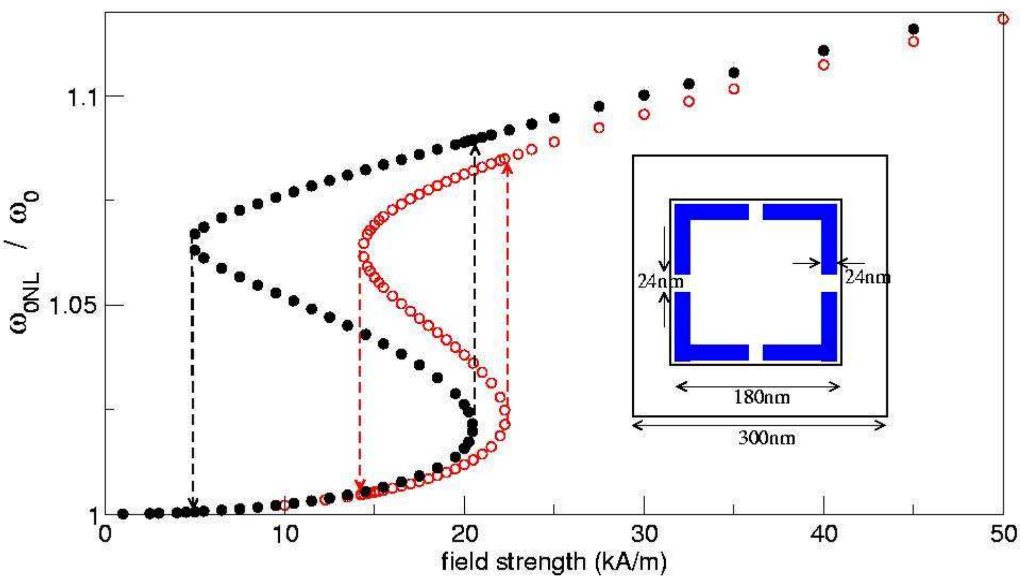

Fig. 2: The non-linear resonance frequency versus the field strength for two values of the dissipation rate : filled circles are for the γ_s of silver and open circles are for $3\gamma_s$. The inset shows the unit cell of the SRR structure with four splits and dimensions for operation in the NIR frequencies. The inner box shows the non-linear material just enclosing the SRR.

Finally, we should mention that the Kerr non-linearity that we have considered here is always possible in a dielectric regardless of the frequency of operation. Hence the effects we have discussed in this paper are possible over the entire electromagnetic spectrum. The local field enhancements in the capacitive gaps of the SRR can be larger at microwave frequencies by about one order of magnitude compared to higher frequencies. Thus one can also use this switching behaviour to switch between negative and positive refractive index in composite thin wire and SRR microwave media which are the only materials where negative refractive index has been experimentally demonstrated so far.

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