Variable Length Per Tone Equalizers for Multicarrier Systems

Said E. El-Khamy, ***Fellow IEEE,*** **Hassan El-Ragal*****, Member IEEE*** **and**
Amr A. El-Sherif***, Student Member IEEE***

Electrical Engineering Department, Faculty of Engineering, Alexandria University
Alexandria 21544, Egypt. Email: elkhamy@ieee.org

Abstract

Per tone equalization was recently proposed as an alternative equalization structure for multicarrier systems. In this paper we propose two NLMS based variable length per tone equalization algorithms. In the first algorithm, the number of equalizer taps used in every subchannel is determined according to channel conditions which results in a reduction in equalization complexity. While the second algorithm uses the variable length equalizers concept to ameliorate the convergence speed of the NLMS initialization algorithm. Simulation results show the enhanced performance of our proposed techniques compared with conventional per tone equalization techniques.

1 INTRODUCTION

Multicarrier modulation (MCM) [1] techniques like orthogonal frequency division multiplexing (OFDM) and discrete multitone (DMT) have been gaining in popularity in recent years. One reason for this surge in popularity is the ease with which MCM can combat channel dispersion, provided the channel delay spread is not greater than the length of the cyclic prefix (CP).

Fig. 1 shows the baseband model of a typical multicarrier modulation system. Each block of bits is divided into N bins, and each bin is viewed as being modulated by a different carrier. An efficient means of implementing the modulation is to use an inverse fast Fourier transform (IFFT). After transmission and reception, an FFT can be used for the demodulation. In order for the subchannels to be independent, the convolution of the signal and the channel is made to appear circular by adding a cyclic prefix (CP) to the start of each data block, which is obtained by prepending the last ν samples of each block to the beginning of the block. If the channel is shorter than the CP, then the output of each subchannel is equal to the input times a scalar gain factor. The signals in the bins can then be equalized by a bank of complex scalars, referred to as frequency domain equalizers (FEQ). If the CP is not as long as the channel delay spread, then inter-channel interference (ICI) and inter-symbol interference (ISI) will be present. A well known technique to combat the ISI/ICI caused by the inadequate CP length is the use of a time-domain equalizer (TEQ) at the receiver front-end. The TEQ is a filter that shortens the channel so that the delay spread of the effective channel impulse response is no longer than the length of the CP. The TEQ design problem has been extensively studied in the literature [2, 3, 4, 5].

2 PER TONE EQUALIZATION TECHNIQUE

A general disadvantage of time domain equalization is that the TEQ equalizes all tones in a combined fashion, which limits system performance. The recently proposed per tone equalization (PTEQ) approach [6] is based on transferring the TEQ operations to the frequency domain (i.e., after the FFT demodulation) which results in a separate T-taps PTEQ for each tone. Since equalization of one tone is now independent of the equalization of other tones, this scheme is able to optimize the SNR for each tone separately. The optimal equalizers' coefficients, which give rise to maximum SNR on each tone i, can be found as the solution to the MMSE problem:

$$\min_{\mathbf{v}_i} J\left(\mathbf{v}_i\right) = \min_{\mathbf{v}_i} E\left\{\left|\overline{\mathbf{v}}_i^T \cdot \underbrace{\left[\begin{array}{c|c} \mathbf{I}_{T-1} & \mathbf{0} \mid -\mathbf{I}_{T-1} \\ \hline \mathbf{0} & F_N\left(i,:\right) \end{array}\right]}_{\mathbf{F}_i} \cdot \mathbf{y}^{(k)} - X_i^{(k)}\right|^2\right\} \tag{1}$$

where $\overline{\mathbf{v}}_i$ denotes the coefficient vector $\mathbf{v}_i$ with its elements in reverse order, $F_N\left(i,:\right)$ is the i^{th} row of the FFT matrix F_N, $\mathbf{y}^{(k)}$ is a vector of received samples at symbol period k and $X_i^{(k)}$ is the subsymbol on tone i at time k.

Direct equalizer coefficient computation, based on the knowledge of the channel impulse response as well as the signal and noise statistics, has an excessively high computational cost. However (1) may be used for a training based initialization of the per tone equalizers. LMS-based schemes [7] for the per tone equalizers have a low computational complexity. However, they have poor convergence speed and have been shown to require an excessively large number of training symbols. On the other hand, RLS-based schemes [8] are faster but they are characterized by high computational complexity.

3 VARIABLE LENGTH PER TONE EQUALIZERS

In general, for a broadband channel, the amount of distortion in each subchannel will not be the same. A number of subchannels will exhibit severe distortion, and hence require relatively long equalizers to be able to reverse the channel effect, while other subchannels may exhibit mild distortion which can be equalized with shorter equalizers. Thus, it can be considered as a waste of computational power to use long equalizers for all subchannels.

Our method exploits this fact and tries to find the best equalizer length for each subchannel based on the amount of distortion present. Our objective is to maximize the SNR in each subchannel, since maximizing the SNR maximizes the bit rate for a given probability of error in xDSL applications or minimizes the overall bit error rate in wireless applications. The proposed algorithm is as follows:

1. Set the length of all equalizers to 2-taps
2. Initialize the equalizers using the NLMS
3. For every subchannel *i*, estimate the SNR_i using:

$$SNR_i = 10\ \log_{10}\left(M \cdot S_X \Big/ \sum_{k=1}^{M} \left| X_i^{(k)} - \hat{X}_i^{(k)} \right|^2 \right) \tag{2}$$

   where S_X is the signal power in subchannel *i* and $\hat{X}_i^{(k)}$ the equalizer output.

4. For each subchannel *i*, if $SNR_i < SNR_{\max}$ (a prescribed SNR value), increase the length of $\mathbf{v}_i$ by 2 taps
5. restart form 2 to initialize only the equalizers whose length has increased
6. algorithm stops when the SNR in all subchannels attain the value $SNR_{\max}$, or when all equalizers reach the maximum allowable length $L_{\max}$, or if increasing the equalizer length increases the SNR by a value not larger than a prescribed value Δ_{SNR}

The second proposed algorithm aims at improving the convergence speed of the NLMS PTEQ initialization algorithm used with fixed length equalizers. This is achieved by employing variable-length filters that can take advantage of the fast convergence speed of the NLMS algorithm when used to train short filters. The algorithm can be summarized in the following steps:

1. Start the NLMS training algorithm with all equalizers set to a short length T_1 and with step-size parameter μ_1. μ_1 could have a relatively large value since we are working with short filters, this results in a relatively fast convergence.
2. After a specified number of iterations, the length of all equalizers is increased to T_2 taps. The initial condition of these new equalizers is chosen as the last state of the previous adaptation stage.
3. To maintain the stability of the NLMS algorithm, the value of the step-size parameter has to be changed to $\mu_2 < \mu_1$ since we are now training longer filters.
4. The algorithm continues in the same manner till we arrive at the desired equalizers' length.

Since our algorithms are based on the NLMS algorithm, it is clear that they have the same computational complexity, i.e., $O(T)$ computations per training symbol per tone.

4 SIMULATION RESULTS

We use the ADSL downstream transmission to test our algorithms, this system uses 512-point IFFT and FFT and cyclic prefix of length 32 [9]. The eight standard carrier-serving-area (CSA) loops are used as test channels [10]. The input signal power spectral density at the transmitter output is set to -40 dBm/Hz. Channel noise is modeled as an additive white Gaussian noise (AWGN) with -140 dBm/Hz power density, plus near-end-cross-talk (NEXT) noise from 15 ADSL and 5 ISDN disturbers [9, 10].

Fig. 2 compares between the SNR distribution obtained by the NLMS initialization algorithm and by our first variable length algorithm over CSA loop 2. The average length of the equalizers is $\langle T \rangle = 6$ taps. For fairness of comparison, we compare the variable length method with the constant length NLMS having length equal to $\langle T \rangle$, and thus having the same overall computational complexity. It is clear that our method outperforms the constant length method in terms of SNR, which is translated in higher achievable bit rate. Table 1 summarizes the results obtained for the constant length and variable length algorithms over the eight CSA loops.

Fig. 3 shows of the achievable bit rate as a function of the number of iterations (symbols) for the NLMS, the RLS and our proposed variable length algorithm (VL-LMS). It can be seen that the RLS algorithm needs about 200 iterations to converge to the optimal rate, whereas our VL-LMS algorithm needs about 400 iterations while the NLMS has not converged after 1000 iterations. The VL-LMS algorithm is advantageous over RLS by its very low computational complexity.

5 CONCLUSIONS

In this paper we presented a novel algorithm for defining the lengths of the per tone equalizers. It is shown that by tailoring the lengths of the equalizers to channel conditions the system outperforms other systems employing constant length equalizers of equivalent overall complexity. Then we proposed a variable length LMS adaptation algorithm which was shown to outperform the NLMS in convergence speed, and that it is comparable to the speed of the RLS algorithm but at a much lower computational complexity.

References

[1] J. A. C. Bingham, “Multicarrier Modulation for Data Transmission: An Idea Whose Time Has Come,” *IEEE Comm. Mag.*, vol. 28, pp. 5-14, May 1990.

[2] N. Al-Dhahir and J. M. Cioffi, “Efficiently Computed Reduced-Parameter Input-Aided MMSE Equalizers for ML Detection: A Unified Approach,” *IEEE Trans. on Info. Theory*, vol. 42, pp. 903-915, May 1996.

[3] P. J. W. Melsa, R. C. Younce, and C. E. Rohrs, “Impulse Response Shortening for Discrete Multitone Transceivers,” *IEEE Trans. on Comm.*, vol. 44, pp. 1662–1672, Dec. 1996.

[4] G. Arslan, B. L. Evans, and S. Kiaei, “Equalization for Discrete Multitone Receivers To Maximize Bit Rate,” *IEEE Trans. on Signal Processing*, vol. 49, pp. 3123-3135, Dec. 2001.

[5] S. E. El-Khamy, H. El-Ragal and A. A. El-Sherif, “A Modified Eigen Approach Method for the Design of Channel Shortening Equalizers in Discrete Multi-Tone Systems,” in *Proc. 22nd National Radio Science Conference, NRSC'2005*, Cairo, Mar. 2005.

[6] K. Van Acker, G. Leus, M. Moonen, O. van de Wiel, and T. Pollet, “Per Tone equalization for DMT-based systems,” *IEEE Trans. Commun.*, Vol. 49, pp. 109-119, Jan. 2001.

[7] S. Haykin, *Adaptive Filter Theory*, 2nd ed. Englewood Cliffs, NJ: Prentice-Hall, 1991.

[8] K. Van Acker, G. Leus, M. Moonen, and T. Pollet, “RLS-based initialization for per-tone equalizers in DMT receivers,” *IEEE Trans. Commun.*, Vol. 51, pp. 885–889, June 2003.

[9] “Asymmetric digital subscriber line (ADSL) transceivers” ITU-T G.992.1, 1999.

[10] “Test procedures for digital digital subscriber line (DSL) transceivers” ITU-T G.996.1, 2001.

Table 1: Bit rate in Mbps over the 8 CSA loops

Loop	Constant-Length PTEQ	Variable-Length PTEQ
1	4.8760	5.5760
2	4.1360	5.600
3	4.2560	4.7680
4	4.1040	4.7080
5	5.3960	5.7120
6	4.0920	4.3960
7	2.9960	4.3120
8	2.9400	3.8240

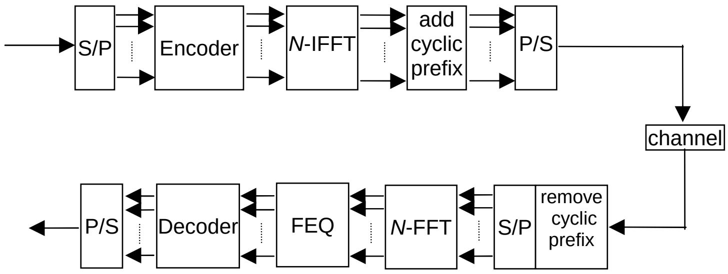

Fig. 1: Multicarrier Modulation transceiver

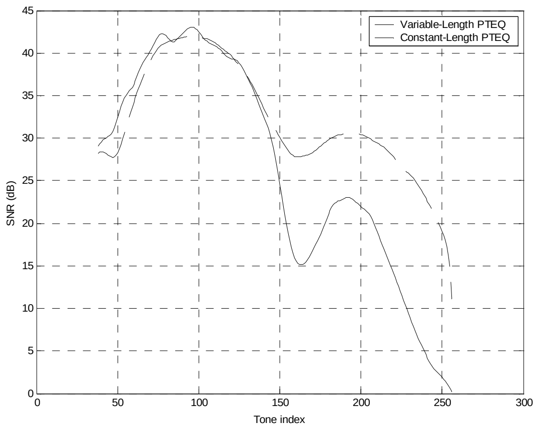

Fig. 2: SNR distribution over the set of used tones

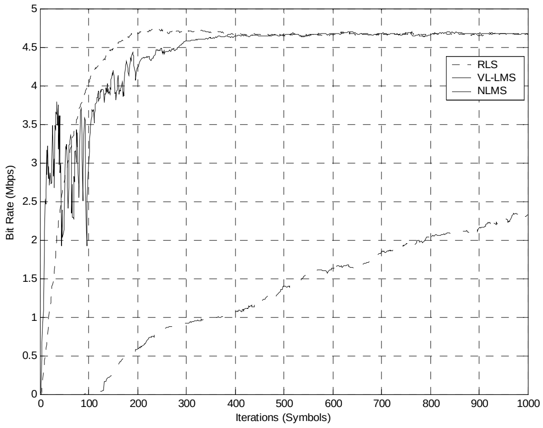

Fig. 3: Achievable rate for the different algorithms