PERFORMANCE OF EQUALIZERS BASED ON SISO ALGORITHMS

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Abstract:

One of the major obstacles to achieving high speed data transmission in a band limited channel is the presence of Inter Symbol Interference (ISI). The Soft In Soft Out algorithms like MAP and SOVA are used for data detection in a highly frequency selective channel representative of severe ISI for uncoded as well as coded data transmission. The performances are almost similar for the uncoded case. The MAP-MAP concatenation seems better than SOVA-SOVA combination by almost 0.4 db for the coded data transmission. We have compared the performance of such algorithms in an elaborate manner.

Keywords: ISI, MAP, SOVA, SISO

I. Introduction

High speed data transmission in a band limited channel in the presence of Inter Symbol Interference (ISI) leads to considerable decrease in noise margin. ISI results due to the imperfect frequency response of the given channel or the multipath nature of the channel as in a mobile environment [4]. The bits initially assigned a given time slot at the transmitter tend to spill into the time slots of other bits after transmission through a band limited frequency selective channel, causing interference with each other. As a result of this, reliable detection of the bits becomes difficult at the receiver.

Equalization is the technique carried out by the receiver to mitigate the effects of ISI. In this paper, we study the performance of Soft-In Soft-Out equalizers in a highly frequency selective channel. We give the performance of equalizers employing either the MAP (Maximum a Posteriori probability) algorithm [2] or the SOVA (Soft Output Viterbi Algorithm)[3] for uncoded as well as coded ISI. Both can accept soft inputs and produce soft outputs in the form of reliability of a given bit usually in the form of Log-Likelihood Ratio (LLR). Section II states the problem and the two algorithms. The performance results as obtained with computer simulations are plotted in section III while section IV gives the conclusion.

II. Problem Formulation and the SISO(Soft In Soft Out) Algorithms

The problem is the classic problem of reliable data detection over a highly frequency selective ISI channel with AWGN that distorts the original data bits. The SISO algorithms have received a great deal of interest in the recent past because of their impressive performance in AWGN channels. Literature talks of the performance of either the MAP or the SOVA. However, a comparison of both of these algorithms as used in either uncoded or coded systems is not available to the best of our knowledge. In this paper, our aim is to provide a quantitative comparison of both algorithms under the same conditions for both scenarios. The uncoded data transmission system that we have simulated is described by

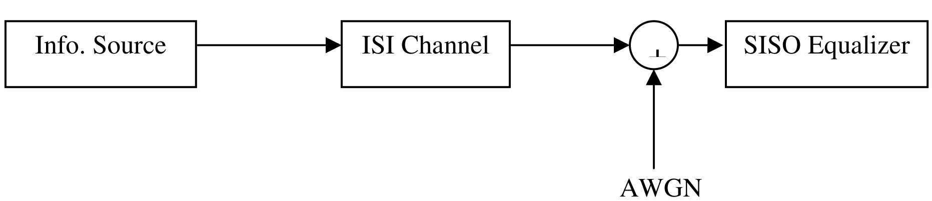

Fig.1. Block Schematic of Communication System using SISO Equalizer

the general block diagram of the SISO receiver as given in Fig.1. The idea behind SISO equalization is to treat the channel causing ISI as the inner encoder that appears in series with any forward error correcting encoder like a convolutional encoder. The whole system becomes a serial concatenated code to which the iterative feedback can be applied at the receiver [1].

In Fig.2, the outer encoder is a Recursive Ssystematic Convolutional (RSC) encoder. The parameters are: rate R=1/2, constraint length K=3 with generator polynomials 7 and 5 expressed in the octal form. The ISI channel is taken to be a 3-tap channel [2] with channel coefficients [0.407 0.815 0.407]. The channel is said to have a memory of two.

The interleaver separates the outer and the inner encoder. This is to decorrelate the adjacent bits. A random interleaver is used in our work.

The whole scheme of this soft equalization is thought of as a serial concatenated code where the outer encoder is an RSC code and the ISI channel is the inner encoder with rate 1. The channel is modeled as an FIR filter with the above coefficients. The transmitter block diagram is shown in Fig.2 in which a part of the receiver is shown. The SISO equalizer is expanded in Fig.3.

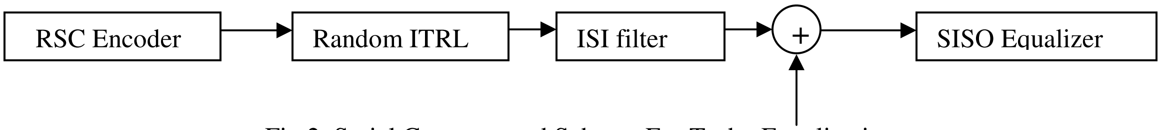

Fig.2. Serial Concatenated Scheme For Turbo Equalization

The concatenated equalizer consists of a MAP equalizer which is followed by a deinterleaver . The receiver using this structure is shown in Fig.3. The MAP equalizer aims in removing the ISI, while the decoder takes care of white noise.

III. Description of MAP and SOVA

Fig.3. Block Schematic of SISO Receiver

The equalizer we have used in our work is based on the MAP algorithm. The aim is to compute the APP value of each bit. As the ISI channel is a rate 2/2 code, we have computed the APP (A Posteriori Probability) values for the 2*N number of coded bits as coming from the outer RSC encoder.

The algorithm proceeds by storing the ISI corrupted noisy received sequence. The trellis for this channel is found to be similar to the one as the one for a NSC encoder of constraint length 3 with generator polynomials 7 and 5. However, the output along each transition is only one sample in contrast to two output bits of the NSC encoder. The values are analog due to the discrete time coefficients of the channel.

The noise corrupting the ISI affected bits is gaussian .The expression for the output at the reciver input is given by

$$\text{rec\_s}(k) = \sum_{l=0}^{L-1} x_l h_{k-l} + n_k \tag{1}$$

where L is the length of the channel, x_k is the kth input bit to the channel, h_ks are the channel coefficients and n_k is the kth gaussian noise sample.

Thus, to calculate the branch transition probability $\gamma(s_k,s_{k+1}/\mathbf{y})$ we write the familiar equation:

$$\gamma(s_k,s_{k+1}/\mathbf{y}) = K*\exp(-(\text{rec\_s}_k - v_k)^2)/2*\sigma^2 \tag{2}$$

where K is defined as(2 $*\pi*\sigma^2$) and may be neglected in further calculations, rec_s_k is the received sample at the k-th instant and v_k is the expected value that occurs due to the transition from state s_k to the state s_{k+1} due to the incoming bit –1 or +1.

The forward recursion α_k is calculated from the previous value of α_{k-1} as follows:

$$\alpha_k = \Sigma\alpha_{k-1} * \gamma(s_k,s_{k+1}/\mathbf{y}) \tag{3}$$

over all the previous states s_{k-1} to the present state s_k.

The backward recursion is computed using (4) over all the possible next states of the trellis.

$$\beta_k(s) = \Sigma\beta_{k+1}(s_{k+1},s_k)*\gamma(s_{k+1},s_k) \tag{4}$$

Now, the APP value of each of the 2*N number of coded bits is computed using (5).

$$\text{llr}(c_k) = \Sigma\alpha_k(\text{rec\_s}_i)*\gamma_k(\text{rec\_s}_i,\text{rec\_s}_j)*\beta_{k+1}(\text{rec\_s}_j) \Big/ \Sigma\alpha_k(\text{rec\_s}_i)*\gamma_k(\text{rec\_s}_i,\text{rec\_s}_j)*\beta_{k+1}(\text{rec\_s}_j) \tag{5}$$

where the numerator is taken over all the transitions for $c_k= +1$ and the denominator is over all the transitions due to $c_k= -1$, i and j represent present and next state.
The decision on a particular bit is taken by looking at the sign of its LLR value. If the LLR is positive, decision is +1 and it is –1 otherwise.

The LLR values coming from the equalizer are deinterleaved and fed to the decoder working on the trellis of the RSC code. As the a priori information to the equalizer in the first iteration is zero, the difference between the equalizer output and the a priori values is equal to the LLR values. The MAP decoder computes the APP values for the systematic as well as the parity bits. But, this time the branch metric calculation is different. As each transition produces two output bits, we multiply the probabilities of each of the bits to compute the branch metric γ at any instant of time k. The probabilities follow from the LLR values by the following formula:

$$P(c_k = \pm 1) = \exp(\pm LLR(c_k) \Big/ (1+\exp(\pm)LLR(c_k) \qquad (6)$$

SOVA

The Soft Output Viterbi algorithm we have implemented in traceback mode is as follows:

***Storage**:*

k (time index, modulo $\delta+1$)

$$c(s_k) = \{c_{k-\delta}(s_k),\ldots\ldots c_k(s_k)\}, \quad 0 \le s_k \le S-1 \qquad (7)$$

(hard decision values, $\hat{u}\{\pm\}$)

$$L(s_k) = \{L_{k-\delta}(s_k),\ldots L_k(s_k)\} \qquad (8)$$

***Recursion**:*

Classical Viterbi step:

For each state s_k

Compute

$$\Gamma(s_{k-1},s_k) = \Gamma(s_{k-1}) + (y_k - x_k)^2 \qquad (9)$$

for both transitions (s_{k-1},s_k)

$$\text{Find } \Gamma(s_k) = \min \Gamma(s_{k-1},s_k). \qquad (10)$$

Store $\Gamma(s_k)$ and the corresponding survivor $\hat{u}(s_k)$.

***Soft-deciding update**:*

For each state s_k

$$\text{Store } \Delta = \max \Gamma(s_{k-1},s_k) - \min \Gamma(s_{k-1},s_k) \; \Delta \ge 0, \qquad (11)$$

Initilization $L(s_k) = +\text{Infinity}$.

For $j = k-\nu$ to $j = k-\delta_m$

Compare the two paths merging in s_k

if $c_j^{(1)}(s_j) \neq c_j^{(2)}(s_j)$ then update

$$L_j := \min(L_j,\Delta) \qquad (12)$$

IV. Performance Results

Simulation parameters used are shown in Table No.1.

Table No.1

RSC Encoder	K=3, R=1/2, g1=[1 1 1], g2=[1 0 1]
Interleaver	Random
ISI Channel	[0.407 0.815 0.407]
Channel Memory	2
Block Length	100,000
Traceback Depth in SOVA	10
Modulation	BPSK
Iteration Number	0

The performance results as obtained by computer simulations for uncoded and coded ISI for both the MAP and SOVA are plotted in Fig.4 and Fig.5 respectively. It is observed that, for the uncoded case, as shown in Fig.1 both the algorithms give almost identical performances. The SOVA used here is just the Viterbi algorithm that produces hard outputs. We find that, VA yields almost identical results as MAP although at a much reduced complexity. For the coded case as in Fig.2, the energy available per bit reduces by ½ and the MAP and the SOVA equalizers perform almost similar. However, the MAP Decoder is better than the SOVA decoder by almost 0.4 dB.upto 7 dB Eb/No which subsequently gets increased for higher values of Eb/No.The power penalty caused by coding on each bit is taken care of by the further block of the decoder. Either of the decoder in the coded case gives a gain of about 3 to 4 db as compared to the equalizer. This shows the impressive performance of SISO modules. The coding gain becomes appreciable from 4 db of Eb/No and onwards for the decoder.

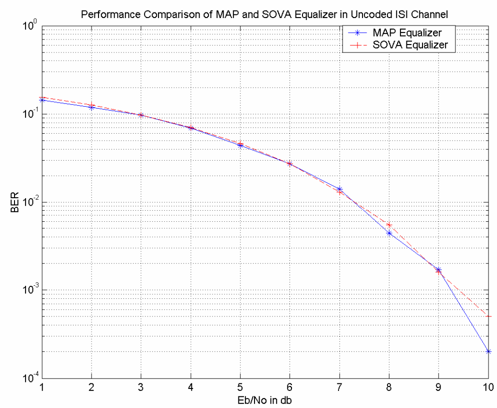

Fig.4 Performance Comparison of MAP and SOVA

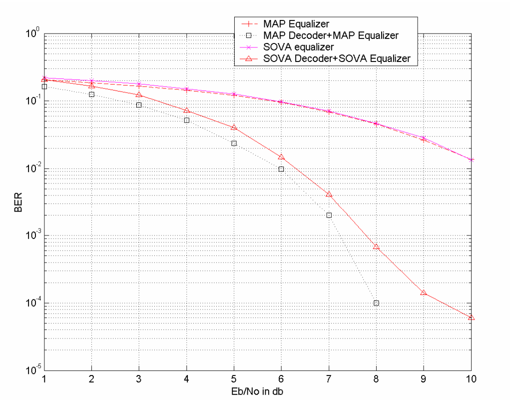

Fig.5 Performance Comparison of for MAP-MAP vrs.SOVA-SOVA for coded ISI

IV. Conclusion

The SISO algorithms based on either the MAP or the SOVA seem to produce impressive performances in terms of coding gain for a highly frequency selective channel as given above. This makes data communication possible even when the channel condition is very poor, where conventional techniques would fail.

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