# ON THE CAPACITY OF ULTRA-WIDEBAND RADIO CHANNELS

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## ABSTRACT

**In general, because of multipath propagation, ultra wideband radio channels exhibit time-variant and frequency-selective fading. Based on Shannons formula for the capacity of a continuous AWGN channel, the capacity of wideband radio channels and the fluctuations of the capacity are evaluated analytically as a function of bandwidth. A relation between the frequency-correlation function of the radio channel and the fluctuations of channel capacity is presented.**

## 1 INTRODUCTION

Shannon has shown that, theoretically, it is possible to transmit information over a given channel with an arbitrary small error probability if the data rate is not higher than the channel capacity. Today, with coded modulation schemes it is possible to transmit with data rates quite close to channel capacity. Therefore, channel capacity is one of the most important parameters of a transmission system.

A radio channel (without co-channel interference) is described by a time-variant impulse response $h(t,\tau)$ or time-variant transfer function $H(t,\omega)$ and additive white Gaussian noise (AWGN) $n(t)$ (thermal noise). Because of the time-variant behaviour also the channel capacity changes with time. The statistical properties of these fluctuations are of great importance for the design of broadband wireless data transmission systems, but have been discussed in literature mainly for narrow-band channels. In [1] the author has shown by evaluation of measurements that even for medium bandwidths in the range of 20 MHz the capacity fluctuations decrease with increasing bandwidth. It had been shown that for bandwidths $B$ much higher than the coherence bandwidth, the relative fluctuations decrease approximately proportional $1/\sqrt{B}$. This means that an ultra-wideband channel exhibits only very small fluctuations of of channel capacity.

In the present paper the temporal fluctuations of the capacity of ultra-wideband radio channels with frequency-selective fading are analyzed on the basis of a simplified channel model. Since high data rate communication systems are considered, the transfer function of the radio channel varies very slowly compared with symbol duration. Because of this reason it is assumed that the instantaneous transfer function of the radio channel can be estimated at the receiver and communicated back to the transmitter via signalling channels. Therefore, it is assumed that the transfer function is known both at transmitter and receiver.

## 2 CALCULATION OF CHANNEL CAPACITY

The calculation of the channel capacity is based on Shannons formula for the continuous AWGN channel. This approach is admissible since the channel changes very slowly compared with the transmitted ultra-wideband signal. In general, because of multipath propagation, a wideband radio channel exhibits frequency-selective fading. In case of a frequency-selective channel the capacity is determined by the ratio of the signal's power spectral density (PSD) $S_x(\omega)$ to the equivalent noise PSD. The transmission channel has to be divided into a large number of narrowband subchannels in each of which the transfer function can be considered as constant. The channel capacity results from integration over all subchannels:

$$C(t) = \frac{1}{2\,\pi} \int_0^\infty \mathrm{ld}\left(1 + \frac{2 \cdot S_x(\omega) \cdot |H(t,\omega)|^2}{k \cdot T_0 \cdot F(\omega)}\right) \mathrm{d}\omega. \tag{1}$$

where $k$ denotes Boltzmann's constant, $T_0$ the reference temperature and $F$ the noise figure which for simplicity is assumed to be frequency-independent.

The transmitted signal shall be limited by its mean power. Under this assumption the maximum of the channel capacity is achieved if the PSD of the transmitted signal is adapted to the transfer function of the radio channel. The optimum method for distribution of power is the well-known water-pouring method [2]. From a qualitative point of view, with this method, most of the power is concentrated within frequency ranges where the attenuation is small. Since the transfer function $H(t, \omega)$ is time-variant, also the optimum signal PSD becomes time-variant.

## 3 FLUCTUATIONS OF CHANNEL CAPACITY

The time-variant transfer function has to be modelled by a random process. Therefore, also the channel capacity has to be modelled as a random process. In the following the fluctuations of the channel capacity will be calculated. As a simple measure of the fluctuations, the variance will be calculated:

$$\sigma_C^2 = \mathrm{E}\{[C(t) - \overline{C}]^2\} = \mathrm{E}\{C(t)^2\} - \overline{C}^2 \tag{2}$$

where E{.} denotes the expectation operation and $\overline{C}$ the average capacity:

$$\overline{C} = \mathrm{E}\left\{\frac{1}{2\,\pi} \int_0^\infty \mathrm{ld}\left(1 + \frac{2 \cdot S_x(\omega) \cdot |H(t,\omega)|^2}{k \cdot T_0 \cdot F(\omega)}\right) \mathrm{d}\omega\right\}. \tag{3}$$

Here, it is assumed that the channel is at least wide-sense stationary so that the expectation of channel capacity as well as its variance do not depend on time. Inserting (1) and (3) into (2) and exchanging the order of integration and expectation yields:

$$\begin{aligned}
\sigma_C^2 \;=\;& \frac{1}{4\,\pi^2} \int_{\omega_1}^{\omega_2} \int_{\omega_1}^{\omega_2} \mathrm{E}\left\{\mathrm{ld}\left(1 + \frac{2 \cdot S_x(\omega) \cdot |H(t,\omega)|^2}{k \cdot T_0 \cdot F(\omega)}\right) \cdot \mathrm{ld}\left(1 + \frac{2 \cdot S_x(u) \cdot |H(t,u)|^2}{k \cdot T_0 \cdot F(u)}\right)\right\} \mathrm{d}\omega\, \mathrm{d}u \\
-& \frac{1}{4\,\pi^2} \int_{\omega_1}^{\omega_2} \int_{\omega_1}^{\omega_2} \mathrm{E}\left\{\mathrm{ld}\left(1 + \frac{2 \cdot S_x(\omega) \cdot |H(t,\omega)|^2}{k \cdot T_0 \cdot F(\omega)}\right)\right\} \cdot \mathrm{E}\left\{\mathrm{ld}\left(1 + \frac{2 \cdot S_x(u) \cdot |H(t,u)|^2}{k \cdot T_0 \cdot F(u)}\right)\right\} \mathrm{d}\omega\, \mathrm{d}u.
\end{aligned} \tag{4}$$

Here it is assumed that the channel is only used in the frequency range between $\omega_1$ and $\omega_2$.

One motivation to introduce ultra-wideband systems is that their transmit PSD may be chosen so low that the ultra-wideband signals do not cause significant interference in existing systems in the same frequency range. The low PSD of the transmitted signal simplifies very much the analysis of fluctuations of channel capacity: The logarithm in the capacity formula may be linearized:

$$\mathrm{ld}(1 + \epsilon) \approx \frac{1}{\ln 2} \cdot \epsilon. \tag{5}$$

Inserting the approximation (5) into (4) yields:

$$\sigma_C^2 = \frac{1}{\pi^2 \cdot k^2 \cdot T_0^2} \int_{\omega_1}^{\omega_2} \int_{\omega_1}^{\omega_2} \left[ \mathrm{E}\left\{ \frac{S_x(\omega) \cdot |H(t,\omega)|^2}{F(\omega)} \cdot \frac{S_x(u) \cdot |H(t,u)|^2}{F(u)} \right\} \right.$$

$$\left. - \mathrm{E}\left\{ \frac{S_x(\omega) \cdot |H(t,\omega)|^2}{F(\omega)} \right\} \cdot \mathrm{E}\left\{ \frac{S_x(u) \cdot |H(t,u)|^2}{F(u)} \right\} \right] \mathrm{d}\omega \, \mathrm{d}u. \tag{6}$$

For simplicity, in the following it is assumed that the PSD of the transmitted signal is constant: $S_x(\omega) = S_0$.

$$\sigma_C^2 = \frac{S_0^2}{\pi^2 \cdot k^2 \cdot T_0^2} \int_{\omega_1}^{\omega_2} \int_{\omega_1}^{\omega_2} \frac{1}{F(\omega)F(u)} \left[ \mathrm{E}\left\{ |H(t,\omega)|^2 \cdot |H(t,u)|^2 \right\} - \mathrm{E}\left\{ |H(t,\omega)|^2 \right\} \cdot \mathrm{E}\left\{ |H(t,u)|^2 \right\} \right] \mathrm{d}\omega \, \mathrm{d}u. \tag{7}$$

The remaining expectations will be expressed as a function of the frequency correlation function of the radio channel. The time-variant transfer function is a complex Gaussian random process. In general for zero-mean Gaussian random variables $z_1$, $z_2$, the following relation holds:

$$\mathrm{E}\left\{ |z_1|^2 \cdot |z_2|^2 \right\} = \mathrm{E}\left\{ |z_1|^2 \right\} \cdot \mathrm{E}\left\{ |z_2|^2 \right\} + \left| \mathrm{E}\left\{ z_1 z_2^* \right\} \right|^2. \tag{8}$$

Using this relation and the definition of the correlation function of the time-variant transfer function $R_{HH}(t_1, t_2, \omega_1, \omega_2) = \mathrm{E}\left\{ H(t_1, \omega_1) \cdot H^*(t_2, \omega_2) \right\}$ Eq.(7) can be written as:

$$\sigma_C^2 = \frac{S_0^2}{\pi^2 \cdot k^2 \cdot T_0^2} \int_{\omega_1}^{\omega_2} \int_{\omega_1}^{\omega_2} \frac{1}{F(\omega)F(u)} \left| R_{HH}(t,t,\omega,u) \right|^2 \mathrm{d}\omega \, \mathrm{d}u. \tag{9}$$

For a wide-sense stationary channel the correlation function $R_{HH}(t,t,\omega,u)$ does not depend on the observation time $t$. The ultra-wideband channel can be assumed to be wide-sense stationary but because of its large bandwidth it cannot be assumed to show uncorrelated scattering. Therefore, the frequency correlation function $R_{HH}$ depends on two frequency variables $\omega$ and $u$. Equation (9) may be evaluated numerically or simplified channel models may be used in order to find analytic results. A simplification may be found by dividing the channel transfer function into two parts:

$$H(t,\omega) = H_0(\omega) \cdot H_\sim(t,\omega). \tag{10}$$

The first factor $H_0(\omega)$ may represent the overall average (deterministic) frequency selectivity which includes e.g. the frequency response of the antennas at transmitter and receiver as well as the frequency-selective free-space attenuation. On the other hand $H_\sim(t,\omega)$ describes the multipath propagation and has to be modelled as a random process. Since the overall frequency selectivity is already described by $H_0(\omega)$, the time-variant part of the transfer function $H_\sim(t,\omega)$ can be assumed to describe a wide-sense stationary uncorrelated scattering (WSSUS) channel. This means that the frequency correlation function of the transfer function $H_\sim(t,\omega)$ depends only on a single variable – the difference of frequencies:

$$R_{H_\sim H_\sim}(t_1, t_2, \omega_1, \omega_2) = \mathrm{E}\left\{ H_\sim(t_1,\omega_1) \cdot H_\sim^*(t_2,\omega_2) \right\} = R_{H_\sim H_\sim}(\omega_2 - \omega_1). \tag{11}$$

Using this result we can rewrite the frequency correlation function:

$$\begin{aligned} R_{HH}(t_1,t_2,\omega_1,\omega_2) &= \mathrm{E}\left\{ H(t_1,\omega_1) \cdot H^*(t_2,\omega_2) \right\} \\ &= \mathrm{E}\left\{ H_0(\omega_1) \cdot H_\sim(t_1,\omega_1) \cdot H_0^*(\omega_2) \cdot H_\sim^*(t_2,\omega_2) \right\} \\ &= H_0(\omega_1) \cdot H_0^*(\omega_2) \cdot \mathrm{E}\left\{ H_\sim(t_1,\omega_1) \cdot H_\sim^*(t_2,\omega_2) \right\} \\ &= H_0(\omega_1) \cdot H_0^*(\omega_2) \cdot R_{H_\sim H_\sim}(t_1,t_2,\omega_1,\omega_2). \end{aligned} \tag{12}$$

Inserting this result into Eq. (9) and assuming that the correlation function $R_{H_\sim H_\sim}(t_1,t_2,\omega_1,\omega_2)$ is nonzero only if $\omega_1$ and $\omega_2$ are very close together, the following result is obtained:

$$\begin{aligned} \sigma_C^2 &= \frac{S_0^2}{\pi^2 \cdot k^2 \cdot T_0^2} \int_{\omega_1}^{\omega_2} \int_{\omega_1}^{\omega_2} \frac{|H_0(\omega) \cdot H_0(u)|^2}{F(\omega)F(u)} \left| R_{H_\sim H_\sim}(t,t,\omega,u) \right|^2 \mathrm{d}\omega \, \mathrm{d}u \\ &\approx \frac{S_0^2}{\pi^2 \cdot k^2 \cdot T_0^2} \int_{\omega_1}^{\omega_2} \int_{\omega_1}^{\omega_2} \frac{|H_0(\omega)|^4}{F^2(\omega)} \left| R_{H_\sim H_\sim}(\omega - u) \right|^2 \mathrm{d}\omega \, \mathrm{d}u. \end{aligned} \tag{13}$$

It is assumed that $F(\omega)$ and $H_0(\omega)$ change only slowly with frequency – therefore, it can be assumed that $F(\omega) \approx F(u)$ and that $H_0(\omega) \approx H_0(u)$.

# 4 CONCLUSIONS AND OUTLOOK

In this paper the fluctuations of the capacity of ultra-wideband radio channels have been calculated analytically. An equation has been found which relates the variance of the fluctuations of channel capacity with the frequency-correlation function of the radio channel.

Numerical evaluations which show the relation between fluctuations of the channel capacity and parameters of the power delay profile will be carried out in future.

# REFERENCES


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[2] R. G. Gallager: *Information theory and reliable communication.* John Wiley & Sons, New York 1968.