# A SUPER-RESOLUTION APPROACH TO RANGE PROCESSING IN HIGH RESOLUTION RADARS

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**ABSTRACT:**

High-resolution radars employ Linear Frequency Modulated (LFM) waveforms for improving the range resolution, the resolving ability of which is proportional to the bandwidth of the waveform. The received waveform is de-ramped, which encodes the ranges of the targets as discrete frequencies. Hence a frequency analysis of the de-ramped output will resolve the targets in range. We model the above problems as those of finding the frequency of discrete tones in noise. Subspace invariance approach like ESPRIT is considered as the solution to the above problem. The proposed method can resolve frequencies, which cannot be resolved by the Raleigh limited Fast Fourier Transform (FFT). As the de-ramping process encodes ranges in terms frequencies, closely spaced targets can be resolved because of improved frequency resolution.

**Keywords: Range Resolution, Linear Frequency Modulation (LFM), De-ramping, Frequency encoding, ESPRIT, AIC**

## 1.INTRODUCTION

Range-resolution is the ability of the radar to resolve two close by targets in range. High-resolution radars employ frequency-modulated waveforms for improving the range resolution. The resolution capability of a Linear Frequency Modulated (LFM) waveform is directly proportional to bandwidth of the waveform. If the range swath of interest is of the order of the resolution capability of the un-coded pulsed waveform, then the intermediate frequency bandwidth can be effectively reduced before sampling by de-ramp processing. In de-ramp processing the input data is mixed with a replica of the transmitter LFM waveform delayed by the two way propagation delay from the Antenna Phase Center (APC) to the center of the range swath. The de-ramping operation produces discrete tones corresponding to the target over the range swath and the frequency of the tones are linearly related to the distance of the target from the scene center. Hence de-ramping operation can be interpreted as the frequency encoding of ranges of the targets over the range swath and a frequency analysis of the de-ramped output will resolve the targets in range, which can be effectively performed by a Fast Fourier Transform (FFT) operation. But in most scenarios the dominant point targets may be closely spaced in range so that it cannot be resolved by the conventional FFT processing because it is Rayleigh limited. The resolution performance deteriorates with decreasing Signal to Noise Ratio (SNR).

This paper gives a technique to model the problem in terms of super- resolution techniques thereby exploiting the underlying data model. The organization of the paper is as follows. In section 2 we discuss de-ramp processing and subsequent frequency encoding of ranges. In section 3 the ESPRIT formulation of detecting tones in noise are discussed. Section 4 presents simulation results and observations.

## 2. DE-RAMP PROCESSING

Consider an LFM waveform x(t) with duration $T_p$ and chirp rate p. It can be represented as

$$x(t)= \exp(j2\Pi f_c t+ j\Pi\, pt^2) \qquad -T_p/2 \leq t \leq T_p/2 \tag{1}$$

Where $f_c$ is the center frequency of the LFM waveform. The bandwidth of such a waveform is given by $B=p\,T_p$ and it can be compressed to 1/B. The range resolution, ρ achieved by such a waveform is given by[3]

$$\rho=ck/2B \tag{2}$$

Where c is the velocity of light and k is the excess bandwidth factor to compensate for the main-lobe broadening introduced by the weighting function. The pulse compression ratio (PCR) is defined as the time bandwidth product and is given by

$$PCR = T_p B = pT_p^2 \qquad (3)$$

When the LFM pulses scatters over a small range swath as in the case of high resolution imaging radars operating in the spotlight mode the analog signal bandwidth can be reduced by the de-ramp processing. It eases the speed of the analog to digital converters and the size of the digital memory. In de-ramp processing the returned analog signal is mixed with a replica of the transmitted waveform delayed by the two way time delay from Antenna Phase Center (APC) to the center of the target [4]. The bandwidth of de-ramped signal is given as

$$B_{IF} = 2r.\Delta r/cT \qquad (4)$$

Where r is the range to the target and $\Delta$r is the range swath of interest.

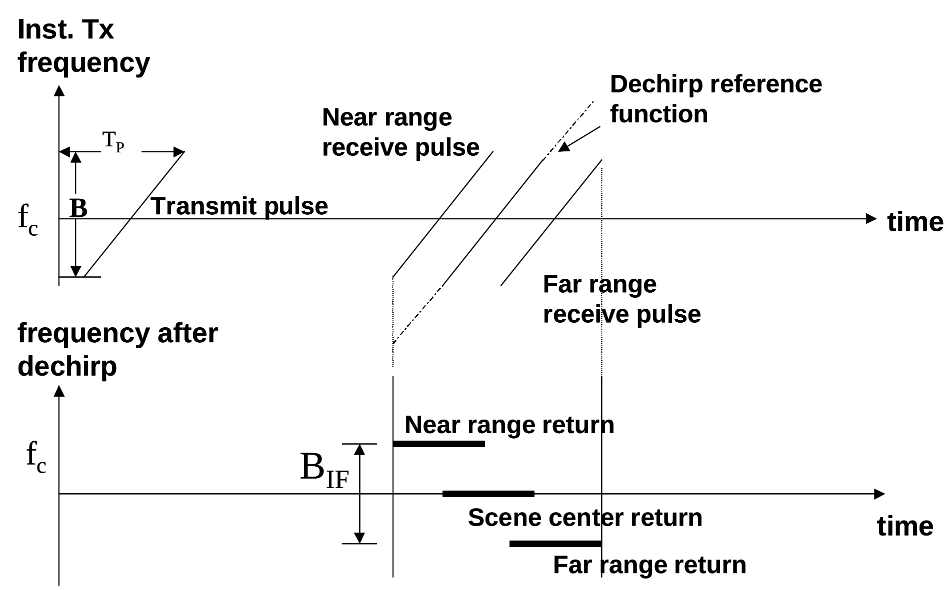



Fig 1: Timing relationship in de-ramp processing

## 3. ESPRIT ALGORITHM

Estimation of Signal Parameters by Rotational Invariant Transforms (ESPRIT) exploits an underlying rotational invariance of signal subspaces spanned by two temporarily displaced data sets. ESPRIT correctly exploits the underlying data model of complex sinusoids in additive noise. The de-ramping operation which encodes the ranges to frequencies, results in complex sinusoids, corresponding to targets, in additive noise and their spectral resolution is proportional to the time aperture over which the samples are collected and the Signal to Noise Ratio (SNR). Conventional processing like the Fast Fourier Transform (FFT) is Raleigh limited and hence difficulty arises in resolving closely spaced targets.

Consider a signal consisting of d complex sinusoids with unknown amplitude and phase in additive noise

$$x(k) = \sum_{i=1}^{d} s_i \exp(jk\omega_i) + n(k) \tag{5}$$

where $\omega_i$ ε (-Π,Π) is the normalized frequency and $s_i$ is the complex amplitude of the $i^{th}$ complex sinusoid. It is assumed that n(k) is stationary zero mean complex white Guassian random process such that $E[\ n(j)\ n^H(k)] = \sigma^2\delta_{jk}$, where $\sigma^2$ is the variance of the additive noise and $(\ .\ )^H$ denotes the Hermition conjugate. To exploit the underlying deterministic nature of the complex sinusoids, m (>d) samples are chosen from the time series and define y = x(k+1)[1]

Then

$$\mathbf{x}(k) = [x(k)\ ,\ldots\ldots\ldots\ldots x(k+m-1)]^T, \tag{6}$$

$$\mathbf{n}(k) = [n(k)\ ,\ldots\ldots\ldots\ldots n(k+m-1)]^T, \tag{7}$$

$$\mathbf{y}(k) = [y(k)\ ,\ldots\ldots\ldots\ldots y(k+m-1)]^T = [x(k+1)\ ,\ldots\ldots\ldots\ldots x(k+m)]^T \tag{8}$$

Using (5) the time series can be written in matrix form as

$$\mathbf{x}(k) = \mathbf{A}\mathbf{s} + \mathbf{n}(k) \tag{9}$$

$$\mathbf{y}(k) = \mathbf{A}\phi\ \mathbf{s} + \mathbf{n}(k) \tag{10}$$

Where $\mathbf{s} = [s_1\ldots\ldots\ldots s_d]$ is d x 1 vector of complex amplitudes of the complex sinusoids, A is a n m x d Vandermonde matrix whose columns $\{\mathbf{a}(\omega_i)\ ; i = 1, 2 \ldots\ldots\ldots d\}$ are given by $\mathbf{a}_i = [1\ \ \exp(j\omega_i)\ldots\ldots\exp(j(m-1)\omega_i)\ ]$ and $\phi = \text{diag}[\ \exp(j\omega_1)\ldots\ldots\ldots\ \exp(j\omega_d)]$ is a d x d diagonal matrix containing the relative phase between adjacent time samples for each of the d complex sinusoids. $\phi$ is unitary matrix relating temporally displaced vectors **x** and **y** and is called rotation operator. The rotation operator has information about the frequencies of the sinusoids.

The auto covariance matrix of x is given by

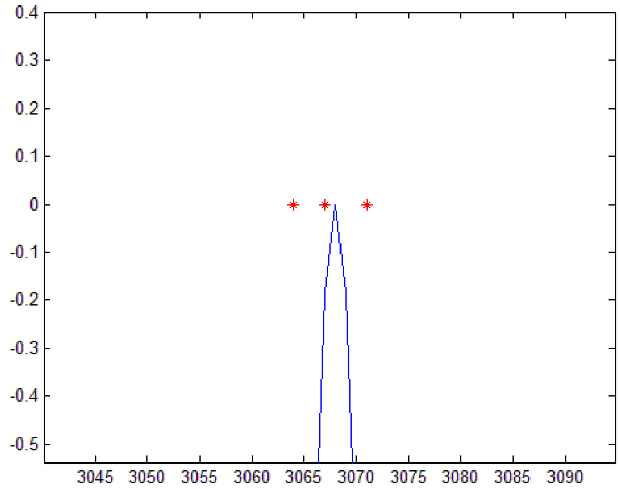
$$R_{xx} = E[\ \mathbf{x}(k)\ \mathbf{x}^H(k)] = AR_SA^H + \sigma^2 I \tag{11}$$

Where I is the identity matrix, $R_S$ is the source covariance matrix and $\sigma^2$ is the variance of the noise.

Exploiting the underlying data model ESPRIT estimates the frequencies of the sinusoids through solving an invariance equation relating the signal subspaces spanned by the vectors **x** and **y .** The signal and noise subspaces of the vectors are obtained by performing a Singular Value Decomposition (SVD) of the data vectors[2].The left Eigen vectors from SVD are partitioned into orthogonal signal and noise subspaces. The invariance equation is solved by the Total Least Square (TLS) method based on minimizing the Frobenius norm[6]. The spatial frequencies are then determined from the rotation operator. ESPRIT algorithm requires the number of complex sinusoids present, and the information can be derived form the model order estimation algorithms like Akaikes Information Criterion (AIC)[5] or minimum description length (MDL).

## 4. RESULTS AND OBSERVATIONS

The returned signal in the simulation model consists of the time overlapping echoes of Linear Frequency Modulated waveform corresponding to the three targets, which was de-ramped and fed to the signal processor. The ESPRIT algorithm was applied to this de-ramped output to determine the frequencies of the overlapping tones.. The model order, which corresponds, to the number of overlapping echoes was determined by Akaikes Information Criterion (AIC). A Total Least Square(TLS) based ESRIT algorithm was applied to resolve the tones. The ranges of the targets were determined by the conventional FFT processing and the proposed methods and is given in fig 2. It was seen that the TLS-ESPRIT outperforms the conventional processing in case of very close by targets and degrading SNR conditions.




**F**ig 2 : Red stars shows three targets resolved by the proposed method and blue corresponds to conventional FFT processing which cannot resolve closely spaced targets

**CONCLUSIONS**

It is concluded from the simulation studies that the super-resolution based de-ramp processing outperforms the conventional FFT based Spectral Analysis Techniques. The super-resolution based algorithms are also effective under low SNR conditions. The method can be effectively used in Synthetic Aperture Radar (SAR) radar processing where the scatterers are closely spaced on the ground.

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