Estimation of Directions of Arrival of Wideband and Wideband Spread Sources.

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Abstract

The demand of communicating at higher data rates can be easily met by using wideband signals [1], rendering the problem of estimating the direction of arrival (DOA) of wideband sources quite important. Multipaths, scatterers etc. present near the transmitter provide spatial bandwidth to these sources, which seriously degrade the performance of traditional DOA estimation techniques, designed assuming point sources. This paper, for the first time[1] talks about the DOA estimation of the sources having both spatial and temporal bandwidth. The interesting aspect of the approach presented is the usage of "spatial only" observations to estimate the DOA of both wideband sources and wideband spread sources.

1 Introduction

Higher data rates in communication can be achieved by using wideband signals [1]. Scatterers present near the transmitter change the point source into spatially spread source. DOA estimation of wideband sources, which also have a spatial bandwidth, thus, becomes an important issue which needs to be addressed. This paper presents an approach for estimation of DOA and other parameters of the resulting non-planar wavefront of wideband sources using a uniformly spaced linear array.

Some of the important contributions to DOA estimation of broadband sources, having a plane wavefront, include the works of Wax et al. [2], Buckley and Griffiths [3] and more recently Agrawal and Prasad [4] amongst others. In [4], Agrawal and Prasad proposed a spatial-only model for the array data, which leads to a practical algorithm for DOA estimation of wideband sources requiring only a 1-D search.

The problem of DOA estimation of wideband sources in its most general form, can also be regarded as a problem of estimation in 2-dimensions (2-D), viz., the spatial and the temporal frequency. This is somewhat analogous to the problem of DOA estimation of narrow band spatially spread sources[2], which is

[1] to the best of author's knowledge

[2] Spread sources are sources which exhibit an angular spread around their mean direction of arrival.

also a 2-D parameter estimation problem, i.e. the estimation of mean direction and the spread around the mean. Direction finding techniques for narrowband scattered source have been recently developed in [7] among others. Recently Agrawal et.al. [5] have used an AR-model for the modified (linearly modulated) array cross covariance lags to estimate the nominal directions and angular spread of such sources.

Motivated by these observations, in this paper, we first propose a simple yet robust technique that enables high-resolution estimation of DOA for wideband point sources observed on a uniform linear array (ULA). It is shown that the modified (linearly modulated) array cross covariance lag function can be modeled as an auto-regressive (AR) process. The roots of the polynomial corresponding to the AR model yield the desired estimates of DOA's of the wideband sources. The method is shown to yield accurate estimates with performance better than that of [4], although at the cost of increased array size requirements.

Next, we address the problem of DOA estimation of wideband spread sources for the first time. The methodology used for wideband sources is generalized to handle wideband spread sources.

2 Wideband Sources

As in [4], we model the broadband sources having an ideal bandpass power spectrum over a given bandwidth. The signal received from P wideband sources by the mth sensor of the linear array comprising of M sensors is given by,

$$y_m(t) = \sum_{p=1}^{P} \int_{f_l}^{f_h} e^{j2\pi f(t+\tau_m(\theta_p))} dS_p(f) + n_m(t), \tag{1}$$

where terms have their usual meaning. For a uniform linear array (ULA) $\tau_m(\theta_p) = m\frac{\Delta sin(\theta_p)}{c}$. Assuming sources to be uncorrelated, and making use of the assumption of an ideal bandpass power spectrum for each source, distributed uniformly over the entire frequency support define $\mathbf{r} = [\mathbf{R}(2,1),\ \mathbf{R}(3,1), \cdots, \mathbf{R}(M,1)]^T$ a vector consisting of the cross-covariance terms. For a uniform linear array (ULA), the linearly modified cross-correlation $[(r)]_{(m)} = m[\mathbf{R}]_{(m,1)}$ can be written as sum of $2P$ as,

$$[\tilde{\mathbf{r}}]_{(m)} = \textstyle\sum_{p=1}^{P} \frac{\rho_p}{j(f_h - f_l)\eta_p} \ \ (e^{jmf_h\eta_p} - e^{jmf_l\eta_p}) \tag{2}$$

Therefore, it satisfies the Yule-Walker equation corresponding to an AR model of order $2P$

$$\sum_{n=0}^{2P} \beta_n \tilde{\mathbf{r}}_{(m+n)} = 0 \qquad m = 1, \cdots, M - 2P - 1 \tag{3}$$

where $\{\beta_n\}$ are the coefficients of the annihilating polynomial $\beta(z) = \beta_0 + \beta_1 z^1 + \cdots + \beta_{2P} z^{2P}$, whose roots are $\{e^{j\eta_p f_h},\ e^{j\eta_p f_l}\}$. The polynomial $\beta(z)$ can be estimated by the forward-backward predictor method [6][3]. Therefore it enables us to estimate the DOA's of wideband sources using simple polynomial rooting.

[3]Several other computationally efficient algorithms based upon $\mathbf{UDU}^H$ decomposition, Levsion-Durbain method etc. can also be used here.

3 Wideband Spread Sources

Next consider the case of wideband sources having an angular spread. The angular spread makes the propagating wavefront non-planar and results in a time-varing perturbation at each sensor. It complicates the signal model, thus necessitating special treatment[7]. Mathematically the signal received at the mth sensor of the M sensor array is given by,

$$y_m(t) = \sum_{p=0}^{P} \int_{f_l}^{f_h} \sum_{i=0}^{\infty} b_{mpi}(t) e^{\left[j2\pi f\left(t+\tau_m(\theta_{pi})\right)\right]} dS_p(f) \tag{4}$$

where $b_{mpi}(t)$ is the perturbation at the mth sensor corresponding to the pth source due to ith multipath. This perturbation results in the source having a spatial spread and hence it can no longer be treated as a point source. For the signals having uniform bandpass power spectrum and also uniform spread of δ_p around θ_p [5] it can be shown that the modified cross-corelation function $([(r)]_{(m)} = m^2[\mathbf{R}]_{(m,1)})$ is consists of four modes. These four modes intern provide the estimates of DOA, can be modeled as an AR process and can be estimated by polynomial rooting as in previous section.

4 Simulation Studies

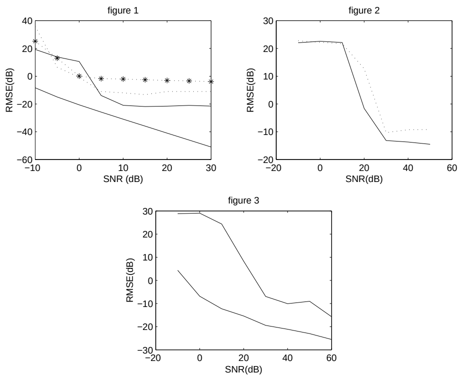
In this section, we give the results of the simulation experiments. The simulations reported here are around a uniform linear array of 12 sensors. The spectral support of the signal is taken to be the normalized frequency interval [.75, 1]. The array covariance matrix is estimated using 100 independent snapshots ($N = 100$). The root mean square error (RMSE) has been used as the performance. All performance curves given below are obtained by averaging the results over 200 independent trials ($K = 200$).

Figure 1 shows the RMSE and the asymptotic CRB [8] for the case of a single source placed at 30^0 as a function of SNR. The suggested method always performs better than [4] and [3]. Figure 2 shows the RMSE in the estimation of DOA as a function of SNR for a source at 30^0 having non-flat (triangular) spectrum. Here the threshold is more as compared to Figure 1, where the source is assumed to have a flat spectrum. But the method works reasonably well at high SNR's.

Next we study the case of a wideband spread source present at 30^0 having an angular spread of 5^0. Figure 4 shows the RMSE in the estimates of DOA and CRB w.r.t. SNR. Again the threshold increases and the RMSE is also larger as compared to the scenario of a point source. This can be simply explained since P wideband spread sources require $4P$ quantities, whereas, P point wideband sources require the estimation of only $2P$ parameters. But still at high SNR we could estimate the DOA.

5 Conclusions

In this paper we have suggested algorithms to obtain the estimates of the DOA's of the broadband and the broadband spread sources. In both cases, the modified covariance lags are shown to satisfy the AR model. The modes obtained



by rooting the AR polynomial give the estimates of the direction. Numerical simulations show that the method is quite robust.

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