NONLINEAR DIFFRACTION. FOCUSING OF SECONDARY WAVES

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INTRODUCTION

An excitation of combination frequency waves in nonlinear medium was investigated in [1-3]. It has been theoretically found that neither availability of dispersion nor availability of anisotropy is a necessary condition for effective nonlinear diffraction. But there are restrictions on a size and a form of the homogeneous medium or on the spatial distribution of the nonlinearity of inhomogeneous medium. It was determined that the propagation direction of the secondary wave is controllable and can be varied within wide range.

It was established, that regardless of the shapes of primary waves phase surfaces, there are so-called focusing surfaces. If the medium is nonlinear only on one of focusing surface the secondary waves are in-phase at a certain point (focus). Theory and application of focusing effect are the objectives of this paper.

THEORY OF FOCUSING EFFECT

Let us suppose that N monochromatic primary waves are superimposed on nonlinear medium, which is localized at the surface S. The frequencies of primary waves are $\omega_1, \omega_2, \ldots, \omega_N$. The fields $E_1, E_2, \ldots, E_N$ of primary waves are functions of time t and coordinates $\mathbf{r}$:

$$E_1(t,\mathbf{r}) = E_1(\mathbf{r}) \exp[\omega_1 t - \varphi_1(\mathbf{r}) - \varphi_{1,0}], \tag{1}$$

$$E_2(t,\mathbf{r}) = E_2(\mathbf{r}) \exp[\omega_2 t - \varphi_2(\mathbf{r}) - \varphi_{2,0}], \tag{2}$$

....

$$E_N(t,\mathbf{r}) = E_N(\mathbf{r}) \exp[\omega_N t - \varphi_N(\mathbf{r}) - \varphi_{N,0}], \tag{3}$$

where the functions $\varphi_1(\mathbf{r})$, $\varphi_2(\mathbf{r}), \ldots, \varphi_N(\mathbf{r})$ describe the shapes of phase surfaces of primary waves; $\varphi_{1,0}, \varphi_{2,0}, \ldots, \varphi_{N,0}$ are the initial phases of primary waves.

Each elementary surface dS is a source of elementary spherical secondary waves at combination frequencies. Let us consider only one of combination frequencies, $\Omega = \nu_1\omega_1 + \nu_2\omega_2 + \ldots \nu_N\omega_N$, where $\nu_1, \nu_2, \ldots, \nu_N$ are integer (some of them can be negative).

The complex amplitude of corresponding elementary secondary spherical wave is

$$dE_{0,\Omega} = \mathbf{P}_\Omega(\mathbf{r}) \exp[\nu_1\varphi_1(\mathbf{r}) + \nu_2\varphi_2(\mathbf{r}) + \ldots \nu_N\varphi_N(\mathbf{r}) + (\varphi_{1,0} + \varphi_{2,0} + \ldots \varphi_{N,0})]\, dS, \tag{4}$$

where complex function $\mathbf{P}_\Omega(\mathbf{r})$ depends on nonlinear properties of medium and on parameters of primary waves.

The field of elementary secondary wave in point $\mathbf{r}'$ has complex amplitude $dE(\mathbf{r},\mathbf{r}')$:

$$dE(\mathbf{r},\mathbf{r}') = dE_{0,\Omega}\, \frac{\exp\left(-jK|\mathbf{r}-\mathbf{r}'|\right)}{|\mathbf{r}-\mathbf{r}'|}, \tag{5}$$

where $K = \frac{\Omega}{c}$, c is velocity of secondary wave.

The phase $\varphi(\mathbf{r}')$ of elementary secondary wave in point $\mathbf{r}'$ is:

$$\varphi(\mathbf{r}') = \nu_1\varphi_1(\mathbf{r}) + \nu_2\varphi_2(\mathbf{r}) + \ldots \nu_N\varphi_N(\mathbf{r}) + (\varphi_{1,0} + \varphi_{2,0} + \ldots. \varphi_{N,0}) + K|\mathbf{r}-\mathbf{r}'|. \tag{6}$$

So, the equation

$$\nu_1\varphi_1(\mathbf{r}) + \nu_2\varphi_2(\mathbf{r}) + \ldots \nu_N\varphi_N(\mathbf{r}) + K|\mathbf{r}-\mathbf{r}'| = \mathrm{const} = \Phi \tag{7}$$

specifies the focusing surface, corresponding to focus point $\mathbf{r}'$.

If the right side of (7) is determined without ambiguity, then the focusing surface is specified unambiguously. But all the surfaces, corresponding to different values of right side of (7), are focusing surfaces with the same focus point $\mathbf{r}'$.

Any point belongs to one of focusing surface. The exceptions are the points of location of primary waves sources.
Each focusing surface corresponds to the certain phase of secondary waves in focus point. If the nonlinear medium is localized in volume V, situated between focusing surface, corresponding to the phase ψ_1, and focusing surface, corresponding to ψ_2, all elementary secondary waves in focus point have phases in the range from ψ_1 to ψ_2. If ψ_1 and ψ_2 differ reasonably little, all elementary secondary waves are nearly in-phase in focus point. In this sense the volume V is a focusing volume. The position of the focus is invariant with respect to a medium evolution within a focusing volume.
If one of the primary waves changes the direction of propagation, the position of the focus will also change. In this sense the focusing surface or focusing volume serves as a lens for any of the primary waves.
The shape of focusing surface depends on the parameters of the primary waves. By way of example let us consider the special case of focusing effect, when N=2, ω_2=0.2ω_1, ν_1=1, ν_2= -1, the first primary wave is plane and the second one is spherical.
Each black or white region in Fig. 1 is a focusing volume. The first primary wave is propagating from left to right. The point *A* is the phase center of second primary wave. The point *F* is focus. Each boundary between the focusing volumes is the focusing surface. One of the focusing surfaces differs from the other focusing surfaces in shape and in dimensions of surrounding focusing volumes. This surface can be called the main focusing surface.
Focus can be an infinitely distant point. For focusing of secondary waves on an infinity it is necessary that both primary waves are spherical, with not coincided centers of curvature of phase surfaces.
The focusing surfaces and focusing volumes in case of focusing on infinity are shown in Fig. 2. The point A_1 is the phase center of the first primary wave. The point A_2 is the phase center of the second primary wave. Here N=2, ω_2=0.99ω_1, ν_1=1, ν_2= -1. The main focusing surface in case of focusing on infinity is a sphere. The center of this sphere coincides with the phase center of the second secondary wave (point A_2 in Fig.2). The radius R_0 of focusing sphere is determined by the following expression:

$$R_0 = \frac{d\left(\omega_1 + \omega_2\right)}{\omega_1 - \omega_2}, \tag{8}$$

where d is a distance between phase centers of primary waves.
Conditions of secondary wave focusing on infinity are conditions under which a combination frequency secondary wave has minimum divergence.
Surfaces of equal phases of elementary sources of secondary waves in case of focusing on infinity are shown in Fig. 3 for any fixed time t. The positions of these surfaces vary with time. Surfaces move from left to right. For the points on axis, the phase velocity is equal to velocity of primary waves. For the other points the phase velocity differ little from

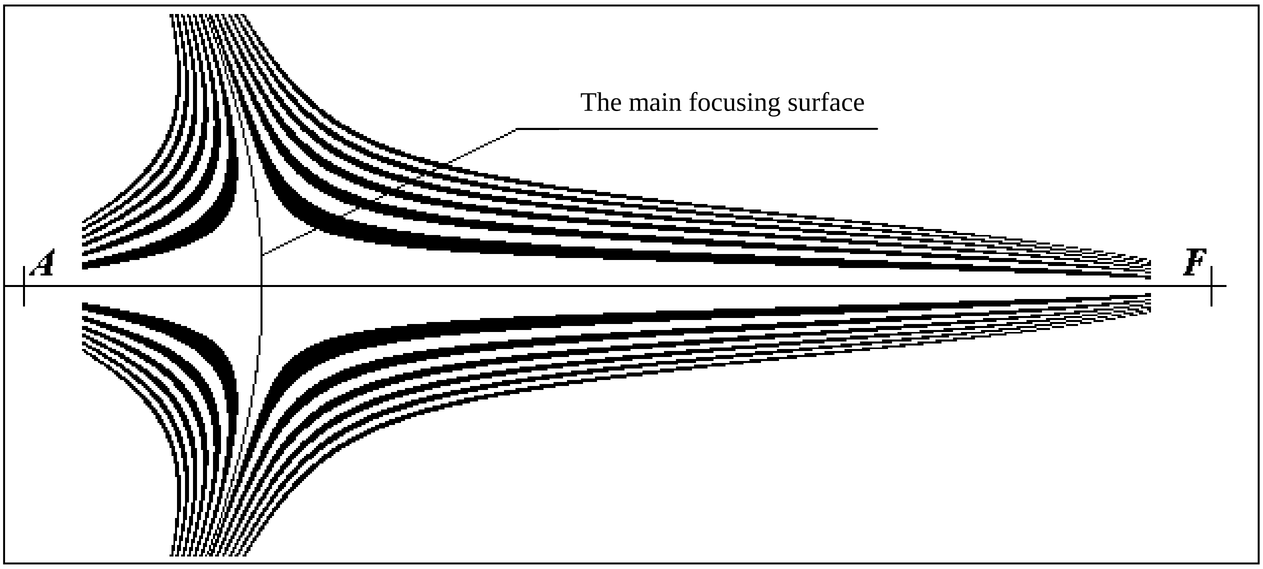

Fig.1. Focusing volumes and focusing surfaces in case of two primary waves. The first primary wave is plane.
It is propagating from left to right. The point *A* is the phase center of second primary wave. The point *F* is focus.

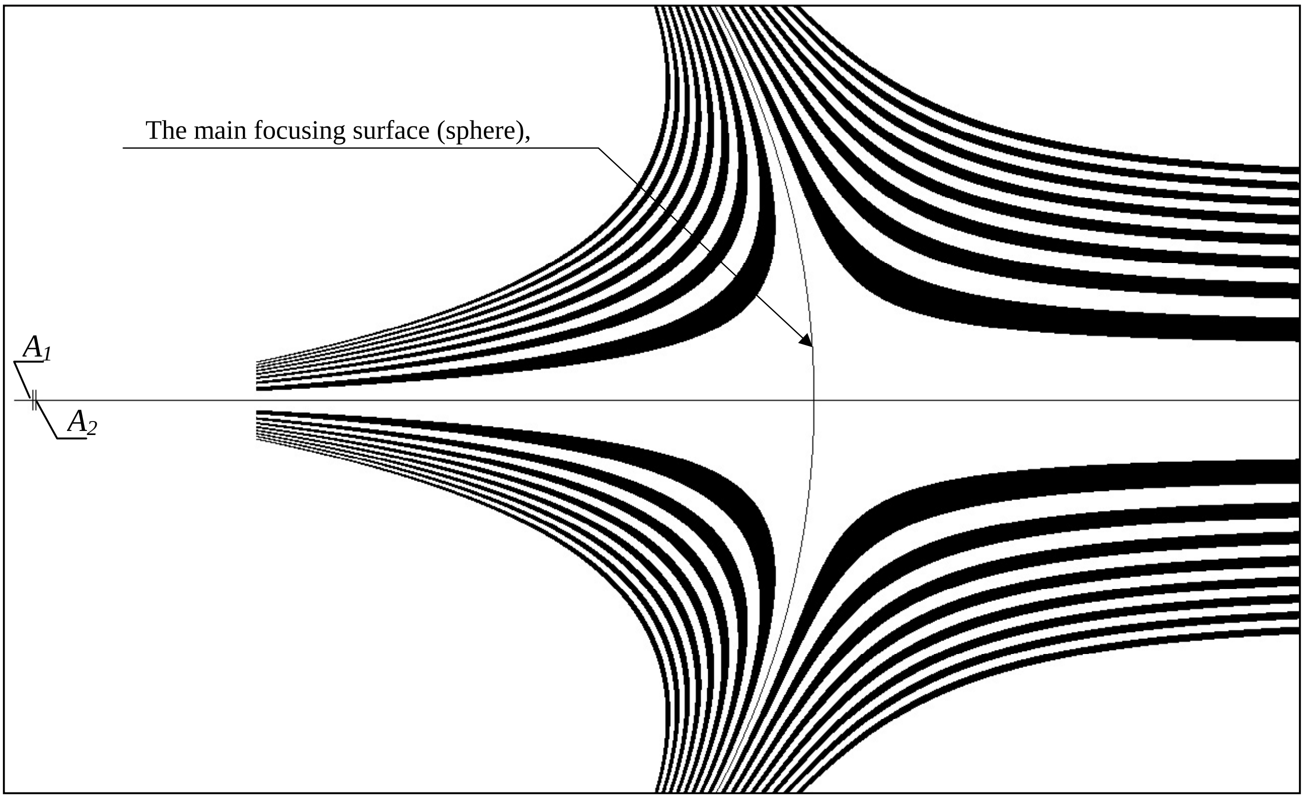

Fig. 2. Focusing volumes and focusing surfaces in case of focusing on infinity. Two primary waves are spherical. The point A_1 is the phase center of the first primary wave. The point A_2 is the phase center of the second primary wave.

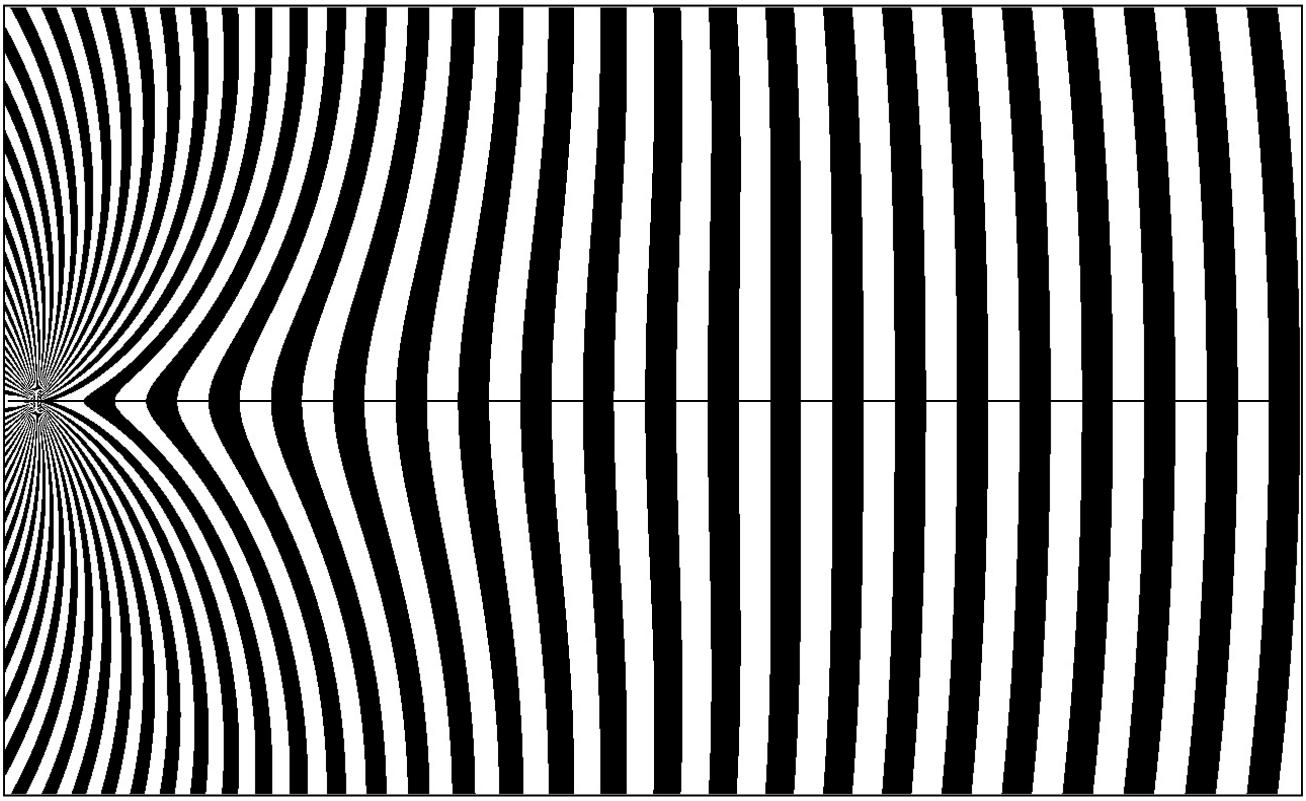

Fig. 3. Surfaces of equal phases of elementary sources of secondary waves in case of focusing on infinity.

the velocity of primary waves. There is a vast region, in which the phase surfaces are near the plane. If the primary waves are electromagnetic ones, then phases of elementary sources of secondary waves in this region are in close proximity to the phase of the plane electromagnetic wave.

APPLICATION OF FOCUSING EFFECT

The focusing effect can be used for phasing of arbitrary located wave exciters or receivers. A case in point is the nonlinear-diffraction phasing of non-rigid transmitting or receiving antenna arrays [4-6]. The focusing on infinity can be used for phasing of transmitting array. Each module of transmitting array contains an auxiliary antenna for primary waves, the converter and the main antenna for secondary wave. The focusing volume is a region of permissible displacements of the modules.

It is reasonable to use two point sources, located at the backside of the array as sources of primary monochromatic spherical waves. The source, having a lower frequency, must be dislocated relatively to the source with a higher frequency by the distance d, determined by (8), in the direction, which coincides with the required direction of the beam.

It is established, that allowable displacements of the modules can attain hundreds of wavelengths for modules, located near the axis, and a few wavelengths for peripheral modules. So, a substantial reduction of the mass of a large-aperture array is possible, due to reduced masses of the structural elements, responsible for the array rigidity (or even due to the elimination of such elements). For instance, the array may be designed as a net with modules in its knots. In addition, the array's weight is reduced due to absence of phase-shifters in it.

The proposed phasing technique is free from coordinate measurements for the modules.

The phasing of non-rigid receiving antenna arrays is discussed in [4,5].

In addition proposed methods may be useful in designing of conformal antenna arrays and antenna arrays with irregular locations of the modules.

The phasing of arbitrary located acoustic sources (receivers) is another example of focusing effect application.

CONCLUSIONS

1. The secondary waves of combination frequencies can be focused by focusing surfaces or by focusing volumes.
2. The focusing surfaces and focusing volumes are specified ambiguously.
3. One of the focusing surfaces differs from the other focusing surfaces in shape and in dimensions of surrounding focusing volumes.
4. The position of the focus depends on parameters of primary waves.
5. Focus can be an infinitely distant point.
6. The focusing effect can be used for nonlinear-diffraction phasing of arbitrary located wave exciters or receivers.
7. The most important application of nonlinear diffraction phasing is the phasing of non-rigid antenna arrays.

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