ON ARTIFICIAL MAGNETO-DIELECTRIC SUBSTRATES WITH MICROSTRIP ANTENNAS: THE ROLE OF FREQUENCY DISPERSION

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ABSTRACT

In the present paper we discuss the effect of artificial magneto-dielectric substrates on the impedance bandwidth properties of microstrip antennas. A realistic dispersive behavior of a practically realizable substrate is embedded into the model, and we show that frequency dispersion of the substrate plays a very important role in the impedance bandwidth characteristics of the loaded antenna. The impedance bandwidths of miniaturized size patch antennas loaded with dispersive magneto-dielectric substrates and high-permittivity substrates are compared. It is shown that practically realizable substrates with dispersive magnetic permeability are not advantageous in antenna miniaturization. This observation is experimentally validated.

INTRODUCTION

During the first heyday of metamaterials, artificial high-permeability materials working at microwave regime have gained increasing attention [1, 2]. The possibility to create artificial magnetism at microwave frequencies has heated the discussion on the possibility to enhance the impedance bandwidth properties of microstrip antennas using magneto-dielectric substrates [3, 4, 5]. According to the work of Hansen and Burke [6], inductive (magnetic) loading leads to an efficient size miniaturization of a microstrip antenna. A transmission-line (TL) model for a normal half-wavelength patch antenna predicts that increase in the permeability of the antenna substrate does not reduce the impedance bandwidth of the miniaturized radiator (when the material parameters are dispersion-free, and \( \mu_{\text{eff}} \gg \epsilon_{\text{eff}}, \mu_{\text{eff}} \gg 1 \)) [6]. However, at microwave frequencies only moderate permeabilities can be achieved with a complex mixture of electrically small inhomogeneities (resonating unit cells). Moreover, to achieve a paramagnetic response, one has to operate rather close to the resonance of the artificial magneto-dielectric substrate. Thus, it is clear that the substrate obeys strong frequency dispersion. To reveal the benefits achieved in practice with artificial magneto-dielectric substrates it is necessary to consider realistic values for the substrate material parameters, and take into account frequency dispersion.

IMPEDANCE BANDWIDTH BEHAVIOR WITH A DISPERSIVE SUBSTRATE

In this section we use a TL-model and calculate the impedance bandwidth properties of a half-wavelength patch antenna loaded with a dispersive artificial magneto-dielectric substrate, and with a dispersion-free reference dielectric substrate. The empty antenna is designed to resonate at 3.0 GHz. A schematic illustration of the analyzed antenna structure and the equivalent TL-model are presented in Fig. 1 (detailed derivation for the model and the antenna dimensions are presented in [7]).

The practically realizable magneto-dielectric substrate is implemented as an array of metasolenoids [2]. The estimated dispersive behavior of \( \mu_{\text{eff}} \) of the substrate is shown in Fig. 2(a). The effective permittivity of the substrate is estimated
to be $\epsilon_{\text{eff}} = 8.5(1 - j0.001)$. The value for the relative permittivity of the reference dielectric substrate (offering the same size reduction) is $\epsilon_{\text{ref}} = 10.1(1 - j0.001)$. To better see the possible effect of frequency dispersion, we will also consider a loading scenario in which the dispersive magneto-dielectric substrate is replaced with a substrate having dispersion-free material parameters $\mu_{\text{eff}} = 1.21(1 - j0.0024)$ (picked up from the dispersion curve at the operational frequency of the loaded antenna) and $\epsilon_{\text{eff}} = 8.5(1 - j0.001)$.

Fig. 2(b) shows the calculated reflection coefficient with different material fillings. The main calculated parameters are gathered in Table 1 ($V$ is the volume of the radiators, $BW$ is the bandwidth, and $Q_0$ is the unloaded quality factor). The obtained result indicates that practically realizable magneto-dielectric substrate offers no advantages over high-permittivity dielectrics (when it comes to impedance bandwidth). If the realistic dispersive behavior of the magneto-dielectric substrate is replaced with a scalar constant permeability, the TL-model predicts wider impedance bandwidth with magneto-dielectrics than with pure high-permittivity dielectrics.

**RELATIVE RADIATION QUALITY FACTOR**

Next an expression explicitly explaining the negative effect of frequency dispersion will be derived. We consider a $\lambda/2$ long section of a transmission line filled with a certain material having material parameters $\mu, \epsilon$. Moreover, we assume the line to be terminated by ideal open ends, and that the losses are only due to radiation losses. Taking into account the frequency dispersion in the material parameters one obtains the radiation quality factor of the line in the following form:

$$Q_r = \frac{\pi Y}{8G_r} \left( \frac{1}{\mu} \frac{\partial(\omega \mu)}{\partial \omega} + \frac{1}{\epsilon} \frac{\partial(\omega \epsilon)}{\partial \omega} \right).$$

(1)

Above $Y$ is the characteristic admittance of the line, and $G_r$ is the radiation conductance. For an antenna (line) having the same dimensions and loaded with a reference dispersion-free magneto-dielectric material we have

$$Q_r = \frac{\pi Y_{\text{ref}}}{4G_r},$$

(2)

Figure 2: $\mu_{\text{eff}}$ for the substrate and the calculated reflection coefficient.

Table 1: The calculated impedance bandwidth characteristics. Practically realizable example.

<table>
<thead>
<tr>
<th>Loading</th>
<th>$V$ (cm$^3$)</th>
<th>$BW_{-6dB}$ (percent)</th>
<th>$Q_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air filling</td>
<td>9.4</td>
<td>12.3</td>
<td>10.9</td>
</tr>
<tr>
<td>Magneto-dielectric</td>
<td>1.5</td>
<td>1.4</td>
<td>93.7</td>
</tr>
<tr>
<td>Reference dielectric</td>
<td>1.5</td>
<td>1.9</td>
<td>69.7</td>
</tr>
<tr>
<td>Dispersion-free $\mu$</td>
<td>1.5</td>
<td>2.5</td>
<td>54.2</td>
</tr>
</tbody>
</table>
where $Y_{\text{ref}}$ is the characteristic impedance of the reference antenna. The following holds for the ratio between the radiation quality factors:

$$\frac{Q_r}{Q_{r\text{ref}}} = \frac{1}{2\mu} \sqrt{\epsilon\mu_{\text{ref}}} \left( \frac{1}{\mu} \frac{\partial (\omega \mu)}{\partial \omega} + \frac{1}{\epsilon} \frac{\partial (\omega \epsilon)}{\partial \omega} \right).$$

(3)

Since the two antennas resonate at the same frequency we have

$$\mu \epsilon = \mu_{\text{ref}} \epsilon_{\text{ref}}.$$  

(4)

If we consider the reference material to be pure dielectric ($\mu_{\text{ref}} = 1$), (3) simplifies to

$$\frac{Q_r}{Q_{r\text{ref}}} = \frac{1}{2\mu} \left( \frac{1}{\mu} \frac{\partial (\omega \mu)}{\partial \omega} + \frac{1}{\epsilon} \frac{\partial (\omega \epsilon)}{\partial \omega} \right).$$

(5)

We will assume that $\epsilon \approx \text{const}$ (this is a good approximation with the metasolenoid array [3]). In this case we obtain

$$\frac{Q_r}{Q_{r\text{ref}}} = \frac{1}{2\mu} \left( \frac{1}{\mu} \frac{\partial (\omega \mu)}{\partial \omega} + 1 \right).$$

(6)

Further, we assume the dispersion in $\mu$ to be covered by the general (lossless) Lorentzian type dispersion rule

$$\mu = 1 + \frac{A\omega^2}{\omega_0^2 - \omega^2},$$

(7)

where $A$ is the amplitude factor ($0 < A < 1$) and $\omega_0$ is the undamped frequency of the zeroth pole pair.

The relative quality factor (given by (6)) is presented with different amplitude coefficients in Fig. 3 ($f_0 = 3.3$ GHz). We can observe that if the static value for $\text{Re}\{\mu_{\text{eff}}\} = 1$, substrate with Lorentzian type dispersion in $\mu_{\text{eff}}$ leads always to larger $Q_r$ than pure dielectrics offering the same size reduction (except in the limiting case with $A = 1$). This result is in line with the TL-model results presented earlier.

**EXPERIMENTAL VERIFICATION**

The estimated dispersive behavior of the manufactured metasolenoid array and a photograph of the prototype antenna are shown in Fig. 4(a). At the operational frequency of the loaded antenna ($F = 2.07$ GHz) we estimate that $\text{Re}\{\mu_{\text{eff}}\} = 1.25$ and $\text{Re}\{\epsilon_{\text{eff}}\} = 8.5$ (the host substrate for the metasolenoids is Rogers R/T Duroid 5870). The relative permittivity for the reference dielectric leading to the same size reduction is $\epsilon_{\text{ref}} = 10.8(1 - j0.0037)$.

Fig. 4(b) and Table 2 show the measured reflection coefficient and gather the main measured parameters. The radiation efficiency $\eta_{\text{rad}}$ has been measured using the Wheeler cap method. We can see that the impedance bandwidth behaves according to the analysis presented in the paper: Practically realizable magneto-dielectric substrate (with

![Figure 3](image-url)  
Figure 3: The relative quality factor. Static $\text{Re}\{\mu_{\text{eff}}\} = 1$.  

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*Note: The raw text contains mathematical expressions and figures that are not rendered properly in the text format. The rendered text is provided for clarity.*
Lorentzian type dispersion rule) does not improve the impedance bandwidth in antenna miniaturization compared to pure dielectrics.

CONCLUSION

Impedance bandwidth properties of a half-wavelength patch antenna loaded with an artificial magneto-dielectric substrate have been studied using a TL-model. It has been shown that with substrates obeying the Lorentzian type dispersion for $\mu_{\text{eff}}$, frequency dispersion cannot be neglected in the analysis. A relation has been derived for the ratio between radiation quality factors of ideally shaped antennas loaded with dispersive magneto-dielectrics and dispersion-free reference dielectrics. The result shows that dispersive magneto-dielectrics lead always to larger radiation quality factor (Lorentzian type dispersive behavior) if static $\text{Re}\left\{\mu_{\text{eff}}\right\} = 1$. The main observation on the negative effect of frequency dispersion on the impedance bandwidth properties has been experimentally validated.

REFERENCES