Nonlinear transmission and spatiotemporal solitons in left-handed metamaterials

Ilya V. Shadrivov[1], Nina A. Zharova[1,2], Alexander A. Zharov[1,3], and Yuri S. Kivshar[1]
[1]*Nonlinear Physics Centre, Research School of Physical Sciences and Engineering,*
Australian National University, Canberra, ACT 0200, AUSTRALIA
E-mail: ivs124@rsphysse.anu.edu.au
[2]*Institute of Applied Physics, Russian Academy of Sciences,*
Nizhny Novgorod 603600, Russia
[3]*Institute for Physics of Microstructures, Russian Academy of Sciences,*
GSP-105, Nizhny Novgorod 603950, Russia

We study, by means of the finite-difference time-domain numerical simulations, wave scattering by a slab of a nonlinear left-handed material. We observe switching of the transmission properties of the slab from totally reflecting to transparent by varying the electromagnetic field intensity. This effect becomes possible due to a hysteresis-type dependence of nonlinear magnetic permeability of metamaterial on the field intensity. We also predict the existence of self-localized nonlinear beams, spatial solitons, which appear as single- and multi-hump beams and can exist due to self-induced domains of negative magnetic permeability.

A number of theoretical studies [1-4] and experimental results [5-7] demonstrated the existence of a new type of microstructured composite materials, which can be characterized, for some parameters, by a negative real part of the magnetic permeability and a negative real part of the dielectric permittivity in the microwave frequency range. These materials are often referred to as left-handed metamaterials (LHMs), double-negative materials, or materials with negative refraction. Properties of the left-handed materials were analyzed theoretically by Veselago long time ago [8], but only very recently such materials were demonstrated experimentally, as the composite structures created by arrays of metallic wires and split-ring resonators.

It has already been noticed that the left-handed metamaterials may possess quite complicated nonlinear magnetic response [9, 10], their properties can be altered by inserting diodes into the split-ring resonators [11], and nonlinear metamaterials can demonstrate interesting features of bistability [12] and the second-harmonic generation [13]. Importantly, the microscopic electric field in such composite structures can become much higher than the macroscopic electric field carried by the propagating electromagnetic wave. This provides a simple physical mechanism for enhancing nonlinear effects in the resonant structure with the left-handed properties. Moreover, any future effort in creating tunable structures, where the field intensity change the transmission of a composite structure, would require the study of nonlinear properties of such metamaterials, which are expected to be quite unusual.

We study numerically the wave transmission through a slab of the left-handed metamaterial assuming that it possesses a hysteresis nonlinear response [9]. We make a step forward in comparison with the recent predictions in Ref. [9], and simulate numerically, with the help of the finite-difference time-domain simulations, a nonlinear microstructured material. When the slab possesses an intensity-dependent nonlinear response due to nonlinear dielectric inclusions in split-ring resonators, we observe the nonlinearity-induced transmission of the slab for larger input powers even the slab is opaque and totally reflecting for low-amplitude wave scattering. In addition, we observe that the spatiotemporal dynamics in the case of the overcritical transmission can be characterized by the generation and propagation of spatiotemporal solitons. We present the results for one- and two-dimensional geometries. We consider a finite slab of a composite structure consisting of a cubic lattice of the periodic arrays of conducting wires and split-ring resonators (SRRs). We assume that the unit-cell size d of the structure is much smaller then the wavelength of the propagating electromagnetic field and, for simplicity, we choose the single-ring geometry of a lattice of cylindrical SRRs. The results obtained for this case are qualitatively similar to those obtained in the more involved cases of double SRRs. This type of microstructured left-handed materials has recently been demonstrated experimentally [5].

We study the scattering of the electromagnetic waves by a slab of the metamaterial assuming that the metamaterial possesses a nonlinear response. We use the finite-difference time-domain (FDTD) numerical simulations which allow the most complete analysis of spatiotemporal effects in the wave scattering. To describe the nonlinearity response of the metamaterial, we employ the effective averaged Maxwell equations in the standard form

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t},$$

$$\nabla \times \mathbf{B} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c}\mathbf{j} + 4\pi\nabla \times \mathbf{M}, \tag{1}$$

where **j** is the current density averaged over the period of the unit cell of the cubic lattice, and M is the magnetization of the metamaterial. We base our numerical simulations on the microscopic model of a nonlinear metamaterial that generalizes the linear model recently introduced in Ref. [14].

One-dimensional scattering

To study the spatiotemporal dynamics of the wave scattering by a slab of a nonlinear metamaterial in the framework of the model introduced above, first we consider a simpler one-dimensional problem that describes the interaction of the plane wave incident at the normal angle from air on a finite slab of the metamaterial. We consider two types of nonlinear effects: (i) nonlinearity-induced suppression of the wave transmission when initially transparent left-handed material becomes opaque with the growth of the input wave amplitude, and (ii) nonlinearity-induced transparency of the slab when an initially opaque composite material becomes left-handed (and, therefore, transparent) with the growth of the input wave amplitude. In our simulations, we assume that the amplitude of the incident wave grows linearly for the first 50 periods, and then it becomes constant. The slab thickness is $1.3 \lambda_0$, where λ_0 is a free-space wavelength. For the selected parameters, the metamaterial is left-handed in the linear regime for the frequency range from $f_1 = 5.787$ GHz to $f_2 = 6.05$ GHz.

Our FDTD numerical simulations show that for the incident wave with the frequency $f_0 = 5.9$ GHz (i.e. inside the left-handed transmission band), the electromagnetic field reaches a steady state independently of the sign of nonlinearity. Both reflection and transmission coefficients in the stationary regime are shown in Figs. 1 as functions of the incident field amplitude, for defocusing and focusing types of nonlinearity. Here and in the rest of the paper, the incident field intensity is normalized to the squared characteristic field, $E_c^2 = U_c^2 / d_g^2$, where U_c is the characteristic nonlinear voltage and d_g is the slit of the SRR resonator.
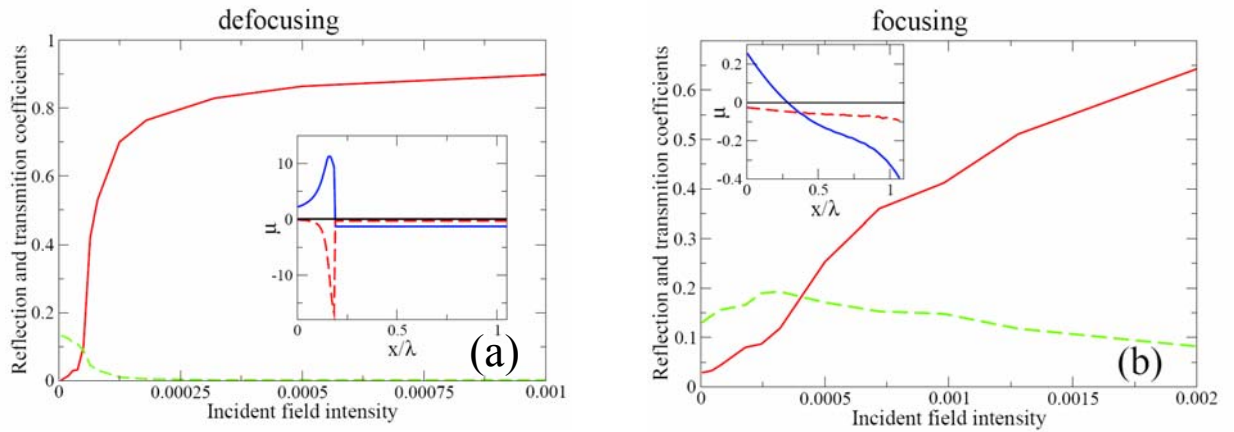

Fig. 1. Reflection (solid red) and transmission (dashed green) coefficients for a slab of nonlinear metamaterial vs. the normalized incident field intensity in a stationary regime, for the case of (a) defocusing nonlinearity, and (b) focusing nonlinearity. Insets show real (solid blue) and imaginary (dashed red) parts of the magnetic permeability inside the slab of a composite material.

In the linear regime, the effective parameters of the metamaterial at the frequency f_0 are: $\varepsilon = -1.33 - 0.01i$ and $\mu = -1.27 - 0.3i$; this allows the impedance matching with surrounding air, so that the reflection coefficient vanishes for small intensities, as shown in Fig. 1(a). Reflection and transmission coefficients differ qualitatively for two types of nonlinearity. For defocusing nonlinearity, the reflection coefficient varies from low to high values when the incident field exceeds some threshold value, see Fig. 1(a). Such a sharp transition can be explained in terms of the hysteresis behavior of the magnetic permeability described by Eq. (8) and discussed in Ref. [9]. When the field amplitude in the metamaterial becomes higher than a certain critical amplitude, magnetic permeability changes its sign, and the metamaterial becomes opaque. Our FDTD simulations show that for the overcritical amplitudes of the incident field, the opaque region of positive magnetic permeability appears inside the slab [see the inset in Fig. 1(a)], and the magnetic permeability experiences an abrupt change at the boundary between the transparent and opaque regions. The dependencies shown in Fig. 1(a) are obtained for the case when the incident field grows from zero to a steady-state value. However, taking different temporal behavior of the incident wave, e.g. increasing the amplitude above the threshold value and then decreasing it to a steady state, one can get different values of the stationary reflection and transmission coefficients, and different distributions of the magnetic permeability inside the metamaterial slab. Such properties of the nonlinear metamaterial slab are consistent with the multi-valued dependence of the magnetic permeability [9].

In the case of focusing nonlinearity [see Fig. 1(b)], the dependence of the reflection and transmission coefficients on the amplitude of the incident field is smooth. First of all, this effect originates from a gradual detuning from the impedance matching condition, and, second, from the appearance of an opaque layer with a positive value of the magnetic permeability for higher powers that is a continuous function of the coordinate inside the slab, as shown in the inset of Fig. 1(b). Now we consider the other type of nonlinear effects mentioned above when an initially opaque composite metamaterial becomes transparent with the growth of the incident field amplitude. We take the frequency of the incident field to be f_0 = 5.67 GHz, so that the magnetic permeability is positive in the linear regime and, correspondingly, the metamaterial is opaque for the incoming waves. In the case of the self-focusing nonlinear response (α=+1), the material properties can be "switched" to the regime with the negative magnetic permeability when the slab becomes left-handed and, therefore, transparent.
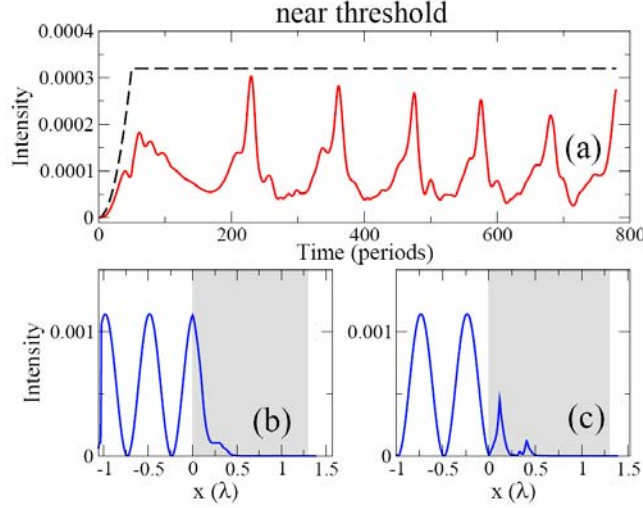

Fig. 2. (a) Temporal evolution of the reflected (solid) and incident (dashed) wave intensity in the strongly nonlinear regime (i.e., for the overcritical amplitude of the incident wave). (b,c) Spatial distribution of the magnetic and electric fields, respectively, at the end of simulation domain; the metamaterial is shaded.

In a strongly nonlinear, overcritical regime, we observe the effect of the dynamical self-modulation of the reflected electromagnetic wave that results from the periodic generation of the self-localized states inside the metamaterial, as oscillating localized states near the interface [see Fig. 2]. Somewhat related effect of the formation of self-focused localized states inside a nonlinear material was previously discussed for the problem of interaction of the intense electromagnetic waves with over-dense plasma [15,16]. Such localized states can be termed as spatiotemporal electromagnetic solitons [17], and they can transfer the energy away from the interface. Figure 3(c, right) shows an example when two localized states enter the metamaterial. These localized states appear at the jumps of the magnetic permeability and, as a result, we observe a change of the sign of the derivative of the electric field at the maximum of the soliton intensity, and subsequent appearance of transparent regions in the metamaterial. Unlike all previous cases, the field structure in this regime does not reach any steady state for high intensities of the incident field.

Two-dimensional scattering

Now we consider the two-dimensional beam scattering by a slab of the nonlinear metamaterial, and present the FDTD results for the nonlinearity-induced transparency of the metamaterial. We launch a TM-polarized beam of the width $2\lambda_0$ at the angle 45° from the left towards the surface of the metamaterial slab of the thickness 0.9 λ_0. Figure 3 (top) shows the snapshot of the magnetic field distribution at the nonlinear stage (simulation time t = 381T, T is the field oscillation period) of simulations. Modification of the metamaterial parameters in the high intensity area results in the formation of the non-stationary spatiotemporal soliton inside the slab [see Fig. 3(top)], which makes possible for the electromagnetic energy to penetrate through the slab. The dynamics of the soliton formation is qualitatively similar to that in the overcritical one-dimensional case discussed above. Figure 3 (bottom) shows the formation of the transparent left-handed domain (shown by black) inside the metamaterial slab, induced by the electromagnetic field. One can see that an initially opaque slab becomes transparent [see black area inside the slab in Fig. 3(bottom)] for the high enough field intensities. The shift of the transparent domain to the left indicates the negative refraction of the beam in the left-handed slab.
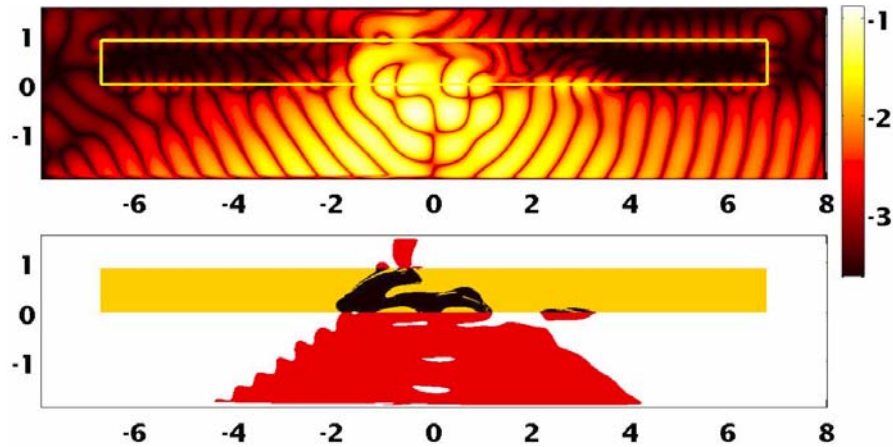
Fig. 3. Magnetic field distribution (in logarithmic scale) for the beam scattering by a metamaterial slab in high-intensity regime (top). Bottom – plot shows the transparent left-handed domain (black) formed in initially opaque metamaterial (yellow) by the beam incident at 45 degrees. Red color indicates the high field areas outside the slab. Coordinates are normalized on the free-space wavelength.

Conclusions

We have demonstrated novel effects associated with the nonlinear response of metamaterials. Using the FDTD numerical simulations, we have studied the spatiotemporal dynamics of the wave scattering by a slab of a nonlinear metamaterial and observed two types of nonlinear effects associated with a change of the metamaterial properties: (i) nonlinearity-induced suppression of the wave transmission, and (ii) the nonlinearity-induced transmission and the generation of spatiotemporal electromagnetic solitons.

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