LATTICE OF DIELECTRIC PARTICLES WITH DOUBLE NEGATIVE RESPONSE

I. A. Kolmakov[1,2], **L. Jylhä**[3], **S. A. Tretyakov**[1], **S. Maslovski** [1],

[1]Radio Lab. / SMARAD, Helsinki University of Technology, P.O. 3000, FI-02015 TKK, Finland,
E-mail: igor.kolmakov@tkk.fi, sergei.tretyakov@tkk.fi

[2]Microwave Microelectronics Lab., Electrotechnical University, 5 Prof. Popov St., 197376 St. Petersburg, Russia,
E-mail: MWLab@eltech.ru

[3]Electromagnetics Lab., Helsinki University of Technology, P.O. 3000, FI-02015 TKK, Finland,
E-mail: liisi.jylha@tkk.fi

ABSTRACT

We study numerically and analytically isotropic arrays of spheres and two-dimensional lattices of cylinders that exhibit double negative response. The analytical expression for the effective permittivity and permeability of the mixture of spheres is improved by taking into account interaction between two sublattices. Numerical simulations are done in Agilent HFSS 5.4. Design parameters for future experiments are presented.

INTRODUCTION

During the last few years, metamaterials with negative real parts of the effective permittivity and permeability, sometimes called *double negative (DNG) media or Veselago media,* have been intensively studied. Some important practical applications, for example in antenna techniques, require isotropic effective permittivity and permeability. Unfortunately, most of the known designs are strongly anisotropic. This work presents a step towards realization of artificial isotropic DNG materials. An isotropic DNG material with spherical magneto-dielectric particles was theoretically investigated in [1]. Lewin's model of the effective permittivity and permeability of mixtures of spherical inclusions in an isotropic host material [2] was used. Due to the symmetry of particles, such materials have isotropic electromagnetic properties at the frequencies lower than the first lattice resonance. A DNG medium consisting of dielectric spherical particles with different radii was proposed in [3]. Two sublattices with large and small spheres act as lattices of electric and magnetic dipoles, respectively.

Different particles with electric and magnetic response can be used for the design of DNG structures. Magnetic activity in dielectric cylinders was shown in [4] and discussed in [1]. We use the same approach [3] to design a two-dimensional DNG medium based on an array of dielectric cylinders. The classical solution of scattering of plane electromagnetic waves by a cylinder was used for the derivation of the effective permittivity and permeability of the medium [8].

MIXING THEORY

Material parameters for a mixture consisting of two sublattices of spheres can be calculated analytically, if the lattice constant is much smaller than the wavelength. The both sets of spheres are made of the same nonmagnetic dielectric material and also the background is assumed to be nonmagnetic. The effective permittivity ε_{eff} and the permeability μ_{eff} for a material with two types of dipoles having two different electric polarizabilities can be calculated from the generalized Claussius Mossotti relation [5]:

$$\frac{\varepsilon_{eff}-\varepsilon_b}{\varepsilon_{eff}+2\varepsilon_b}=\frac{n_e\alpha_e}{3\varepsilon_b}+\frac{n_m\alpha_e}{3\varepsilon_b}, \qquad \frac{\mu_{eff}-1}{\mu_{eff}+2}=\frac{n_m\beta_m}{3\mu_0}, \tag{1}$$

where ε_b and μ_0 are the permittivity and the permeability of the host dielectric, n_m and n_e are the volume densities of spheres in the magnetic resonance and in the electric resonance, respectively, α_m and α_e are the electric polarizabilities of spheres in the magnetic resonance and in the electric resonance mode, and β_m is the magnetic polarizability of spheres in the magnetic resonance. In [1, 3] it was assumed that $\alpha_m = 0$. Neglecting α_m leads to an incorrect low frequency limit for the effective permittivity, because the remaining static electric polarizability of spheres in the magnetic resonance modes is not taken into account. Also the resonant frequency shifts slightly. The error depends on the electrical contrast between inclusions and the environment and it usually increases when the contrast increases.

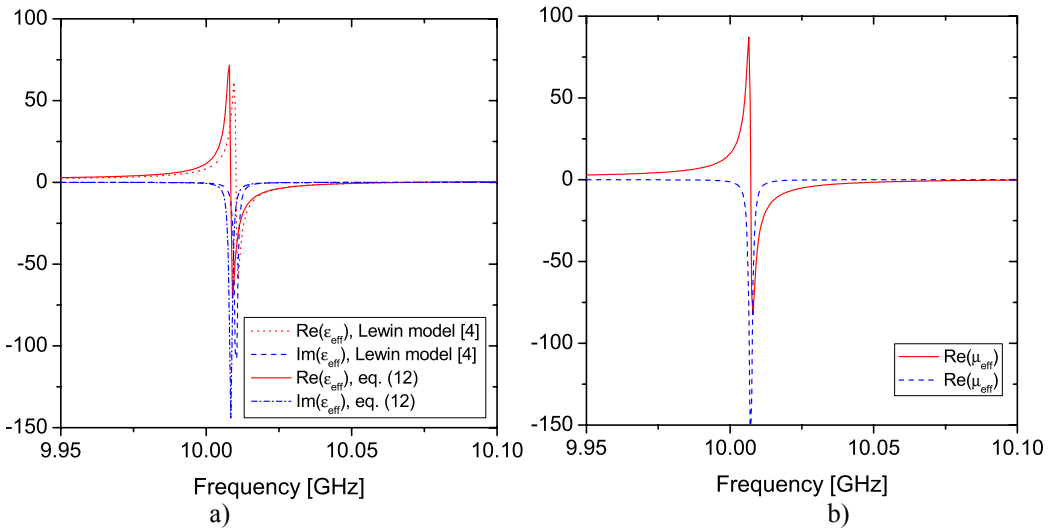


Fig. 1. The relative effective permittivity (a) and the relative effective permeability (b) of a material with two types of spheres as function of the frequency ($\varepsilon_i = 44(1-j1.25\cdot10^{-4})$, $\varepsilon_b = \varepsilon_0$, $r_e = 3.15$ mm, $r_m = 2.25$ mm, $f_e = 0.065$, $f_m = 0.024$).

Frequency dependent polarizabilities α and β for a sphere are calculated using the Mie theory [2, 6]. Finally, effective medium models for a composite consisting of two set of resonating spheres read:

$$\frac{\varepsilon_{eff}-\varepsilon_b}{\varepsilon_{eff}+2\varepsilon_b}=\frac{f_e}{G(\Theta_e)}+\frac{f_m}{G(\Theta_m)}, \qquad G(\Theta)=\frac{\varepsilon_i F(\Theta)+2\varepsilon_b}{\varepsilon_i F(\Theta)-\varepsilon_b}, \tag{2}$$

$$\frac{\mu_{eff}-1}{\mu_{eff}+2}=f_m\frac{F(\Theta_m)-1}{F(\Theta_m)+2}, \qquad F(\Theta)=2\frac{sin\Theta-\Theta cos\Theta}{(\Theta^2-1)sin\Theta+\Theta cos\Theta}, \tag{3}$$

where ε_i is the permittivity of particles, f_e is the volume fractions of spheres in electric resonance, f_m is the volume fractions of spheres in magnetic resonance, $\Theta_e = k_0 r_e \sqrt{\varepsilon_i \mu_i}$ and $\Theta_m = k_0 r_m \sqrt{\varepsilon_i \mu_i}$, where r_m is the radius of spheres in the magnetic resonance mode and r_e is the radius of spheres in the electric resonance mode. Equations (2, 3) are similar to those given in [8, 9] if the second term in the right-hand side of the first equation in (2) is set to zero.

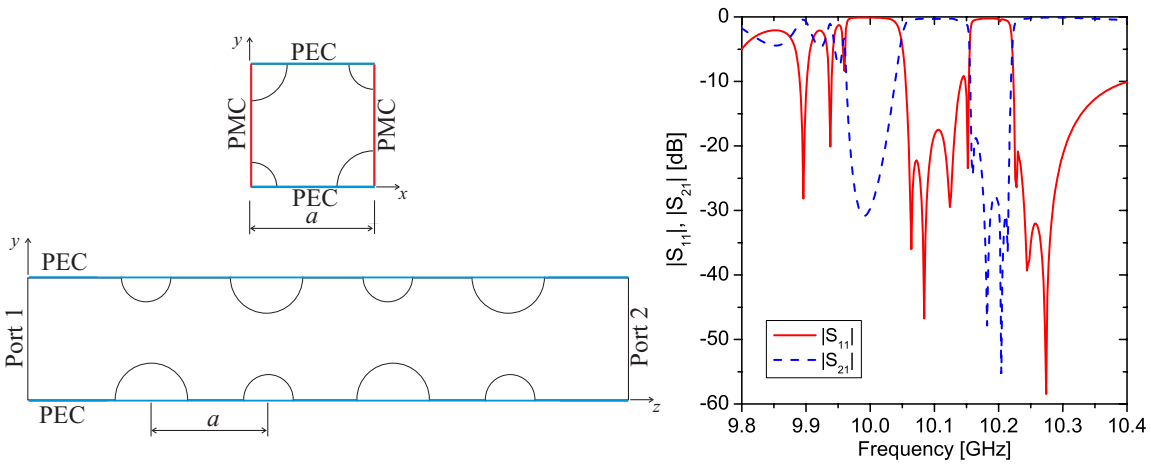
NUMERICAL SIMULATIONS

Array of Spheres

Numerical simulations were made using the finite element method based on Agilent HFSS 5.4 electromagnetic modeling software. Radii of spheres were corrected for keeping the resonant frequency near 10 GHz. An illustration of the calculation domain is shown in Fig. 2(a). PEC denotes that the electric field is normal to the surface and PMC that the electric field is tangential to the surface. In the simulations, mirror planes cut spheres into four parts. These boundary conditions do not disturb the first three resonant modes of the spheres, which have the same symmetry. The results for scattering matrix elements $|S_{11}|$ and $|S_{21}|$ are shown in Fig. 2(b). The effective medium model (3, 4) predicts, that this medium should have negative material parameters from 9.92 GHz to 9.98 GHz. According to convergence studies, numerical simulations overestimate the resonant frequency by about 1%. When the frequency shift caused by numerical error is taken into account, according to the mixing theory the material parameters in Fig. 2 (b) should be negative around 10.05 GHz. Fig. 2 (b) displays that there is a pass band around this frequency range. By studying the phase of the electric field, it was found that the wave is a backward wave around this frequency range. The wavelength was longer than the lattice constant, which indicates that the medium was rather an effective medium than an EBG structure.

Array of Cylinders

Array of dielectric rods with different radii is a 2D analogue of artificial media based on spherical particles. Radii of cylinders are calculated from the analytical model and optimized in HFSS. An array of cylinders with small and large



a) b)

Fig. 2 (a) A cross section of simulation domain. (b) Numerically calculated S-parameters for a slab consisting of four layers of spheres ($\varepsilon_i = 44(1-j1.25\cdot10^{-4})$, $\varepsilon_b = 1$, $r_e = 3.18$ mm, $r_m = 2.28$ mm, lattice constant $a = 10$ mm).

radii is arranged one by one along the direction of propagation. The layout of the structure is shown in Fig 3 (a). In the simulations the symmetry of structure was taking into account. Scattering parameters of a lattice with 7 cylinders are presented at Fig. 3(b). Desired pass band can be seen from 10.12 GHz to 10.22 GHz. Because the lattice constant is close to the half wavelength in free space, the effective medium approach is not suitable for extracting effective dielectric parameters. Interaction between particles is strong enough and resonant frequency is split. Number of resonant frequencies at the pass-band corresponds to the number of eigenwaves, which can be excited in the structure [7]. Wavelength of each eigenwave can be found from the field distribution at resonant frequencies. Set of wave numbers (β) and resonant frequencies for each mode satisfy the dispersion equation for an infinite array of cylinders [7]. Points at dispersion characteristic on Fig. 3(c) were calculated from the resonant frequencies of a mixture with different numbers of cylinders. Negative slope of dispersion characteristic is obtained, which demonstrates DNG properties of the designed medium.

CONCLUSION

In this study, a possibility to realize an isotropic material with negative real parts of both ε and μ using a mixture consisting of two sets of resonating dielectric spheres and cylinders with different radii was considered. A corrected effective medium model for the effective permittivity of a mixture of resonating dielectric spheres was presented. This improved formula takes into account interaction between spheres in the electric and magnetic resonance modes. If the interaction is neglected, the frequency band for negative permittivity is shifted. As a result there is a mismatch between negative values of the permittivity and permeability. This leads at least to greater losses in the DNG frequency band or in some cases this band may not exist at all. Results of electromagnetic simulations are close to the analytical results. The frequency band where the mixing theory predicts negative material parameters is in a good agreement with numerical studies. The material composed of dielectric spheres behaves as an effective medium at least in some part of the DNG frequency band. In contrast, an array of dielectric rods with the same permittivity and resonant frequency behaves more like an EBG around the frequency band where the material parameters are negative. It appears that the effective medium theory can be used to match the resonant frequencies so that there exists a DNG frequency band. However, it should be studied numerically if the material behaves like an effective medium or more like an EBG structure in this DNG frequency band.

Depending on practical application, dielectric spherical particles or dielectric rods can be chosen to design a DNG medium. Design of artificial dielectrics by using dielectric rods is simpler since stacked dielectric tablets can be used. Dielectric cylinders manufactured from high quality dielectrics are widely used as resonators in microwave filters.

ACKNOWLEDGEMENTS

This work has been coordinated and partially funded by the *Metamorphose* Network of Excellence. Financial support of the Academy of Finland and TEKES through the Center-of-Excellence program is acknowledged. Contribution of

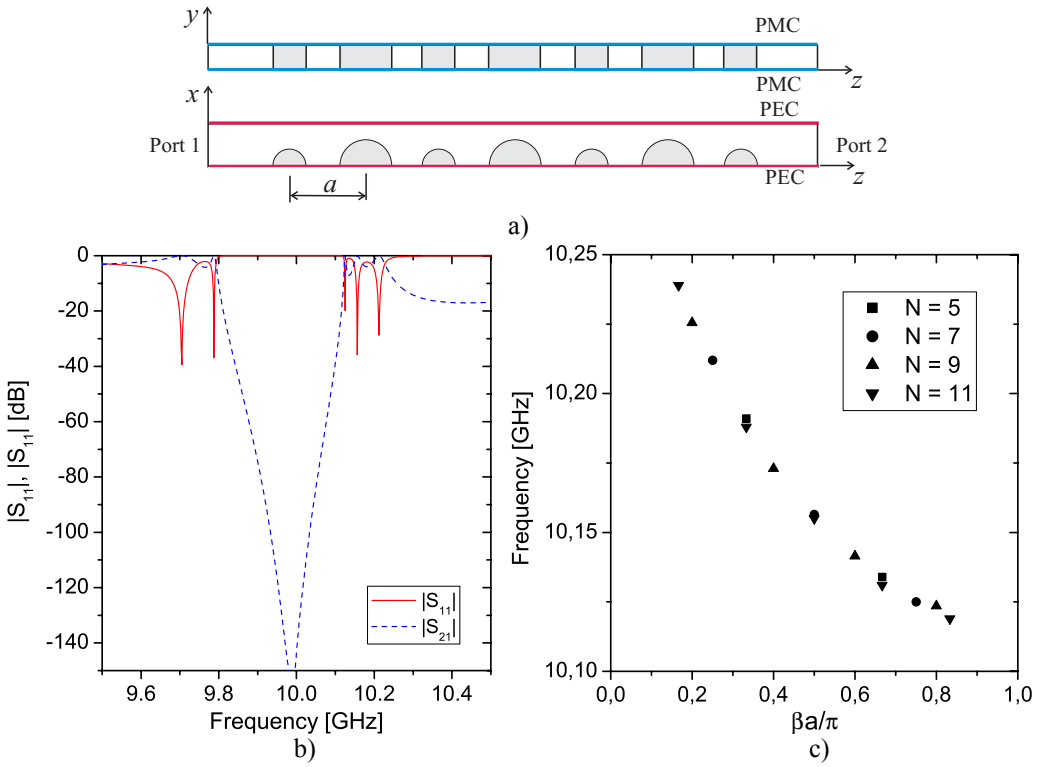


Fig. 3. 2 (a) Cross section of simulation domain. (b) Numerically calculated S-parameters for a lattice of cylinders ($\varepsilon_i = 44(1-j1.25\cdot10^{-4})$, $\varepsilon_b = 1$, $r_e = 2.71$ mm, $r_m = 1.73$ mm, lattice constant $a = 15$ mm). (c) Dispersion curves for a lattice of cylinders calculated for different numbers of cylinders (N)

I. Kolmakov has been partially funded by an INTAS Young Scientist Fellowship grant (Ref. Nr. 03-55-1879) and contribution of L. Jylhä by Finnish Academy of Science and Letters, Vilho, Yrjö and Kalle Väisälä Foundation.

References

[1] C.L. Holloway, E.F. Kuester, J. Baker-Jarvis, P. Kabos, "A double-negative (DNG) composite medium composed of Magnetodielectric spherical particles embedded in a matrix," *IEEE Trans. on Antennas and Propagation,* vol. 51, pp. 2596-2603, 2003.

[2] L. Lewin, "The electrical constants of a material loaded with spherical particles," *Proc. Inst. Elec. Eng.,* Part 3, 94:65-68, 1947.

[3] O.G. Vendik, M.S. Gashinova, "Artificial double negative (DNG) media composed by two different dielectric sphere lattices embedded in a dielectric matrix," *Proc. of 34th European Microwave Conference,* pp. 1209-1212, Amsterdam, The Netherlands, October 2004.

[4] S. O'Brien, J.B. Pendry, "Photonic band-gap effects and magnetic activity in dielectric composites," *J. of Phys.: Condens. Matter,* vol. 14, pp. 4035-4044, 2002.

[5] A. Sihvola, *Electromagnetic mixing formulas and applications*, IEE Electromagnetic Waves Series 47, The Institution of Electrical Engineers, 1999.

[6] C. F. Bohren and D. R. Huffman, *Absorption and Scattering of Light by Small Particles*, John Wiley and Sons, University of California, Berkeley, 1983.

[7] E. Shamonina, V. A. Kalinin, K. H. Ringhofer, L. Solymar, "Magnetoinductive waves in one, two and three dimensions," J. of Appl. Phys., vol. 92, pp. 6252-6261, 2003.

[8] N. A. Khizhniak, "Artificial anisotropic dielectrics formed from two-dimensional lattices of infinite bars and rods" (in Russian), Sov. Phys. Tech. Phys., Zh. Tekh. Fiz., vol. 29., pp. 604-614, 1959.