

## Pole-zero synthesis of metamaterial surfaces

A. Cucini, M. Nannetti, M. Caiazzo, F. Caminita and S. Maci  
University of Siena, Department of Information Engineering, Siena, 53100, Italy  
<http://www.dii.unisi.it>

In recent years, a great interest has been devoted to the study of the dispersion properties of planar structures realized by frequency selective surfaces (FSS) on grounded dielectric slab. This kind of structures are used to realize artificial magnetic conductors (AMC) [1], electromagnetic band-gap (EBG) surfaces, or surfaces which exhibit “soft” and/or “hard” equivalent boundary conditions [2]. It is well known that an approximate model of an FSS may be given in terms of a quasi-static LC impedance placed in a TE or TM equivalent transmission line. In absence of losses, the FSS impedance is purely reactive. Patch-type FSS are described by a series LC impedance. At frequencies below the resonant frequency, the FSS is capacitive. This capacitance resonates in parallel with the inductance provided by the section of short-circuited transmission line, thus, providing an AMC surface. In a certain frequency band, the same structure can provide an inhibition to the surface wave (SW) propagation, acting as an EBG surface. Similar considerations are valid for aperture-type FSS which can be described by a parallel LC circuit. This simple circuitual representation contains the essential physics able to qualitatively describe the basic concepts of the AMC properties. However, the lack of description of important aspects such as the wavenumber dependence of the equivalent circuit or the coupling between TE and TM polarizations, imposes a more rigorous generalization of the model.

Recently, a method has been introduced for the efficient synthesis of the FSS admittance matrix for the study of dispersion properties [3]. This method is based on the application of the Foster’s reactance theorem, which implies that FSS admittance functions of frequency satisfy the pole-zero analytical properties of the driving point LC admittance functions. The identification of the poles and zeros of the FSS equivalent admittance allows to reconstruct the surface response over a large frequency band. In this paper, the method is generalized in order to deal with a wide class of FSS-based artificial surfaces. A new derivation of the FSS admittance matrix is proposed and some general properties of this kind of structures are pointed out. The FSS admittance matrix will be drawn in the hypothesis that at the most only the dominant Floquet wave (FW) is propagating in free space. Otherwise, the formulation can be easily extended to an arbitrary number of ‘accessible’ modes. In particular a Method of Moment formulation is proposed for patch and aperture-type FSS and relationship between the FSS admittance matrix and the Method of Moment matrix is derived. On the basis of the MoM solution, a two-port admittance matrix is defined with the ports corresponding to the dominant TE and TM FW of the exact Floquet expansion. The admittance matrix is then characterized by the resonance of its eigenvalues for a few values of the wavenumber. The identification of a set of dispersion curves associated with the poles and zeros of the FSS and their regularity allows the interpolations of these curves by low-order polynomials. Furthermore, the eigenvalues of the FSS admittance matrix respect the Foster’s reactance theorem and thus the properties of lumped element LC-driving point functions of frequency. These properties allow the approximation of the eigenvalues in terms of rational functions. The consequent closed form expression is applied in order to formulate the dispersion equation for surface waves. The dispersion of a leaky wave mode, not treated here, is addressed in [17] and is still under investigation.

We would like to remark that the full-wave analysis for each wavenumber is very efficient, since it implies the inversion of a moderate size MoM matrix; however, obtaining accurate information on the continuous spectrum requires a large amount of computational time. The main peculiarity of the method presented here is the possibility of reconstructing the dispersion diagram in the continuous wavenumber domain over a large frequency range, starting from the response of the structure in a few points of the spectral domain. This procedure also opens very interesting possibilities for the Green’s function synthesis, which requires an integration of the continuous spectrum. Several numerical examples have been performed using different type of FSS-based metamaterial surfaces.

### References

- [1] F.-R. Yang et al., “A novel TEM waveguide using uniplanar compact photonic-bandgap (UC-PBG) structure,” *IEEE Trans. Microwave Theory Tech.*, Vol. 47, pp. 2092-2098, Nov.1999.
- [2] P.-S. Kildal, “Artificially soft and hard surfaces in electromagnetics,” *IEEE Trans. Antennas Propagat.*, Vol. 38, No. 10, pp. 1537-1544, Oct. 1990.
- [3] S. Maci, M. Caiazzo, A. Cucini, and M. Casaletti, “A pole-zero matching method for EBG surfaces composed of a dipole FSS printed on a grounded dielectric slab,” *IEEE Trans. Antennas Propagat.*, Vol. 53, No. 1, pp. 70-81, Jan. 2005.