TE MODE SPECTRA OF COAXIAL WAVEGUIDES WITH MULTIPLE LONGITUDINAL POLAR RIDGES

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INTRODUCTION

In the field of electron devices for generation of high power microwaves and millimetrewaves, coaxial gyrotron has become a topic of intense research and development activities for more than two decades. The resonating structure generally consists of a longitudinally tapered coaxial waveguide. Recently better mode control has been achieved by using tapered over-moded coaxial waveguide with longitudinally corrugated inner conductor. The ridge dimensions in these structures are the effective parameters for optimization [3,4,5].

The methods employed so far for the analysis of coaxial resonators with corrugated inner conductor have been surface-impedance approach and singular integral equation. Here, we have analyzed and computed the TE eigenvalue spectra of two “cold” structures using the Ritz-Galerkin technique. This method was used earlier by one of the authors [1] to determine the TE/TM mode spectra of quadruple-ridged circular waveguide. The structures considered here are coaxial waveguides with arbitrary number of identical polar ridges of arbitrary angular width, placed uniformly along the periphery of (a) the inner conductor (RIC) and (b) the outer conductor (ROC). The formulation is validated against coaxial data [2], obtained in the limit of vanishing ridge height and vanishing groove width.

TE MODE ANALYSIS OF MULTIPLE-RIDGED COAXIAL WAVEGUIDE

For both the configurations, shown in Fig. 1, the structural parameters are as follows.

\[ 2\Phi_0 = \text{Angular width of a ridge} \quad \Psi_0 = \text{Angular width of a groove} \quad N = \text{Number of ridges or grooves} \]

Multiple Polar Ridges on the Inner Conductor of the Coaxial Waveguide (RIC)

Cylindrical coordinate system \((\rho, \phi, z)\) has been used for the analysis. There are two regions as shown in Fig.1(a).

Region I is the annular region : \(t \leq \rho \leq a, 0 < \phi < 2\pi\). Region-II is the region inside the \(i\)-th groove, defined by \(a \leq \rho \leq b\) and \(\theta_{0i} \leq \phi \leq \theta_{0i} + \Psi_0\), where \(\theta_{0i} = (i-1)(\Phi_0 + \Psi_0) + \Phi_0 / 2\) for both ROC and RIC.

Let the Hertzian vector for TE-to-z modes be written as

\[
P_{\rho\phi} = \hat{z} g(\rho, \phi) \xi(z) \quad (1)
\]

Then

\[
\left(\nabla_i^2 + k_c^2\right) g(\rho, \phi) = 0 \quad (2)
\]

\[
\left(\frac{\partial^2}{\partial z^2} + \gamma^2\right) \xi(z) = 0 \quad (3)
\]

\[
\gamma = \left(k_o^2 - k_c^2\right)^{1/2} \quad \text{if} \quad k_o > k_c
\]

\[
= -j \left(k_c^2 - k_o^2\right)^{1/2} \quad \text{if} \quad k_o < k_c
\]

The basis fields are then given by the following expressions where we have used the notation \(r_{z} = \hat{\rho} \rho + \hat{\phi} \phi\).
\( e_r (r_i) = \nabla g (r_i) \times \hat{z} \)
\[
= \hat{\rho} \left( \frac{1}{\rho} \frac{\partial g}{\partial \phi} \right) + \hat{\phi} \left( -\frac{\partial g}{\partial \rho} \right)
\] (5)

\( h_r (r_i) = (\gamma / j \omega \mu_0) \hat{z} \times e_r (r_i) \) (6)

Considering the TE boundary conditions, we can write the following expressions for \( g (r_i) \) in two regions.

In region I,
\[
g_1 (r_i) = \sum_{n=0}^{\infty} \{ [J_n(k_c \rho)N'_n(k_c t) - N_n(k_c \rho)J'_n(k_c t)] \times (A_{1n} \cos n\phi + B_{1n} \sin n\phi) \}
\] (7)

In region-II inside i-th groove,
\[
g_2^{(i)} (r_i) = \sum_{p=0}^{N} \eta_2^{(i)} \left[ J_v(k_c a)N'_v(k_c b) - N_v(k_c a)J'_v(k_c b) \right] \cos \nu (\phi - \theta_{0i}), \quad \text{where} \quad \nu = \frac{p \pi}{\Psi_0}
\] (8)

Aperture field \( e_{g\phi} \) at the i-th aperture is denoted as \( E_{g\phi}^{(i)} (\phi) \) and it is expressed as,
\[
E_{g\phi}^{(i)} (\phi) = \sum_{l=0}^{L} C_l^{(i)} \cos \frac{l\pi}{\Psi_0} (\phi - \theta_{0i})
\] (9)

where \( C_l^{(i)} \) is the unknown expansion coefficient associated with the i-th basis function. Applying continuity of tangential electric fields over the i-th aperture we write \( e_{\phi,gt}^{(i)} = E_{g\phi}^{(i)} (\phi) \) over \( \theta_{0i} \leq \phi \leq \theta_{0i} + \Psi_0 \) at \( \rho = a \).

Multiplying both sides by \( \cos \frac{p \pi}{\Psi_0} (\phi - \theta_{0i}) \) and integrating over \( \theta_{0i} \) to \( \theta_{0i} + \Psi_0 \) with respect to \( \phi \), we obtain
\[
\eta_2^{(i)} = -k_c^{-1} \left[ J'_v(k_c a)N'_v(k_c b) - N'_v(k_c a)J'_v(k_c b) \right]^{-1} C_p^{(i)}
\] (10)

Next, we match \( e_{\phi,gt} \) and \( E_{g\phi}^{(i)} (\phi), (i=0 \text{ to } N) \), over the boundary \( \rho = a \). In writing \( e_{\phi,gt}^{(i)} = \sum_{i=1}^{N} E_{g\phi}^{(i)} (\phi) \), we note that the left hand side vanishes over the ridge surfaces at the matching boundary. Then multiplying both sides by \( h_m \left( \sin m\phi, \cos m\phi \right) \) and integrating over \( \phi = 0 \) to \( 2\pi \), we obtain
\[
A_{im} = -[k_c \psi_m (2\pi)J_m(k_c a)N'_m(k_c t) - N_m(k_c a)J'_m(k_c t)]^{-1} \times \sum_{j=1}^{N} \sum_{l=0}^{L} C_l^{(i)} P_{lm}^{(i)}
\] (11)
\[
B_{im} = -[k_c \psi_m (2\pi)J_m(k_c a)N'_m(k_c t) - N_m(k_c a)J'_m(k_c t)]^{-1} \times \sum_{j=1}^{N} \sum_{l=0}^{L} C_l^{(i)} Q_{lm}^{(i)}
\] (12)

where, \( P_{lm}^{(i)} = \int_{\theta_{0j}}^{\theta_{0j} + \Psi_0} \cos \frac{l\pi}{\Psi_0} (\phi - \theta_{0j}) \cos m\phi d\phi \); \( Q_{lm}^{(i)} = \int_{\theta_{0j}}^{\theta_{0j} + \Psi_0} \cos \frac{l\pi}{\Psi_0} (\phi - \theta_{0j}) \sin m\phi d\phi \) (13)

Finally we apply the continuity of \( g_1 \) and \( g_2^{(i)} \) at \( \rho = a \) over i-th aperture and write
\[\sum_{m=0}^{\infty} \{[J_m(k_\alpha)N_m'(k_\beta) - N_m(k_\alpha)J'_m(k_\beta)] \times (A_{tm} \cos m\phi + B_{tm} \sin m\phi)\} = \sum_{\mu=0}^{\infty} \eta_{2p}^{(j)} [J_v(k_\alpha)N'_v(k_\beta) - N_v(k_\alpha)J'_v(k_\beta)] \cos \frac{p\pi}{\Psi_0} (\phi - \theta_{0i}) \]  

(14)

Substituting for the expansion coefficients from (10)-(12) and taking inner product with \( \cos \frac{q\pi}{\Psi_0} (\phi - \theta_{0i}) \), \( q=0, 1, \ldots, Q \), the above equation can be written as

\[\sum_{j=1}^{N} \sum_{l=0}^{L} C_l^{(j)} \left\{ \sum_{m=0}^{\infty} F_m^{-1}(k_\alpha, k_\beta) \left[ (\varepsilon_m (2\pi))^{-1} P_{lm}^{(j)} P_{qm}^{(i)} + (\pi)^{-1} Q_{lm}^{(j)} Q_{qm}^{(i)} \right] \delta_{lj} \delta_{qi} \psi_{\Psi_0} \left[ Z_{\mu}^b(k_\alpha, k_\beta) \right]^{-1} \right\} = 0 \]  

(15)

where, \( F_m(x, y) = x \frac{J_m(x)N_m'(y) - N_m(x)J'_m(y)}{J_m'(x)N_m'(y) - N_m'(x)J'_m(y)} \) and \( Z_{\mu}^b(x, y) = x \frac{J_{\mu}^v(x)N_{\mu}'(y) - N_{\mu}^v(x)J_{\mu}'(y)}{J_{\mu}'(x)N_{\mu}'(y) - N_{\mu}'(x)J_{\mu}'(y)} \)  

(16)

In matrix form, (15) can be written as \([H]C = 0\), where \(H\) is a \([(Q+1)N] \times [(Q+1)N]\) matrix and

\[C = \begin{bmatrix} C_0^{(1)} & C_1^{(1)} & \cdots & C_Q^{(1)} & \cdots & C_0^{(2)} & C_1^{(2)} & \cdots & C_Q^{(2)} & \cdots & C_0^{(N)} & C_1^{(N)} & \cdots & C_Q^{(N)} \end{bmatrix}^T\]

For non-trivial solutions, we set the determinant of \([H]\) to zero, the roots of which yield the TE eigenvalues.

**Multiple Polar Ridges on the Outer Conductor of the Coaxial Waveguide**

The structure is shown in Fig.1(b). Following the same procedure as described above, the following equation is obtained.

\[\sum_{j=1}^{N} \sum_{l=0}^{L} C_l^{(j)} \left\{ \sum_{m=0}^{\infty} F_m^{-1}(k_\alpha, k_\beta) \left[ (\varepsilon_m (2\pi))^{-1} P_{lm}^{(j)} P_{qm}^{(i)} + (\pi)^{-1} Q_{lm}^{(j)} Q_{qm}^{(i)} \right] \delta_{lj} \delta_{qi} \psi_{\Psi_0} \left[ Z_{\mu}^h(k_\alpha, k_\beta) \right]^{-1} \right\} \]

This equation also can be written in matrix form and the TE eigenvalues are obtained as above.

**NUMERICAL RESULTS AND DISCUSSIONS**

On the basis of the theoretical analysis, a MATLAB code has been written. Eigenvalues have been computed for two, three, four, and five number of ridges of different widths along the periphery of the inner / outer conductor. Eigenvalues are plotted against ridge height, normalized with respect to the radius of outer conductor. When two ridges are introduced on the inner / outer conductor, all azimuthally dependent coaxial modes show mode splitting. Azimuthally symmetric TE modes do not show such feature. When three or four ridges are introduced, only TE \(31\) and TE \(21\) modes of coaxial waveguide give rise to two branches. Some of the eigenvalue spectra are shown in Fig.2.

**CONCLUSION**

In conclusion, in this paper we have analyzed and computed the TE eigenvalue spectra of two structures ROC and RIC using Ritz-Galerkin technique, as boundary-value problems with the aim to ultimately generate data that would be useful in R&D activities. From the eigenvalue spectra it is observed that the most interesting effect of the introduction of ridges on the modal eigenvalues is the splitting of coaxial modes. Only the azimuthally dependent modes display this splitting, while the circularly symmetric modes do not show this characteristic.

**REFERENCE**


Fig. 1. Coaxial waveguides with multiple polar ridges: (a) Ridges on Inner Conductor (RIC), (b) Ridges on Outer Conductor (ROC)

Fig. 2. Eigenvalue spectra for the structure (a) RIC with three ridges (b) RIC with four ridges (c) ROC with three ridges (d) ROC with four ridges