FINITE-DIFFERENCE FREQUENCY-DOMAIN ANALYSIS OF NOVEL PHOTONIC WAVEGUIDES

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ABSTRACT

This paper concerns the study of novel waveguide structures in photonics, in particular, photonic crystal fibers (PCFs) and photonic wire waveguides. One common feature among these guides is that they are not perfect waveguiding structures in that they possess leakage or confinement losses in the propagation. We present efficient and accurate analysis of these waveguides using a finite-difference frequency-domain (FDFD) method with perfectly matched layer (PML) absorbing boundary conditions. We formulate the eigenvalue problem for waveguide mode solutions using the finite difference method but based on the Yee mesh. To obtain high-accuracy results, proper matching of interface conditions near the dielectric interface is performed.

INTRODUCTION

We present an efficient and accurate method for studying two classes of photonic waveguides which are of much current interest: the photonic crystal fibers (PCFs) [1] and the photonic wire waveguides [2]. Both of them possess leakage or confinement losses in the propagation due to their waveguiding structures being not perfect. Their propagation constants are thus complex and accurate determination of these complex values is essential in designing and applying these waveguides. The method we employ is our recently developed finite-difference frequency-domain (FDFD) method with perfectly matched layer (PML) absorbing boundary conditions [3]. The formulation of the eigenvalue problem for waveguide mode solutions using the finite difference method is relatively simple compared to the popularly employed finite element methods and others. In our method, the finite difference scheme is based on the Yee mesh, which is the well-known mesh used in the finite-difference time-domain (FDTD) simulation [4]. Most importantly, the PMLs are employed to surround the domain of computation so that the confinement losses of leaky waveguides can be calculated. The FDFD eigenvalue problem formulation is derived directly from Maxwell's equations. It is attractive in that the obtained mode fields can easily be incorporated into the FDTD computation. To obtain high-accuracy results, proper matching of interface conditions near the dielectric interface through the Taylor series expansion of the fields is performed [3].

PCFs are one successful application of the photonic crystal concept and have been intensively studied in the past few years. The technology of fabricating PCFs has also advanced rapidly. The family of PCFs include general holey fibers composed of array of air holes running down the fiber length [1], the photonic-band-gap (PBG) fibers making use of the PBG effect in the cladding [5], and the air-core fibers in which the light is guided in air region surrounded by a PBG cladding [6]. We are able to efficiently determine the guided modes and their confinement losses of these PCFs using the FDFD method with accuracy comparable to other more sophisticated methods such as the multipole method and the finite element method [3]. In this paper, we will show the analysis results of PBG fibers with different numbers of air-hole rings and coupling characteristics of two-core PCFs.

Photonic wire waveguides, or photonic wires, are high index contrast optical waveguides with core region of sub-micron cross-sectional sizes. Currently, silicon-on-insulator (SOI) structures are most often considered, in which the silicon core is situated on a SiO_2 layer. Such structure is promising for achieving future high-density integrated photonic circuits. Light can be strongly confined within the core region due to the high index contrast and low-radiation-loss bends with radius in the micron range are possible. However, sidewall surface roughness due to the fabrication process in such sub-micron structures becomes significant and may cause unacceptably high propagation losses. Many efforts have been made in reducing such sidewall roughness. For example, propagation loss as low as 2.4 dB/cm has recently been achieved based on fabrication with deep ultraviolet lithography and dry etching [2]. In addition, there is intrinsic substrate leakage loss because the substrate below the SiO_2 layer and the core have the same refractive index since they are both made of silicon. Therefore, the propagating modes are leaky ones with complex propagation constant even in the absence of surface roughness scattering loss. According to the calculation conducted by Dumon *et al*. [2] based on an eigenmode expansion solver with PML boundary conditions, 1–1.5 dB/cm of the 2.4 dB/cm loss was due to substrate leakage. In this paper, we analyze the same structure and calculate the leakage loss using our FDFD method. Waveguides with different values of core width and SiO_2 layer thickness are studied. Our calculation predicts smaller leakage losses compared with the 1–1.5 dB/cm value. Our results have been carefully checked with those calculated using a mode solver based on the finite-element imaginary-distance beam propagation method (FE-ID-BPM) with PMLs [7] and very good agreement has been obtained.

THE FDFD METHOD

The cross-section of a general waveguide problem we consider is shown in Fig. 1 with the computational window surrounded by PML regions I, II, and III, each having thickness of d, with x and y being the transverse directions and z being the direction of propagation. Regions I and II have the normal vectors parallel to the x-axis and y-axis, respectively, and regions III are the four corner regions. In the PML region, using the stretched coordinate transform [8], Maxwell's equations can be written as

$$\begin{aligned} \nabla' \times \overline{E} &= -j\omega\mu_0\overline{H} \\ \nabla' \times \overline{H} &= j\omega\varepsilon_0\varepsilon_r\overline{E} \end{aligned} \tag{1}$$

where ω is the angular frequency, μ_0 and ε_0 are the permittivity and permeability of free space, respectively, ε_r is the relative permittivity of the medium considered, and the modified differential operator ∇' is defined as

$$\nabla' = \hat{x}\frac{1}{s_x}\frac{\partial}{\partial x} + \hat{y}\frac{1}{s_y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}. \tag{2}$$

In (2), $s_x = s$ and $s_y = 1$ in region I, $s_x = 1$ and $s_y = s$ in region II, and $s_x = s_y = s$ in region III, with the parameter s defined in our study as

$$s = 1 - j\frac{3\lambda}{4\pi nd}\left(\frac{\rho}{d}\right)^2 \ln\frac{1}{R} \tag{3}$$

where λ is the wavelength, n is the refractive index of the medium in the adjacent computational domain, ρ is the distance from the beginning of the PML, and R is the theoretical reflection coefficient for the normal incident wave at the interface of the PML and the computational domain [3], [9].

As detailed in [3], using the two-dimensional (2-D) Yee's mesh and under the $\exp[-j\beta z]$ field dependence assumption with β being the modal propagation constant, the curl equations (1) can be written into six component equations in terms of the six components of the electric and magnetic fields and involving $\partial/\partial x$ and $\partial/\partial y$ differential operators for the whole problem with $s_x = s_y = 1$ in the non-PML region. Then, by applying the central difference scheme for the differential operators, the six component equations can be converted into two matrix equations, and finally into an eigenvalue matrix equation in terms of the transverse magnetic fields with β^2 being the eigenvalue. The eigenvalue equation is solved by using the shift inverse power method. To achieve better accuracy, an improved scheme has been proposed through the Taylor series expansion of the fields and by using interpolation and extrapolation to approximate the fields on both sides of the dielectric interface in order to properly fulfill interface conditions [3].

ANALYSIS OF PHOTONIC CRYSTAL FIBERS

We first consider the loss properties of a PBG fiber, or a honeycomb PCF, with 1-ring, 2-ring, 3-ring, and 4-ring air holes in the cladding region, as shown in Fig. 2, with pitch $a = 1.62$ μm and hole radius $r = 0.205a$. At the edges of the computational window, the PMLs are adopted as indicated. The confinement losses of the fundamental guided mode at different wavelengths are shown in Fig. 3. The confinement loss, L, in dB/m is evaluated from the imaginary part of the complex effective index, $\mathrm{Im}[n_{eff}]$, through the relation $L = (20 \times 10^6)(2\pi/\lambda)\,\mathrm{Im}[n_{eff}]/\ln(10)$, where the complex effective index is defined as β divided by the free-space wavenumber. One can see from Fig. 3 that the increase of air-hole rings help the confinement of light in the core region, which results in smaller losses than those with less air-hole rings. In Fig. 3, the solid lines are our results obtained by the FDFD method, which have a good agreement with the circular dots from the finite element analysis [10]. We then consider a kind of two-core PCFs having three-ring air holes surrounding the two cores with the computational window illustrated in Fig. 4. We are able to determine accurately the effective indices and the losses of the even and odd modes for the two-core PCF. After finding out the effective indices of the even and odd modes, we can obtain the coupling coefficient defined as half the difference between the real parts of the propagation constants of the even and odd modes. Here we only present in Fig. 5 the *x*-polarized coupling coefficients versus wavelength for $r/a = 0.25$, 0.3, and 0.35.

ANALYSIS OF PHOTONIC WIRES

Fig. 6 depicts the cross-section of the SOI photonic wire. The thickness of the silicon guiding core is fixed as 220 nm. The width w of the core and the thickness d of the SiO_2 layer will be varied in the analysis. The refractive indices of silicon and SiO_2 layer are assumed to be 3.5 and 1.45, respectively, and the wavelength is taken to be 1.55 μm. The fundamental transverse-electric (TE) mode is calculated. The FDFD analysis is to be checked with the FE-ID-BPM analysis. Fig. 7 shows the computational domain with horizontal width $X = 5$ μm along with the nonuniform finite element mesh division. In the FDFD calculation, uniform grid division with grid size of 20 nm has been used. Note that due to the symmetry of the waveguide structure, only half of the cross-section needs to be considered. The reflection coefficient in (3) is taken to be $R = 10^{-8}$. We have found that the calculated results are consistent with those obtained when smaller R is assumed. Fig. 8 shows the calculated leakage loss in dB/cm versus the thickness of the SiO_2 layer for five different silicon core widths from 300 nm to 500 nm for the case with $X = 5$ μm. The solid lines are obtained using the FE-ID-BPM while the black dots are from the FDFD method. The agreement between the two methods is very good. Here, proper matching of interface conditions near the dielectric interface through the Taylor series expansion of the fields has been performed in the FDFD calculation [3] and loss values down to 10^{-12} dB/cm has been achieved. It is seen that the loss decreases exponentially with the thickness of the SiO_2 layer and at $d = 1$ μm, the loss is about 2 dB/cm for $w = 350$ nm and is about 10^{-2} dB/cm for $w = 500$ nm. These values are significantly different from those shown in [2] with the corresponding values being 10 dB/cm and 1 dB/cm, respectively. We have also considered larger domain with $X >$ 5 μm and found similar results as in Fig. 8. The size of the computational domain as shown in Fig. 7 is thus good enough.

CONCLUSION

We have demonstrated that the developed FDFD method with PML absorbing boundary conditions can provide accurate modal analysis of various optical waveguides with leakage losses. Honeycomb PCFs having different numbers of air-hole rings, two-core PCFs, and photonic wires of sub-micron cross-sectional sizes have been considered as numerical examples.

ACKNOWLEDGEMENTS

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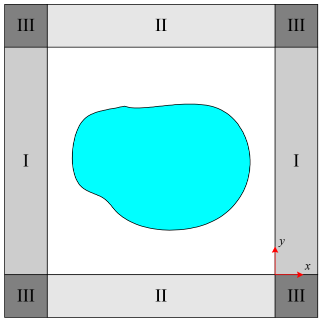

Fig. 1. The cross-section of an arbitrary waveguide structure with the PMLs placed at the edges of the computational domain.

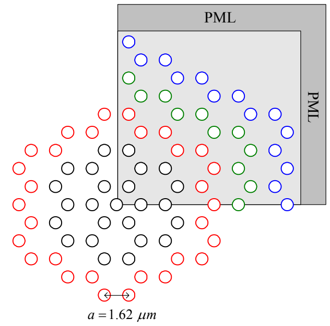

Fig. 2. The cross-section of honeycomb PCFs with different numbers of air-hole rings and the computational domain with PMLs.

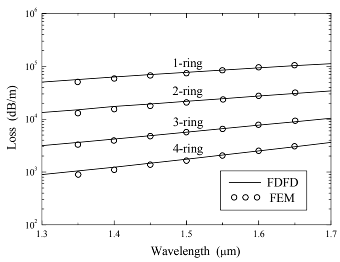

Fig. 3. Confinement loss versus wavelength for the fundamental guided mode of the honeycomb PCF of Fig. 2.

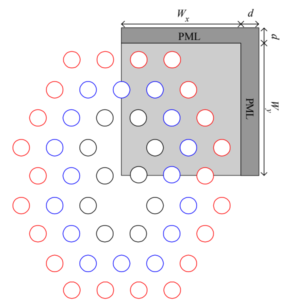

Fig. 4. The cross-section of a two-core PCF and the computational window with PMLs.

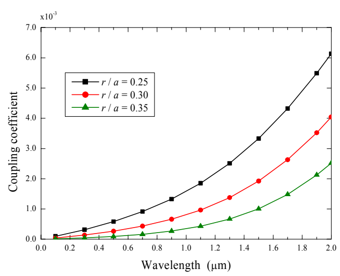

Fig. 5. Coupling coefficient versus wavelength for the two-core PCF of Fig. 4 with $r/a = 0.25$, .03, and 0.35.

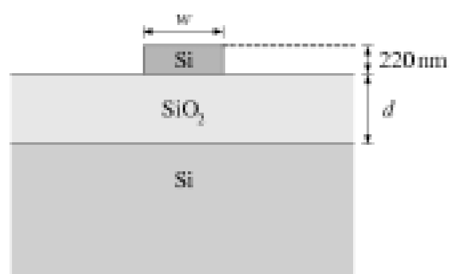

Fig. 6. Cross-section of SOI photonic wire waveguide.

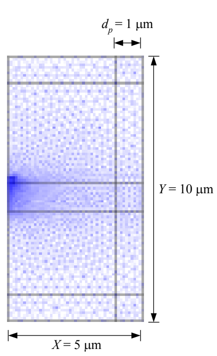

Fig. 7. The computational domain for the structure of Fig. 6 with finite element mesh division included.

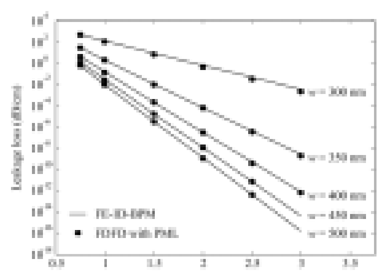

Fig. 8. Calculated substrate leakage loss of the fundamental TE mode on the photonic wire versus the SiO_2 layer thickness for four different silicon core widths.