Complete 2D and 3D Bandgaps in 1D Left-Handed Periodic Structures

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Photonic crystals are artificial materials with a periodic modulation in the dielectric constant, which can create a range of forbidden frequencies, the photonic bandgap [1]. Photons with frequencies within the bandgap cannot propagate through the medium. This unique feature can alter dramatically the properties of light, enabling control of spontaneous emission in quantum devices and light manipulation for photonic information technology [2]. Photonic bandgap structures can also be found in nature, and they explain the colour diversity of some of the living creatures [3]. Complete two-dimensional (2D) and three-dimensional (3D) bandgaps can be realized in photonic crystals, where the refractive index is periodically modulated in two or three dimensions, respectively [1]. Such modulation is necessary to satisfy the Bragg condition simultaneously for all propagation directions, requiring that phase accumulation per period is close to a multiple of π, so that the waves reflected at different interfaces between regions with low and high refractive indices interfere constructively and wave propagation is inhibited for any incidence angle.

Manufacturing of 3D photonic crystals remains a technological challenge due to the requirements of large index contrast and high fabrication precision. The simplest periodic structure, both in geometry and manufacturing, is a one-dimensional (1D) stack of two types of layers, which differ in the dielectric constant [4]. However, such structures may only possess partial band gaps for certain ranges of incident angles, acting as an omni-directional reflector [5-7] for electromagnetic waves launched from a low-index medium. Electromagnetic waves emitted by a source placed inside the structure can still propagate along the layers and possibly in other directions, and therefore *no complete band gap exists in one-dimensional dielectric periodic structures*.

Here we present the study of the scattering properties of one-dimensional periodic structures containing layers made of the so-called left-handed metamaterials (LHMs) - artificially created composites that are characterized by simultaneously negative dielectric permittivity and negative magnetic permeability. Such materials are transparent and can bend light in the opposite direction to normal reversing the way in which refraction usually works [8]. We overview the physical phenomena, such as wave guiding by optically dense dielectric layers and transmission at Brewster angle, which do not allow for the existence of the complete band gaps in conventional 1D periodic structures. We demonstrate that in structures with LHMs such phenomena can be suppressed and it becomes possible to archive a complete band gap in the structures containing alternating LHM and dielectric layers. We show that such a gap can result in complete localization of the point source radiation inside the structure. We consider a one-dimensional periodic structure created by layers (with the thickness d_1 and d_2) of two different materials with dielectric permittivities $\varepsilon_{1,2}$ and magnetic permeabilities $\mu_{1,2}$, respectively, as shown in Fig. 1.

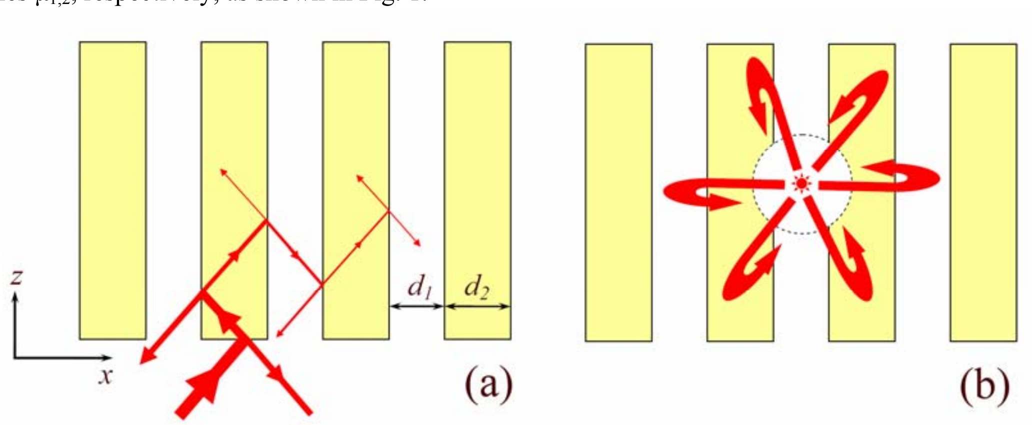

Figure. 1. (a) Schematic of the left-handed periodic structure and the ray diagram for the end-fire wave propagation. (b) Schematic of the suppression of source radiation in the structure with complete band gap.

First, we study the propagation of the TE-polarized electromagnetic waves, which have the component of the electric field parallel to the layers ($E = E_y$); all results can be easily generalized for the case of the TM polarized waves. We consider the wave propagation in the (x; z) plane characterized by the wavevector $k = (k_x; 0; k_z)$. The TE-polarized waves are described by the linear Helmholtz-type equation for the electric field component,

$$\Delta E + \frac{\omega^2}{c^2}\varepsilon\mu E - \frac{1}{\mu}\frac{\partial\mu}{\partial x}\frac{\partial E}{\partial x} = 0 \tag{1}$$

where Δ is the two-dimensional Laplacian. In a one-dimensional periodic structure, the propagating waves have the form of Bloch modes, for which the electric field amplitudes satisfy the periodicity condition, $E(x + \Lambda; z) = E(x; 0) \exp(iK_b + ik_z z)$, where $\Lambda = d1 + d2$ is the period of the structure. Here K_b is the dimensionless Bloch wave number which defines the wave transmission across the layers, and its dependence on the wavevector component along the layers (k_z) can be found explicitly for two-layered periodic structures as [4, 9, 10]

$$2\cos(K_b) = \mathrm{Tr}(M) = 2\cos(k_1 x d_1)\cos(k_2 x d_2) - \left(\frac{k_{2x}\mu_1}{k_{1x}\mu_2} + \frac{k_{1x}\mu_2}{k_{2x}\mu_1}\right)\sin(k_{1x}d_1)\sin(k_{2x}d_2) \tag{2}$$

Here Tr(M) is the trace of the transfer matrix M characterizing the wave scattering in a periodic structure [4], $k_{jx} = k_j(1 - k_z^2/k_j^2)^{1/2}$ are the x-components of the wavevector in the first (j = 1) and second (j = 2) media, and $k_j = \omega\varepsilon_j\mu_j/c$ are wavenumbers in each media. Solutions of the dispersion relation (2) with both real k_z and K_b correspond to Bloch waves which can propagate through the periodic structure, whereas complex k_z or K_b indicate the presence of band gaps in the spectrum where the wave propagation is inhibited. A complete band gap occurs if for all real k_z, the K_b remains complex. In order to emphasize the importance of our findings presented below, first we recall the basic physics, which explains why one-dimensional periodic structures containing materials of the same type (i.e., normal dielectrics) do not possess a complete three-dimensional band gap. Analyzing the effects associated with the wave scattering in Bragg gratings, [11], we come to the conclusion that the only phenomenon which always allows for the wave propagation in the 1D dielectric periodic structures, and which cannot be suppressed by a choice of the structural parameters, is the wave guiding by optically dense layers. Indeed, it is well known that a dielectric waveguide with the core made of an optically dense medium always supports a fundamental mode. However, as was shown recently, [12] the fundamental mode can be absent if the core is made of LH metamaterial. In presence of materials of different types, i.e. both left-handed and right-handed, one more mechanism of wave guiding becomes possible. The single interface between LH and RH material can support either TE or TM waves [13]. Thus, we need to analyze different waveguiding regimes to find a possibility to avoid any wave propagation in the periodic structure for the construction of the complete band gap. We use the results presented in Ref. [13] and indicate on the plane of parameters of LHM on the Fig. 2 (a) the area where TE surface modes exist.

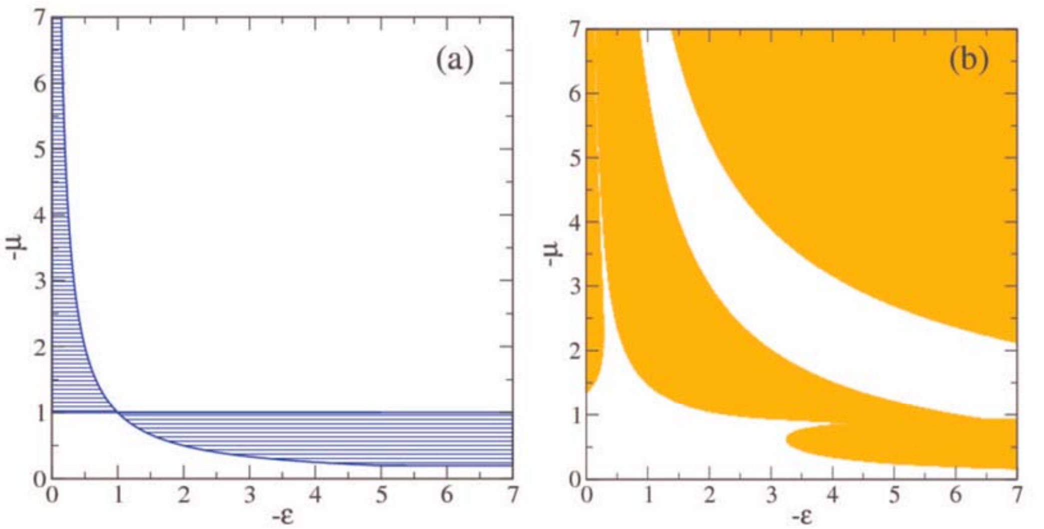

Figure 2. (a) Region of the TE polarized *surface wave* existence on the plane of parameters. (b) Shaded is the region of parameters for which the TE polarized *modes guided by an optically dense layers* exist.

We assume that the right-handed dielectric is vacuum, i.e. $\varepsilon_1 = \mu_1 = 1$. Regions of LHM parameters for which the waveguiding is possible by either LHM layer when $\varepsilon_2\mu_2 > \varepsilon_1\mu_1$ or by the vacuum gap for $\varepsilon_2\mu_2 < \varepsilon_1\mu_1$ are shown in Fig. 2 (b). As it was predicted in Ref. 12, the fundamental mode in the left-handed waveguide may be absent and it is this property of a LH waveguide that eventually allows us to introduce a novel type of one-dimensional periodic structures with a complete band gap. The condition for the guided waves to exist, defined by the dispersion relation for the modes in a slab waveguide, [12], has a simple physical meaning: the round-trip accumulation of phase due to wave propagation across the layer, including the phase retardation upon the total internal reflection, should be equal to a multiple of 2π. The phase change due to the total internal reflection is negative for both types of waveguides, and depending on the angle of incidence, it varies from 0 to $-\pi$. A difference between the conventional and LHM waveguides appears due to the phase accumulated by the wave propagating across the layer. In usual dielectrics, the wave is forward, i.e. the phase accumulated along the direction of energy flow is positive. As a result, there always exists an angle of wave propagation, such that the total phase change vanishes, and at least one mode always exists in a dielectric waveguide. In a LHM waveguide, the wave is backward, and the phase change is negative. Then, one can choose the parameters in such a way that no guided modes exist.

Existence of the band gap requires constructive interference of the waves reflected from different interfaces in the structure, as a result the band gap cannot appear if there is no reflection at each particular interface. It is well known, that there is no reflection for one of the wave polarizations between two different media at Brewster angle, and the complete band gap cannot be opened when the complete transmission at such angle is possible. The Brewster angle for the TE polarized waves scattering at the interface of the LHM and vacuum exists for the parameters shown on the Fig. 3 (a). We present the combined figure showing the regions of the metamaterial parameters for which one of the three discussed above mechanisms of the wave propagation is possible (see Fig. 3 (b)). If the complete band gap do exist in one-dimensional structures with negative refraction, it should appear in the white area of the Fig. 3 (b).

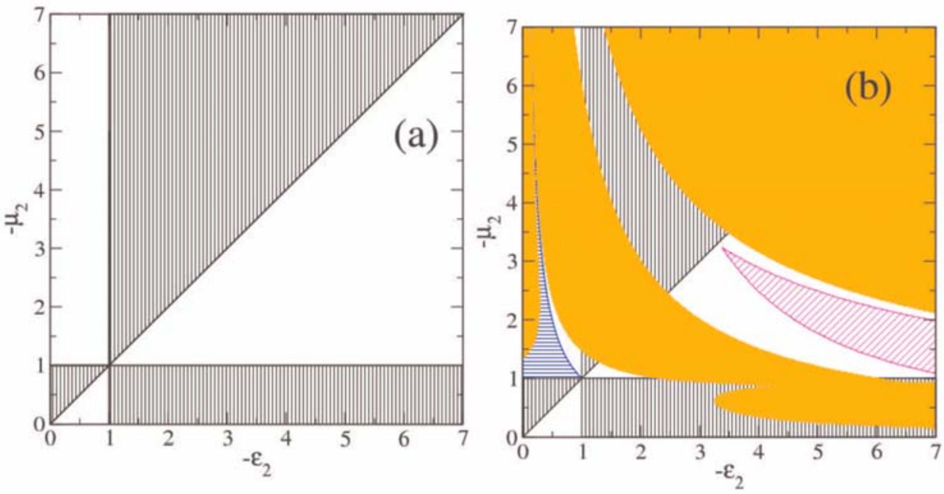

Figure 3. (a) Shaded is the area of metamaterial parameters for which the complete TE wave transmission at each interface appears at Brewster angle. (b) Parameter plane combining three simplest ways of electromagnetic wave transmission, including surface wave guiding, dielectric slab guiding and the transmission at Brewster angle. The magenta area indicates the region of the complete band gap for TE polarized waves.

Indeed, the detailed analysis of the dispersion relation of the Bloch modes shows that the complete band gap for TE-polarized waves in considered layered structures do exist, and the region of corresponding parameters is shown in the Fig. 3 (b). However, there is no complete bandgap for the TM-polarized waves propagating in the same structure. In the optimal case we have only one angle of propagation possible (i.e. the only value of k_z with real K_b). This is the Brewster angle for which there is no reflection of TM waves at the interfaces. From the electromagnetic duality principle we can easily find the structure with LHM metamaterial having complete bandgap for the TM polarized waves [14].

After the study of the two-layer periodic systems and the properties of the complete bandgaps supported by one-dimensional hybrid structures, we are able to suggest the case when the complete bandgap may appear for both polarizations thus allowing the existence of the *absolute bandgap*. Indeed, to do this we should consider more sophisticated case of a three-layer periodic structure in order to suppress the conditions for the existence of the Brewster angle, which prevents us from creating a complete bandgap in two-layer structures. The Brewster-angle transmission resonance can be easily eliminated by introducing a third layer in the structure, thus allowing for the existence of a complete three-dimensional bandgap for all waves propagating inside a specially designed one-dimensional structure. To demonstrate this quite unique property, we choose the structure with the parameters $\varepsilon_1 = \mu_1 = 1$, $\varepsilon_2 = \mu_3 = -6$, $\varepsilon_3 = \mu_2 = -1.38$, $d_1 = 1.5\lambda/2\pi$ and $d2 = d3=0.7\lambda/2\pi$, where λ is a free-space wavelength of electromagnetic wave. The choice of this symmetry simplifies the analysis. Such structure possesses *an absolute three-dimensional bandgap.* The Green's function corresponding to this three-layer structure, which describes the electric field of the dipole, is presented in Fig. 4. One can clearly see, that the dipole radiation is completely suppressed, and it indicates the presence of the complete band gap.

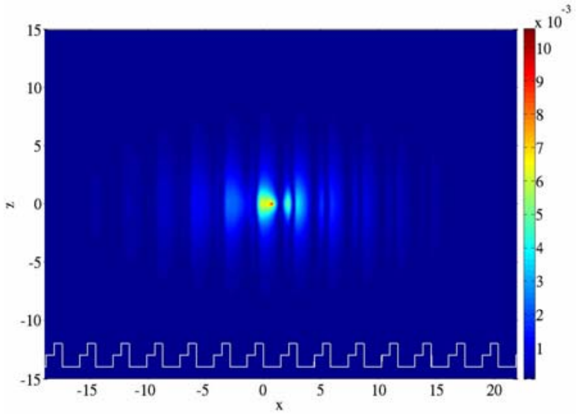

Figure 4. The Green's function of a one-dimensional three-layer periodic structure possessing *an absolute bandgap*.

In conclusion, we have revealed a novel and highly nontrivial property of left-handed metamaterials with negative refraction: A one-dimensional periodic structure containing layers made of a left-handed metamaterial can trap light in three dimensions due to the existence of a complete photonic bandgap. This finding is in a sharp contrast with the fundamental concepts of the conventional physics of photonic crystals where complicated structures with two- and three-dimensional periodicity are required. We believe that our results suggest new directions for the future applications of metamaterials for microwaves, Terahertz frequencies, and visible light as fabrication technologies become available.

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