GREEN'S FUNCTIONS-BASED WAVELETS AND FRAMES IN COMPUTATIONAL ELECTROMAGNETICS

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In the past two decades sophisticated methods have been devised for constructing bases and orthogonal wavelets that rely on the existence of a family of nested subspaces of finite energy functions. Such a family of functions, commonly referred to as a multiresolution analysis (MRA), possesses numerous desirable properties which can be exploited from both theoretic and computational point of view. In particular, band limited wavelets and wavelets with finite support have emerged as dual realizations in spectral domain and real space, respectively. Daubechies' and spline wavelets should be mentioned as two prominent examples for wavelets with finite support. Encouraged by these developments we have posed the following question: Is it conceptually possible and technically feasible to design orthonormal bases, or, more generally, wavelets, or, even more generally, frames, for solving boundary value problems in computational electromagnetics? In this contribution we give an affirmative answer to this question and introduce an easy-to-use recipe for constructing such wavelets and their generalizations employing spectral domain dyadic Green's functions. Procedural details are organized as follows: (i) The Maxwell's equations in differential form are diagonalized for both free space and media with fairly general material constitutions. (ii) The diagonalized forms are transformed into spectral domain obtaining corresponding algebraic eigenvalue equations. (iii) The eigenpairs (eigenvalues and eigenfunction) are determined for asymptotically large values of the wavenumber. (iv) Dyadic Green's functions together with their far-field asymptotic tails in spectral domain are determined in two fundamentally different ways and the equivalence of the results has been established. The dominant terms in the series expansions of the Green's functions describe their respective static limits, while higher-order asymptotic terms correspond to their quasi static limits which we can conveniently obtain to any order desired. (vi) Utilizing the asymptotic expansion terms of various Green's functions in spectral domain, and applying Meyer's orthogonalization technique we construct genuinely physics-based wavelets. Thereby, a deep insight is that the resulting wavelets are spline wavelets of various orders once transformed into spatial domain. (vii) The static and quasi static limits computed and subtracted from Green's functions lead to nearly perfect band limited regularized functions in slowness (inverse velocity) domain. These residual Green's functions can be described in terms of spline wavelets in spectral domain. (viii) Furthermore, we explain the construction of frames and dual frames (overcomplete set of analyzing and synthesizing functions) and other wavelet-like translational shift-invariant orthonormal functions from the branch points of the Green's functions. (ix) For completeness we also address wavelets constructed from radial functions example of which are infinite domain Green's functions in spatial domain. (x) With a brief discussion on field quantization and the construction of coherent states for electromagnetic field analysis we conclude our discussion.