SPECTRAL DOMAIN: A NUMERICAL RAY LAUNCHING METHOD FOR THREE-DIMENSIONAL STRUCTURES

Elias Rachid
Ecole Supérieure d'Ingénieurs de Beyrouth, Université Saint Joseph, Beirut, Lebanon
Email: elias.rachid@fi.usj.edu.lb

ABSTRACT

In this paper, an original method for propagation modeling within waveguides, cavities, or structures with dielectric or conducting curved interfaces is presented, validated and calibrated. This method is based on the discretization of the plane wave spectrum of the source fields, truncated by the guide or cavity aperture (without any approximation, neither far field nor asymptotic). From this decomposition, fields at any observation point in the cavities or waveguides are expressed as a discrete sum of all contributions from ray tubes tracked within the propagation environment. This method called shooting Spectral Rays and Tracking (SRT) is calibrated in contexts with multiple reflections, and applied to waveguides with plane or curved interfaces. The obtained results are validated by comparison with modal solutions. The calibration regards the number of reflections and the number of launched rays. Compared to other ray based methods used in similar contexts ("Shooting and Bouncing Rays" SBR, "Generalized Ray Expansion" GRE), the proposed method appears to be appealing in terms of numerical efficiency and precision.

INTRODUCTION

Ray-based methods have demonstrated their usefulness in many application fields, from optics to seismology and acoustics. Combined with powerful computing resources, they allow computation of fields in anisotropic nonuniform media, limited by nonuniform boundaries. Most of these methods rely on a far field assumption, and the source is represented as a radiating point source, from which rays are traced to each observation point. The major issue is then the computation of ray paths from point to point in a complex environment. For reflector antennas or radar cross section computations, another method not relying on the same far field assumption has become very popular in the last ten years, the SBR method [1]. In this approach, an incident plane wave is sampled into ray tubes which are tracked in the environment following Geometrical Optics (GO) rules. The back scattered or radiated field is obtained via summation of the fields radiated by each spatial subaperture delimited by each exiting ray tube. This method is limited to a source field in the form of a plane wave and does not take into account the field diffracted by the edges of the source aperture. The latter limitation degrades the accuracy of the method for appertures of relatively small dimensions [2]. The GRE method [3] is able to remedy these limitations, but at the expense of a larger number of rays to track: in this method, the source aperture is divided into a number of subapertures, and the far field assumption is then used for each of these apertures to represent its radiated fields in the form of rays. A cone of rays is launched from each subaperture and tracked within the environment. The summation of all the ray fields is thus 2D times 2D summation.
The SRT method is another ray tracking method, which allows starting from arbitrary source distributions, with a computational effort limited to one 2D discrete summation per observation point once the source plane wave spectrum (PWS) is known. This method is based on the discretization of radiating fields not in the spatial domain, but in the spectral domain. In the case of a plane aperture source, rays are tracked backwards from the observation point to the source plane, each ray representing a sample of the source (PWS). This method can be viewed either as a generalization of the SBR method, taking advantage of the dual spatial-spectral significance of rays, or as an analog of the GRE method in the spectral domain.
We have already applied this method in the context of multiple reflections in a conducting parallel plate waveguide [4] and dielectric lens antenna analysis [5, 6]. We propose here to demonstrate its potentialities in the context of open cavities or circular waveguides analysis. We shall present the formulation of the method, and then validate its concept in cases where exact solutions are known. It will be shown analytically that in the case of a circular waveguide, the SRT representation of propagated fields amounts to the summation of discretized PWS integrals multiplied by the divergence factor and the reflected coefficient. The plane wave spectra represent the excitation fields present in the guide aperture.
Numerical results will be shown for the case of cylindrical waveguide. Comparisons between the exact modal solution and numerical results obtained with the SRT method will be presented in the cases of a metallic waveguides excited by the fundamental mode. Calibration results regarding the number of rays to be tracked, and the accuracy of the solution as a function of the waveguide separation and the number of reflexions taken into account will be presented. Finally, comparisons between SRT and physical optics results for cylindrical parabolic antenna will be presented.

THEORETICAL FORMULATION

The SRT method is based on a PWS representation of the fields radiated by a source in plane z=0 truncated by the cavities or waveguide aperture. It is implemented in 3 parts:

1- PWS decomposition of the source field analytic or function integration: $\tilde{E}(k_x, k_y)$ are determined as the inverse Fourier transform of the truncated source [7].
2- For a given observation point P, finding the ray paths (or tubes) from the discretized initial PWS through P, and determining the corresponding source field. Each tube is tracked according to Snell law. Fresnel's transmission and reflection coefficients for perpendicular and parallel polarizations at incident points on the interfaces and divergence factors of transmitted and reflected pencils, must be taken into account.
3- Evaluation and summing up all the ray contributions at the observation point.

The incident field at any incident point I(0,0,z) of a curved interface can be approximated at point $r = (\vec{\xi} = (x, y), z)$ using the field at point (0,0,z) [8,9] by:

$$\vec{E}^{\,i}(r) = \vec{E}^{\,r}(I)\left(\frac{\det Q_i(z_i)}{\det Q_i(0)}\right)^{1/2} e^{-jk_i\left(z_i + \frac{1}{2}\vec{\xi}_i^{\,T} Q_i(z_i)\vec{\xi}_i\right)} \tag{1}$$

The second term of equation (1) denotes the divergence factor (DF) at I, and the exponential one represents the phase shift term. $Q_i(z)$ is the curve matrix of the equiphase wave at z.

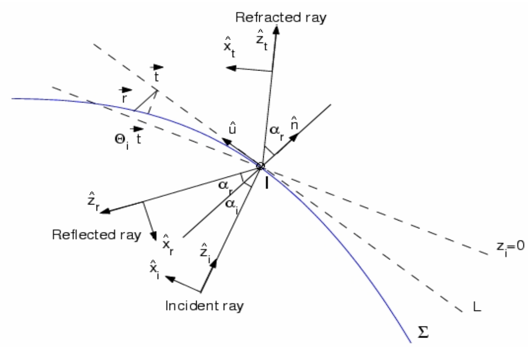


Fig.1. Transformation by a curved dielectric interface $\sum$ for a paraxial ray propagated in the direction of $\hat{z}_i$ (wave-front $Q_i(z)$) and arrived to the incident point I.

The reflected and refracted rays in Fig. 1 have Q_r and Q_t propagate in the direction of $\hat{z}_r$ and $\hat{z}_t$. $(\hat{x}_i, \hat{y}_i, \hat{z}_i)$, $(\hat{x}_r, \hat{y}_r, \hat{z}_r)$ and $(\hat{x}_t, \hat{y}_t, \hat{z}_t)$ denote respectively the unit vectors axes for incident, reflected and refracted rays $(\hat{y}_i = \hat{y}_r = \hat{y}_t = \hat{v} = \hat{n} \wedge \hat{z}_i)$ .The surface of the interface around I can be approximated locally by the parabolic equation as [10,11]:

$$\vec{r}(t) = \vec{t} - \frac{1}{2}\left(\vec{t}^{\,T} Q_\Sigma \vec{t}\right)\hat{n} \tag{2}$$

$\vec{r}(t)$ denotes the vector that connects the incident point I to the one on the surface, and $\vec{t}$ represents the projection of $\vec{r}$ in the tangent plane to the surface at I. Q_Σ and $\hat{n}$ represent respectively the curve matrix and the normal unit vector of Σ at I. $\hat{n}$ is pointed towards outside surface. The reflected and refracted fields at any incident point I of a curved interface can be shown as a function of the incident field by:

$$\vec{E}^{\,r,(t)}(x_{r,(t)}, y_{r,(t)}, z_{r,(t)}) = \underline{\underline{R}},(\underline{\underline{T}})\,\vec{E}^{\,i}(I)\left(\frac{\det Q_{r,(t)}(z_{r,(t)})}{\det Q_{r,(t)}(0)}\right)^{1/2} e - jk_{r,(t)}\left(z_{r,(t)} + \frac{1}{2}\overrightarrow{\xi_{r,(t)}}^{\,T} Q_{r,(t)}(z_{r,(t)})\overrightarrow{\xi_{r,(t)}}\right. \tag{3}$$

$\underline{\underline{R}}$ and $\underline{\underline{T}}$ represent respectively Fennel’s reflected and transmitted matrix coefficients at I.

A discussed previously in the above, the SRT is useful for analyzing the EM scattering by relatively arbitrarily shaped open-ended cavities containing smooth or nonsmooth cross sections as well as tapers. In this case, the PWS which is calculated for the source field truncated by the aperture is launched into the cavity and tracked using the GO rules. The central ray in each tube may then be tracked via the four adjacent rays which propagate along the four corners of the tube. Consider the first reflection at P_1 of the p^{th} ray tube witch enters the cavity; let $E^r_{1,p}$ denote the electric field of this once reflected ray which is evaluated at the second point P_2 on the cavity wall:

$$\vec{E}^{r}_{1,p}(P_2) = \underline{\underline{R}} DF^{r}_{1,p}\, \vec{E}^{i}(P_1) e^{-jkt_1} \tag{4}$$

where $DF^r_{1,p}$ is the divergence factor of the p^{th} reflected ray tube at P_2 and $t_1 = \overline{P_1 P_2}$. $\vec{E}^i(P_1)$ is the incident electric field at P_1, it can be expressed as follows:

$$\vec{E}^{i}(P_1) = \sum_{n}\sum_{p}\{\tilde{E}_x(\vec{k}_{n,p})\hat{x} + \tilde{E}_y(\vec{k}_{n,p})\hat{y} + \tilde{E}_z(\vec{k}_{n,p})\hat{z}\} d^2 k_t \tag{5}$$

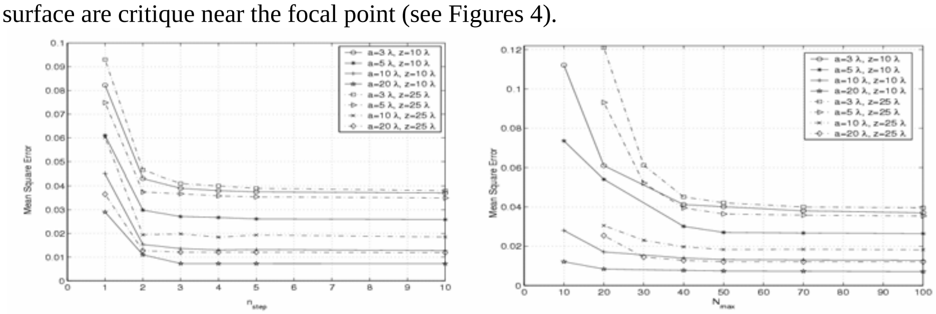
with $(\hat{x}, \hat{y}, \hat{z})$ represent the unit vectors of Cartesian coordinates, and $d^2 k_t$ denotes the differential spectral surface.

The field $\vec{E}^{r}_{1,p}(P_2)$ witch is incident at P_2 undergoes a second reflection at P_2 to arrive at the third reflection point P_3, and so on. Thus the field at the $(n+1)^{th}$ point of reflection P_{n+1} is, after undergoing n previous reflections , given by(with $t_n = \overline{P_n P_{n+1}}$):

$$\vec{E}^{r}_{n,p}(P_{n+1}) = \underline{\underline{R}}_{n,p} DF^{r}_{n,p}\, \vec{E}^{r}_{n-1,p}(P_n)\ e^{-jkt_n} \tag{6}$$

NUMERICAL VALIDATION

The truncated fundamental mode $J_0(x_{01}(\rho/a)$ is applied to the cylindrical conducting waveguide defined above, and the E-field is calculated along this waveguide using the SRT method. Practically, the number of reflection N must be taken a maximum value noted $N_{\max}$ that after this number the convergence is obtain. If the waveguide walls are not perfectly conducting, convergence would be reached with small number of reflections. The number of rays launched into the waveguide is regarding as a function of $N_{\max}$, the waveguide radius, the sampling rate per period n_{step}, the distance of the observation point and, the wavelength. Figures 2 present the Mean Square Error (MSE) for different values of a at $z = 10\lambda$ and $z = 25\lambda$ vs. n_{step} with $N_{\max} = 50$ and, vs. $N_{\max}$ with $n_{step} = 5$. The accuracy is obtain for n_{step} =5 and $N_{\max}$ =50. The MSE decreases when the waveguide aperture increases. This is due to the approach of the local approximation of the surface near the incident point and the wide aperture taken into account. Figures 3 show the evolution of the E-field and the Normalized Absolute Error (NAE) of the truncated fundamental mode $J_0(x_{01}(\rho/a)$ for $a = 10\lambda$ and for two distances $z = 10\lambda$ and 25λ . Part of the error is caused by evanescent waves, which are not taken into account in this method. However their contribution can be shown to be rather negligible.
The RST method is applied to calculate the reflected E-field by a cylinder parabolic antenna especially near the focal point and for plane wave source distribution. The results are compared to those calculated by the physical optic method. The maximum value of NAE between the E-field curves calculated by SRT method and the physical optic approximation is about 0.03. The divergence factor and the differential spectral surface are critique near the focal point (see Figures 4).



(a) (b)

Fig.2. MSE Curves at $z = 10\lambda$ and $z = 10\lambda$ for the truncated mode $J_0(x_{01}(\rho/a))$: (a) $N_{\max} = 50$ reflections, (b) $n_{step} = 5$.

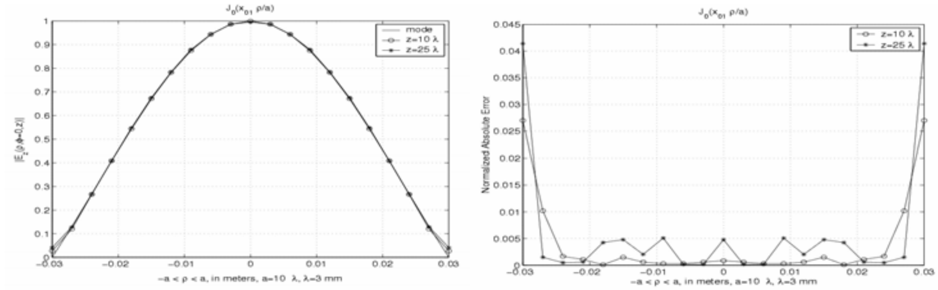


Fig.3. Electrical field and NAE of truncated mode $J_0(x_{01}(\rho / a)$ in the perfect conductor cylindrical waveguide.

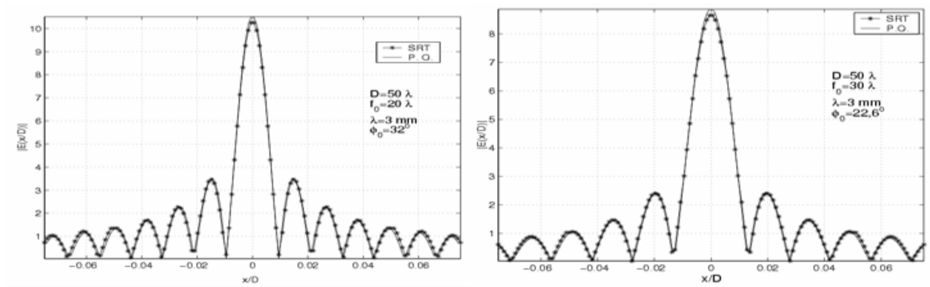


Fig.4. E-field curves in focal plan of the parabolic antenna calculated SRT method and physical optic approximation

CONCLUSION

A new ray tracking method is introduced. Its similarities and differences with existing methods was presented. The method will validate both conceptually and numerically, in cases where exact solutions are known. Further developments include its calibration when boundary interfaces are curved, and when the source fields are not represented by a PWS. The SRT method could represent an interesting alternative to existing methods when fields have to be known at a limited number of predefined observation points, for only one 2D summation per point is needed. Being able to calculate fields at predefined points also allows for the use of Fast Fourier Transform to calculate either the far field of the exit aperture, or the PWS in this aperture, in the course of a marching on procedure. This method is currently extended to a wider class of problems

REFERENCE

[1] H. Ling, R. Chou, S.W. Lee, "Shooting and bouncing rays: calculating the RCS of an arbitrarily shaped cavity", *IEEE Trans. Antennas Propagat., vol. 37, pp. 194--205, Feb. 1989*.

[2] H.Ling, R. Chou, S.W. Lee: "Rays versus modes: Pictorial display of energy flow in an open$-$ended wave guide " *IEEE Trans. Antennas and Propagation, vol. AP 35, pp.605--607, May 1987*.

[3] P.H. Pathak, R.J. Burholder, R.C. Chou, G. Crabtree, ``A genaralized ray expansion method for analyzing the EM scattering by open-endedwaveguide cavities", Antennas *Propagat. Soc. Int. Symp. Dig., 1989, pp.840-843*.

[4] E. Rachid, C. Letrou, ``A spectral domain ray: An Alternative method for devices modelling", *27th General Assembly of Radio Science URSI., August 2002, The Netherland no 1827*.

[5] I.A. Ehtezazi, C. Letrou, ``A spectral domain ray tracing method for quasi-optical devices modelling", *Antennas Propagat. Soc. Int. Symp. Dig., 1998, pp.1086-1089*.

[6] I. Ehtezazi Alamdari, "Modélisation des sysèmes quasioptiques pour une méthode numérique de Lancer de Rayons Spectraux", *Thèse doctorat de l'Université Marne la Vallée,Janvier 2000*.

[7] G. A. Deschamps, "Ray techniques in electromagnetics", *Proc. IEEE, vol-60, pp.1035--19 Sept. 1972*.

[8] S.W. Lee, M.S. Sheshadri, V.Jammnejad and R. Mittra, "Refraction at a curved dielectric interface: geometrical optics solutions", *IEEE Trans. Microwave Theory Tech., vol. MTT 30, pp.12--19 Jan. 1982*.

[9] D.H. Martin and J.W.Bowen, "Long-wave optics" *IEEE, Microwave Theory Tech, vol MTT-41 pp.1676-1689, oct.1993*.

[10] E. Rachid, "Etude d'une méthode numérique de lancer de Rayons dans le domaine spectral. Application à la modélisation de structures tridimensionnelles.", *Thèse doctorat de l'Université Marne la Vallée, Juin 2004*.

[11] H.Shirai, L.B. Felsen, "Rays, modes and beams for plane wave coupling into a wide open-ended parallel waveguide", *Wave Motion, vol.9, pp.301-317, 1987*.